Estratégia
Militares

Estratégia

Militares

Trigonometria I

Matemática

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Elementos Básicos da Trigonometria

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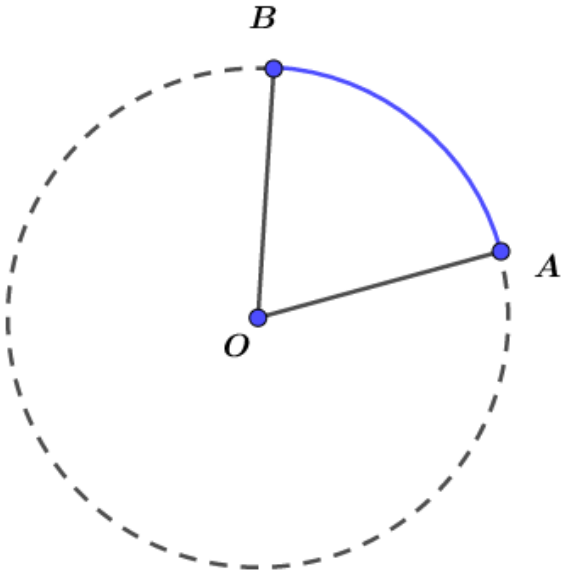
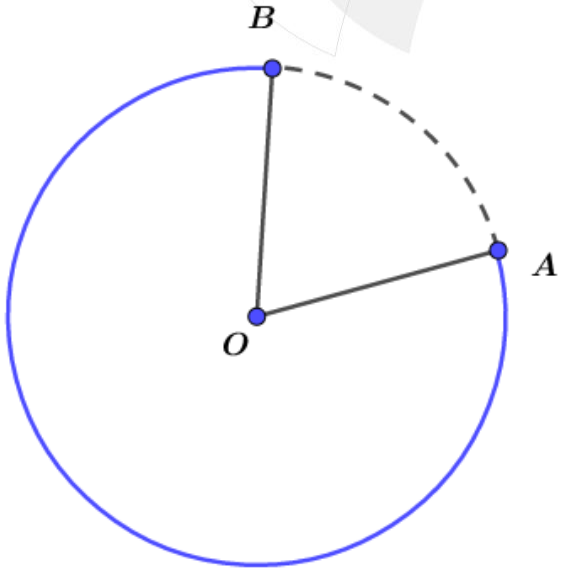
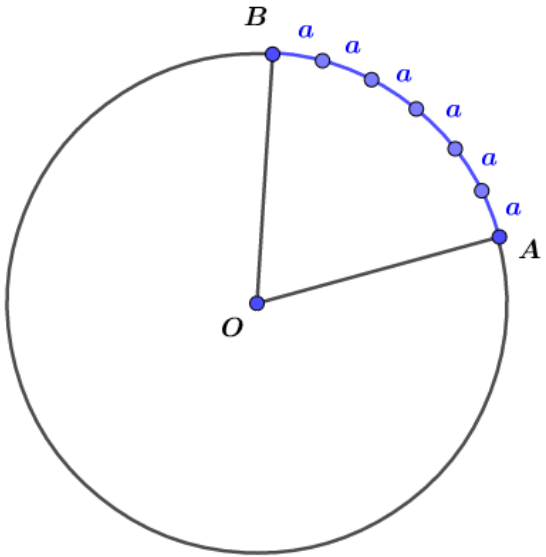
Arco de circunferência

