Calcule o determinante de cada matriz:

1. $\begin{vmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 2 & 3 \\ 1 & 3 & -1 & 4 \\ 2 & 0 & 5 & 4 \end{vmatrix} =$ $\cancelto{0}{0.A_{11}} + 1.A_{12} + 1.A_{13} + \cancelto{0}{0.A_{14}}$

$A_{12} = (-1)^{1+2} \cdot \left|\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 2 \\ 1 & -1 & 4 & 1 & -1 \\ 2 & 5 & 4 & 2 & 5 \end{array}\right| = -1 \cdot (-4+16+15+6-20-8)$

$A_{12} = -5$

$A_{13} = (-1)^{1+3} \cdot \left|\begin{array}{ccc|cc} 1 & 1 & 3 & 1 & 1 \\ 1 & 3 & 4 & 1 & 3 \\ 2 & 0 & 4 & 2 & 0 \end{array}\right| = 1 \cdot (12+8-18-4)$

$A_{13} = -2$

$\det = 1.A_{12} + 1.A_{13}$

$\det = 1 \cdot (-5) + 1 \cdot (-2)$

$\boxed{\det = -7}$

2. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 5 & 0 \\ 6 & 7 & 8 & 0 \\ 0 & 0 & 9 & 10 \end{vmatrix} =$ $\cancelto{0}{0.A_{21}} + \cancelto{0}{0.A_{22}} + 5.A_{23} + \cancelto{0}{0.A_{24}}$

$A_{23} = (-1)^{2+3} \cdot \left|\begin{array}{ccc|cc} 1 & 2 & 4 & 1 & 2 \\ 6 & 7 & 0 & 6 & 7 \\ 0 & 0 & 10 & 0 & 0 \end{array}\right| = -1 \cdot (70 - 120)$

$A_{23} = 50$

$\det = 5.A_{23}$

$\det = 5 \cdot 50$

$\boxed{\det = 250}$

3. $\begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 2 & 3 & -2 \end{vmatrix} =$ $1.A_{11} + \cancelto{0}{0.A_{12}} + \cancelto{0}{0.A_{13}} + \cancelto{0}{0.A_{14}}$

$A_{11} = (-1)^{1+1} \cdot \left|\begin{array}{ccc|cc} 2 & 0 & 0 & 2 & 0 \\ 2 & 1 & 0 & 2 & 1 \\ 2 & 3 & -2 & 2 & 3 \end{array}\right| = 1 \cdot (-4)$

$A_{11} = -4$

$\det = 1.A_{11}$

$\det = 1 \cdot (-4)$

$\boxed{\det = -4}$

4. Calcule o x:

$\begin{vmatrix} x^2 & 0 & x & -1 \\ 3 & 0 & 2 & 8 \\ 5 & 0 & 2 & 10 \\ 1 & 1 & 1 & 10 \end{vmatrix} = 0$ $\cancelto{0}{0.A_{12}} + \cancelto{0}{0.A_{22}} + \cancelto{0}{0.A_{32}} + 1.A_{42}$

$A_{42} = (-1)^{4+2} \cdot \left|\begin{array}{ccc|cc} x^2 & x & -1 & x^2 & x \\ 3 & 2 & 8 & 3 & 2 \\ 5 & 2 & 10 & 5 & 2 \end{array}\right| = 1 \cdot (20x^2 + 40x - 6 + 10 - 16x^2 - 30x)$

$A_{42} = 4x^2 + 10x + 4 \quad \div 2$

$A_{42} = 2x^2 + 5x + 2$

$2x^2 + 5x + 2 = 0$

$\underline{-2} + \underline{-1/2} = -b/a = -5/2$

$\underline{-2} \cdot \underline{-1/2} = c/a = 2/2 = 1$

Veja que o coeficiente angular é $\neq$ de zero

$\boxed{x = -2 \text{ ou } x = -1/2}$

5. Para que valores de m a matriz tem determinante diferente de zero? $\begin{pmatrix} 0 & 1 & 2 & m \\ 0 & 1 & 0 & 2 \\ 0 & 2 & 0 & m \\ m & 0 & 1 & 2 \end{pmatrix}$ $= \cancelto{0}{0.A_{11}} + \cancelto{0}{0.A_{21}} + \cancelto{0}{0.A_{31}} + m.A_{41}$

$A_{41} = (-1)^{4+1} \cdot \left|\begin{array}{ccc|cc} 1 & 2 & m & 1 & 2 \\ 1 & 0 & 2 & 1 & 0 \\ 2 & 0 & m & 2 & 0 \end{array}\right| = -1 \cdot (8 - 2m)$

$A_{41} = 2m - 8$

$m.A_{41} \neq 0$

$m.(2m-8) \neq 0$

$m \neq 0$ $\quad$ $2m - 8 \neq 0$

$2m \neq 8$ $\quad$ $m \neq \frac{8}{2}$

$m \neq 4$

$\boxed{m \neq 0 \text{ e } m \neq 4}$

6. Dadas as matrizes $A = \begin{bmatrix} x^2 & 1 \\ 1 & 2x \end{bmatrix}$, $B = \begin{bmatrix} 0 & x & 1 \\ 1 & -1 & x \\ x & 1 & 0 \end{bmatrix}$ e $C = \begin{bmatrix} 1 & 0 & 1 & x \\ 2 & 3 & 4 & 5 \\ x & 0 & 1 & 0 \\ 0 & 0 & x & 1 \end{bmatrix}$, calcule o valor de x para que se tenha

$$\det A + \det B = \det C.$$

$\det A = \begin{vmatrix} x^2 & 1 \\ 1 & 2x \end{vmatrix} = 2x^3 - 1$

$\det B = \left|\begin{array}{ccc|cc} 0 & x & 1 & 0 & x \\ 1 & -1 & x & 1 & -1 \\ x & 1 & 0 & x & 1 \end{array}\right|$

$\det B = x^3 + 1 + 1x$

$\det C = \cancelto{0}{0.A_{12}} + 3.A_{22} + \cancelto{0}{0.A_{32}} + \cancelto{0}{0.A_{42}}$

$A_{22} = (-1)^{2+2} \cdot \left|\begin{array}{ccc|cc} 1 & 1 & x & 1 & 1 \\ x & 1 & 0 & x & 1 \\ 0 & x & 1 & 0 & x \end{array}\right| = 1 \cdot (1 + x^3 - 1x)$

$A_{22} = x^3 - x + 1$

$\det C = 3.A_{22}$

$\det C = 3 \cdot (x^3 - x + 1)$

$\det C = 3x^3 - 3x + 3$

$\det A + \det B = \det C.$

$\overbrace{2x^3 - \cancel{1}} + \overbrace{x^3 + x + \cancel{1}} = \overbrace{3x^3 - 3x + 3}$

$\cancel{3x^3} + x = \cancel{3x^3} - 3x + 3$

$3x + x = 3$

$4x = 3$ $\quad$ $\boxed{x = \frac{3}{4}}$