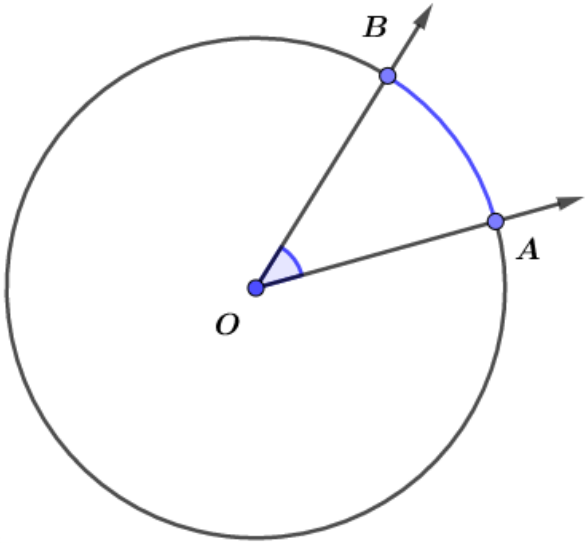




Medida de um arco



Medida de um ângulo



Classificação dos ângulos

Unidades de medidas de ângulos

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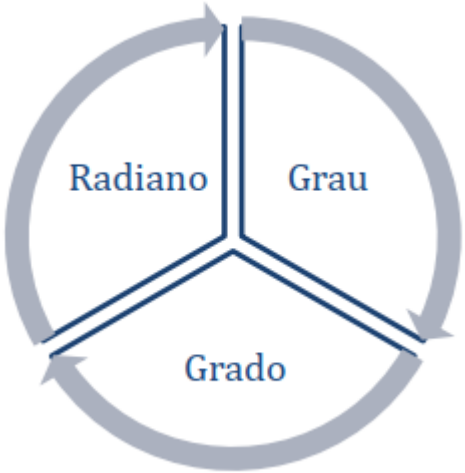
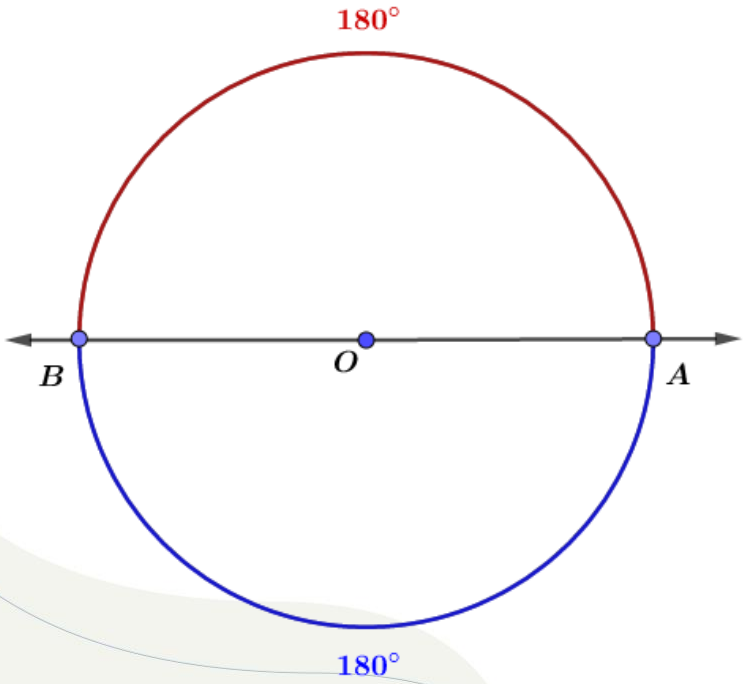
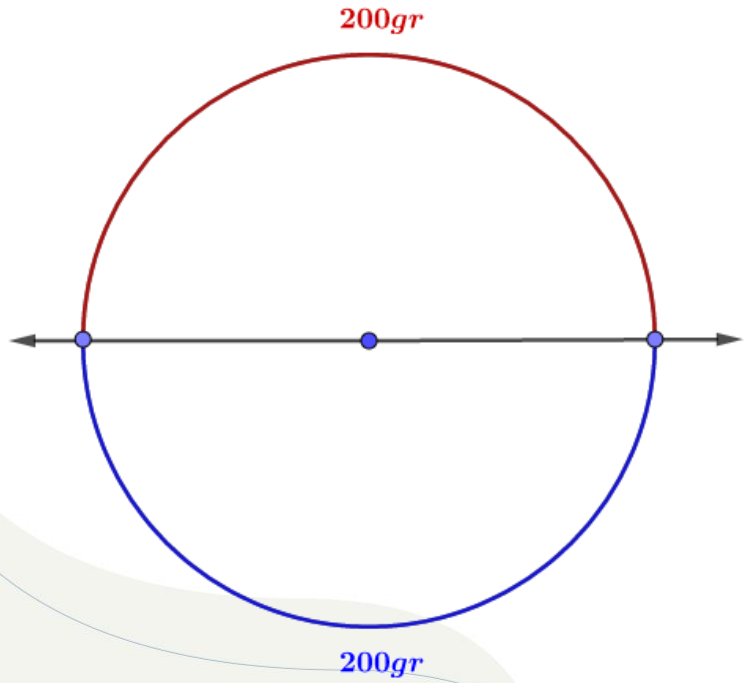
Unidades usuais de medidas de ângulos



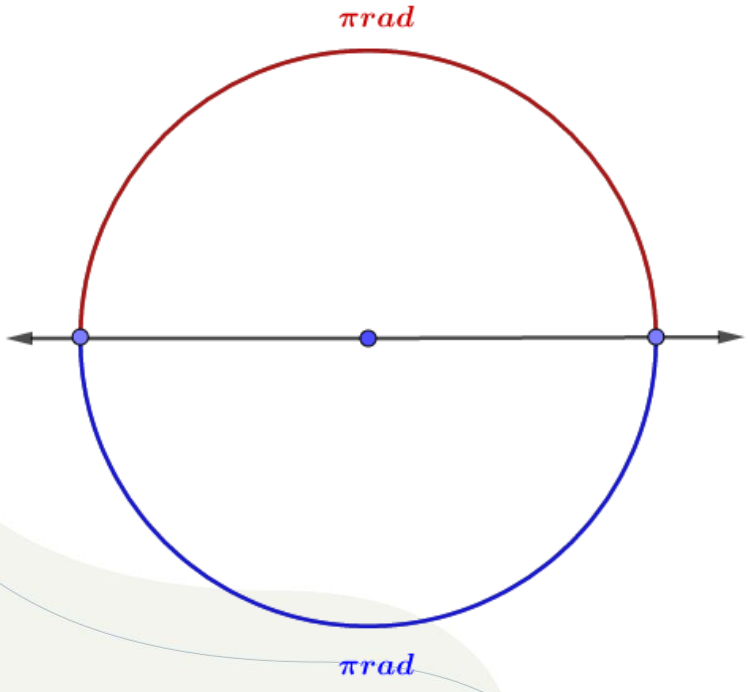
Grau



Grado



Radiano



Relação entre ângulos

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Exemplo

Encontre o ângulo complementar, suplementar e replementar de $36°$.

Equivalência das medidas dos arcos

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Equivalência das medidas dos arcos

Grau	Grado	Radiano
$360°$	$400gr$	$2\pi\ rad$
$180°$	$200gr$	$\pi\ rad$

Conversão de unidades de medida

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Exemplo

Converta $45°$ em grados e em radianos.

Exemplo

Transforme para radianos os seguintes ângulos dados em graus:

a) $120°$
b) $135°$
c) $150°$
d) $210°$
e) $225°$
f) $240°$
g) $300°$
h) $315°$
i) $330°$
j) $360°$

Exemplo

Transforme para radianos os seguintes ângulos dados em graus:

a) $120°$
b) $135°$
c) $150°$
d) $210°$
e) $225°$
f) $240°$
g) $300°$
h) $315°$
i) $330°$
j) $360°$

Exemplo

Converta $\frac{3\pi}{10}\ rad$ em graus.

Teorema de Pitágoras

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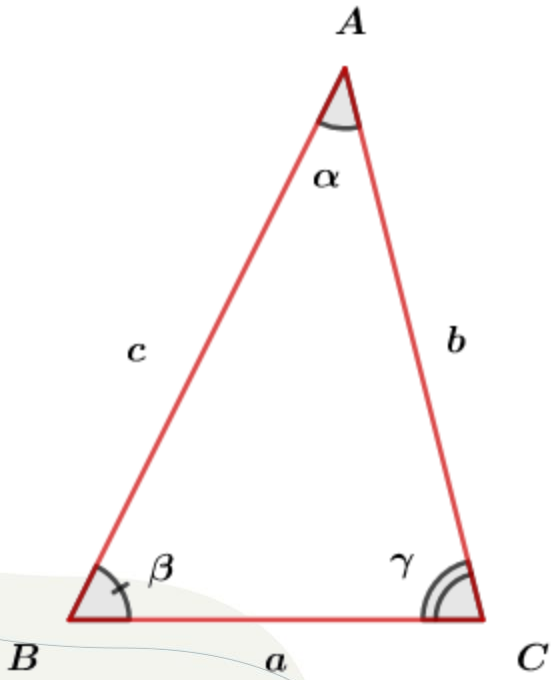
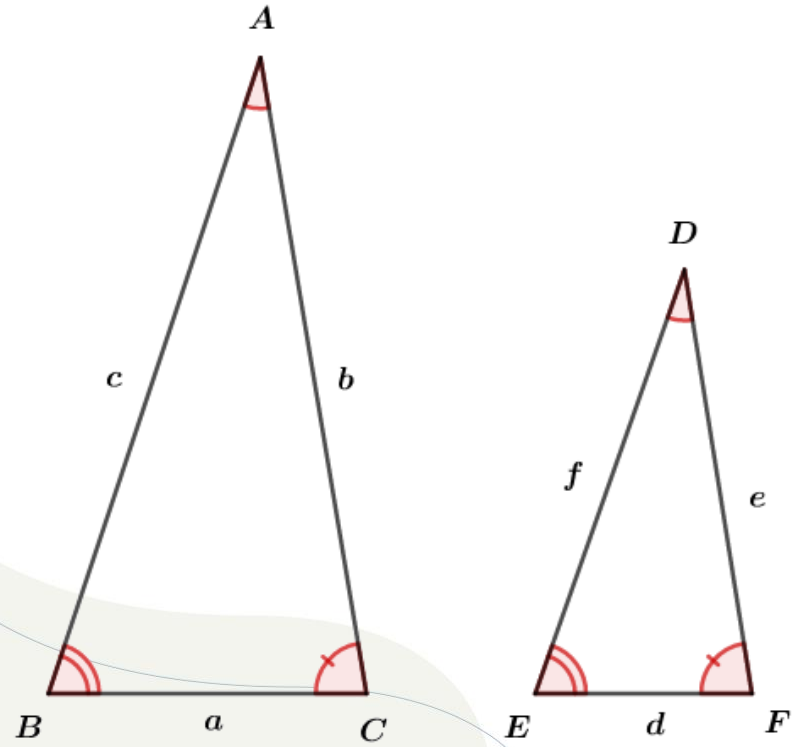
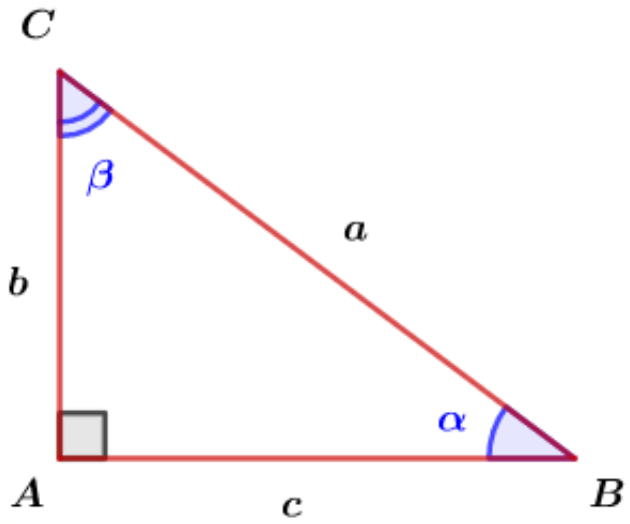
Conceitos básicos de triângulos



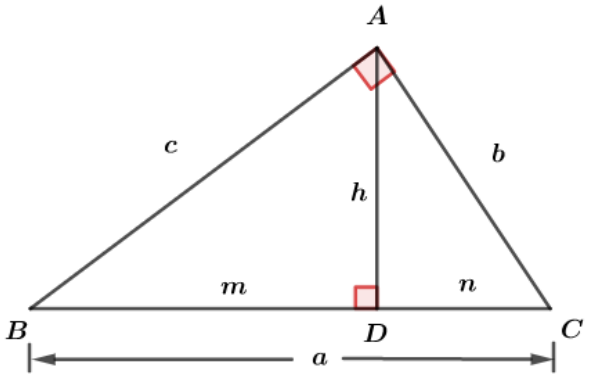
Semelhança de triângulos

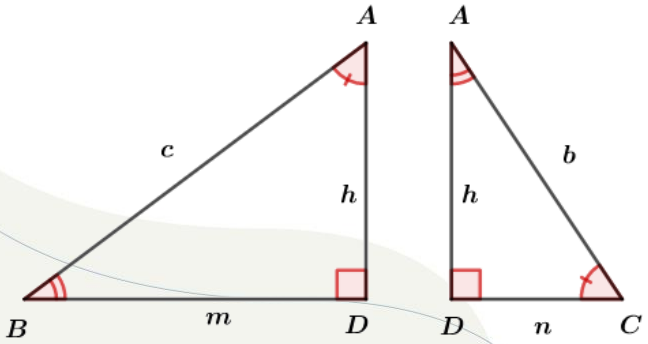


Teorema de Pitágoras



Teorema de Pitágoras





Exemplo

Exemplo

Razões Trigonométricas

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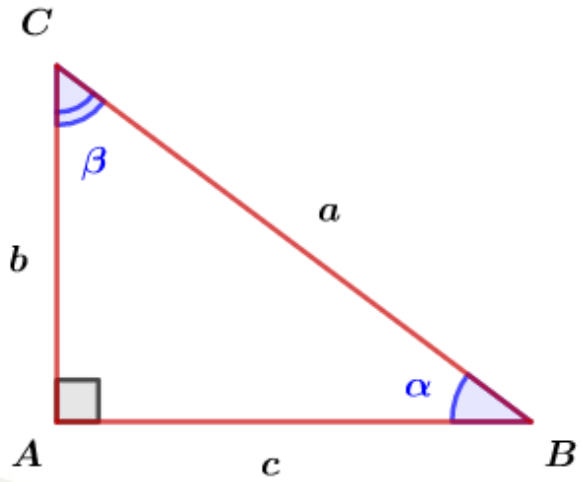
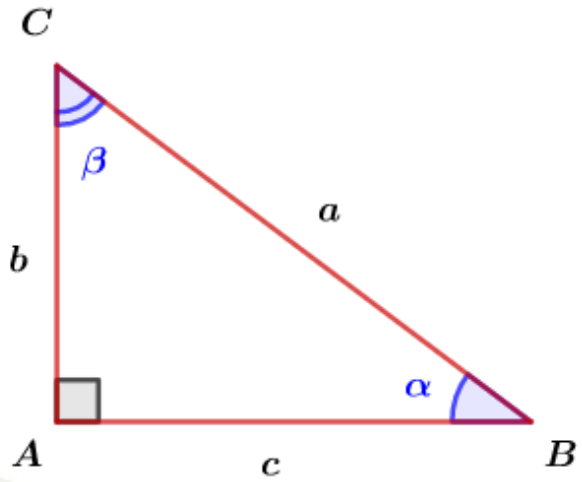
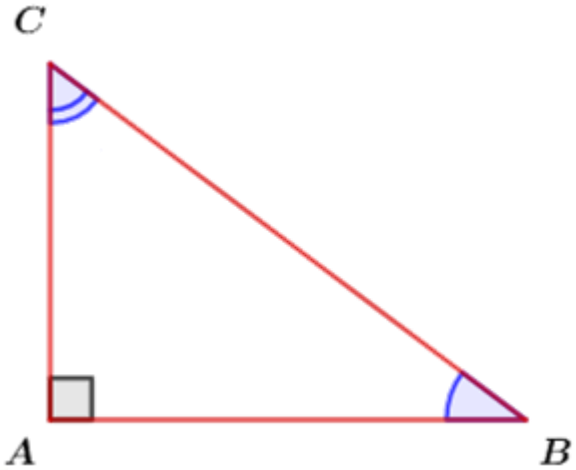
Razões Trigonométricas



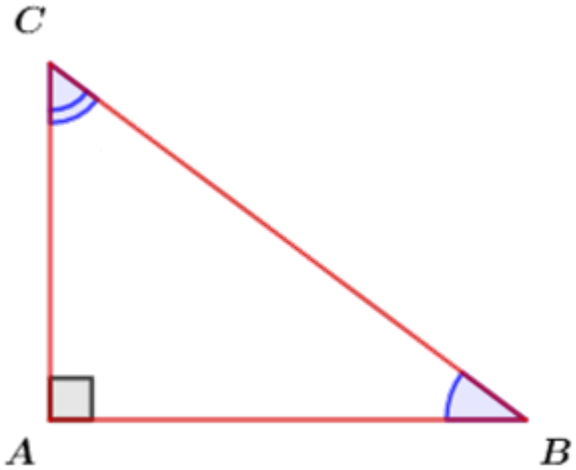
Razões Trigonométricas



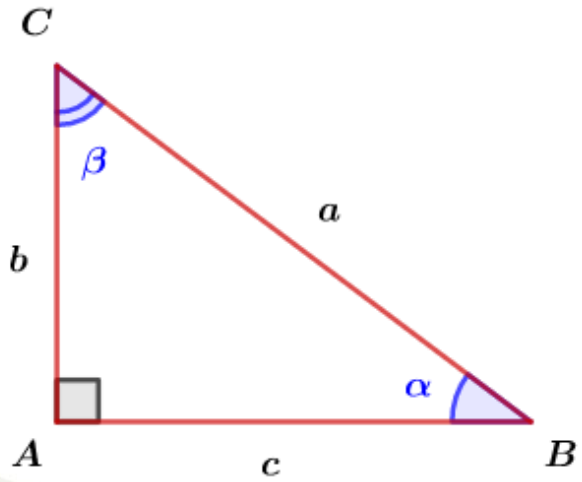
Exemplo



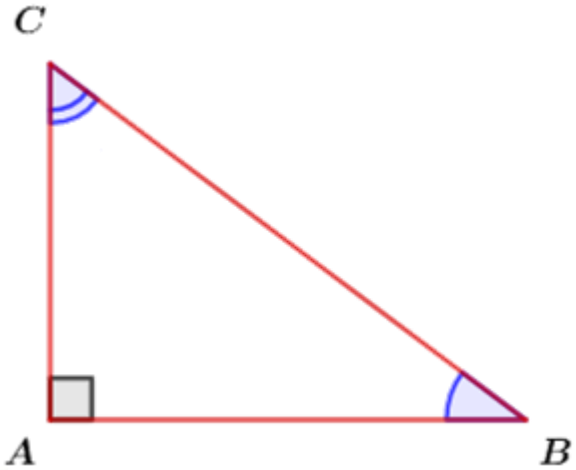
Exemplo



Outras Razões Trigonométricas



Exemplo



Relação Fundamental

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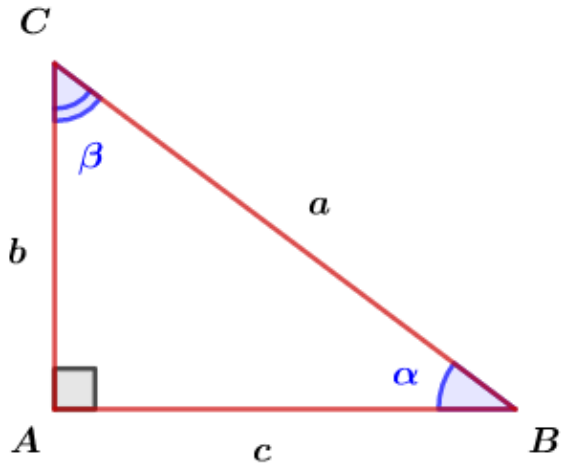
Relação Fundamental



Relação Fundamental

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Relação Fundamental

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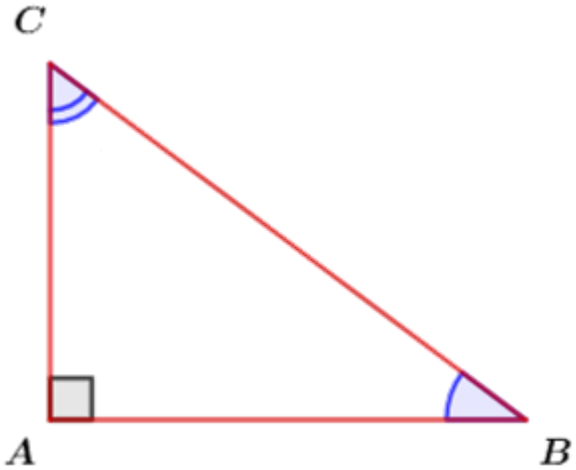
Exemplo



(ESA/2012)

A soma dos valores m que satisfazem as igualdades $\operatorname{sen} x = \frac{m+1}{m}$ e $\cos x = \frac{m+2}{m}$ é:

a) 5

b) 6

c) 4

d) -4

e) -6

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(EEAR/2015) [Adaptada]

Ao simplificar a expressão $(1 + \cos x)(1 - \cos x)$, tem-se

a) 2

b) $\text{sen}^2\, x$

c) $\cos^2 x$

d) $2 + \cos^2 x$

Ângulos Complementares

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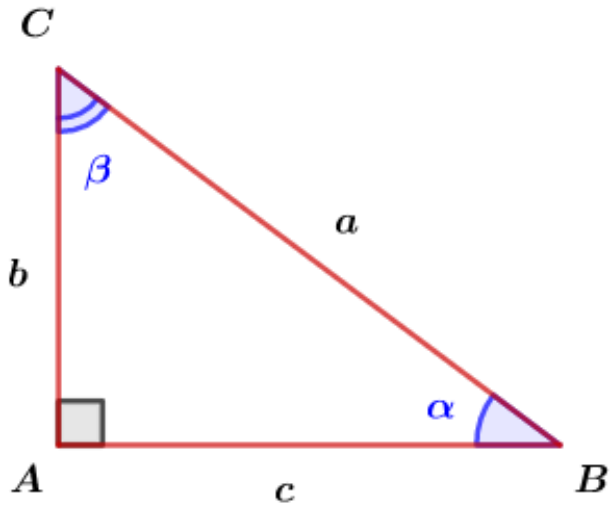
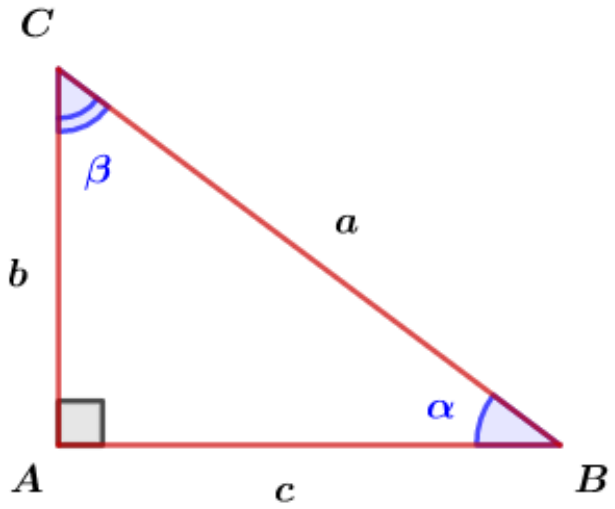
Ângulos complementares

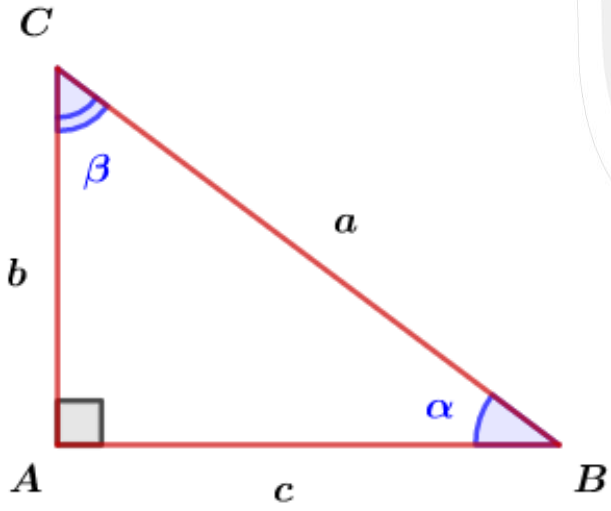


Ângulos complementares



Ângulos complementares

$\alpha + \beta = \frac{\pi}{2}$
$sen\alpha = cos\beta$
$sen\beta = cos\alpha$
$tg\alpha = \frac{1}{tg\beta}$
$tg\beta = \frac{1}{tg\alpha}$



Arcos notáveis

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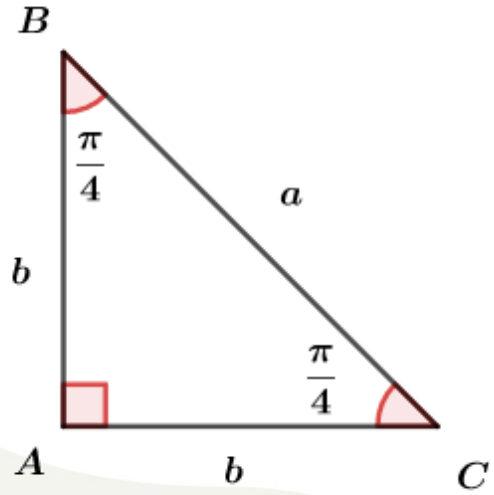
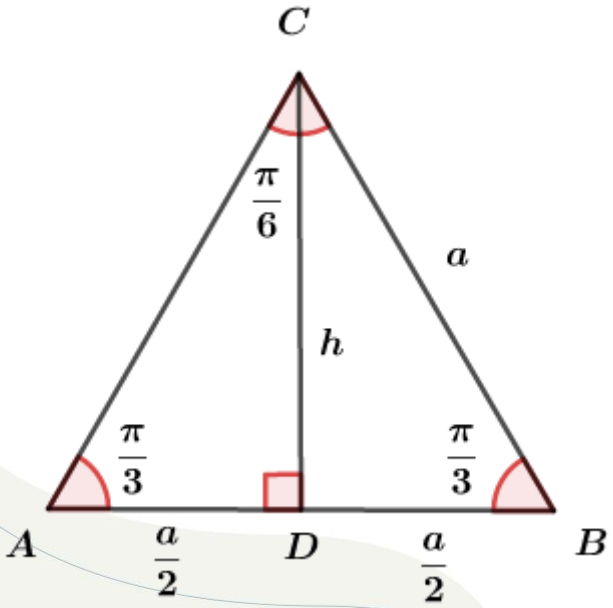
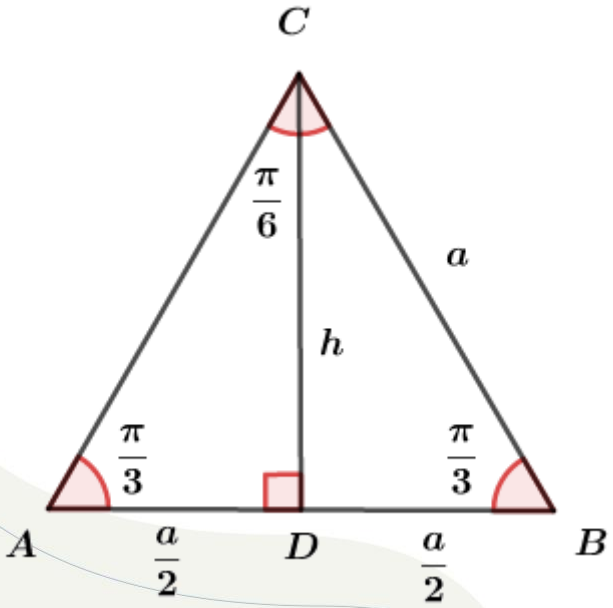
45°



30° e 60°



30° e 60°



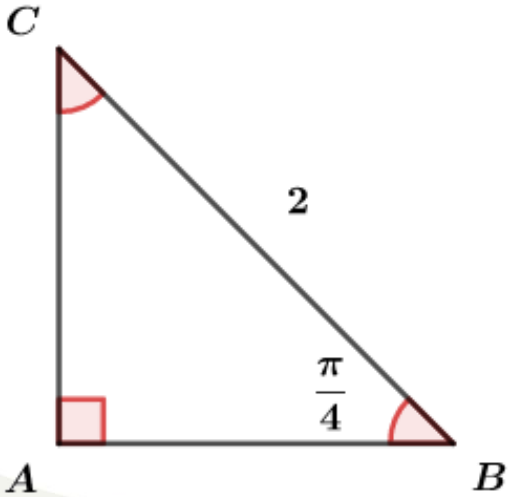
Ângulos Notáveis

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
Seno	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
Cosseno	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
Tangente	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

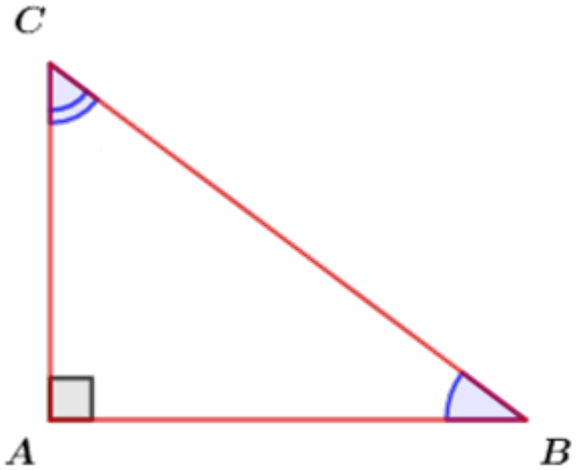
Ângulos Notáveis

Seno	
Cosseno	
Tangente	

Exemplos



Exemplos



Ciclo Trigonométrico

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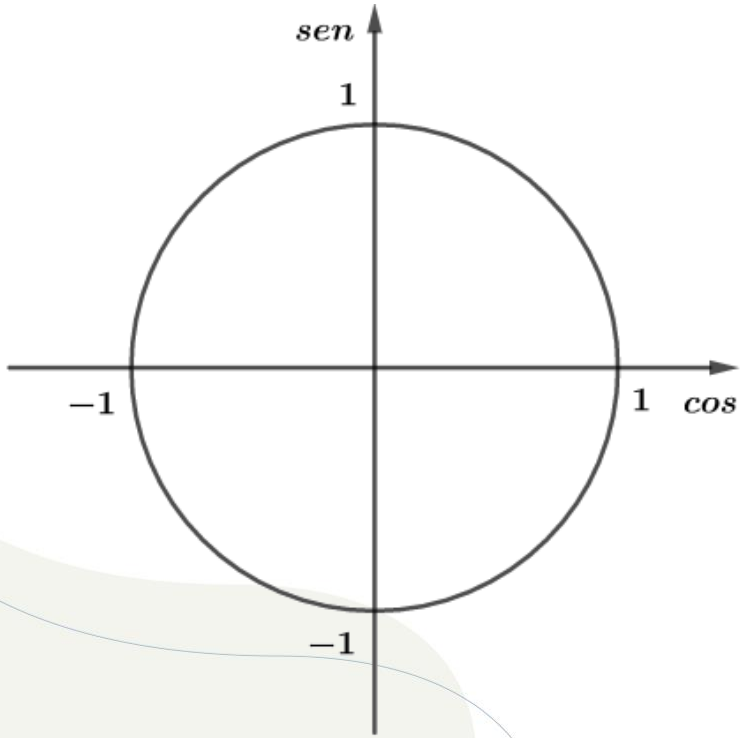
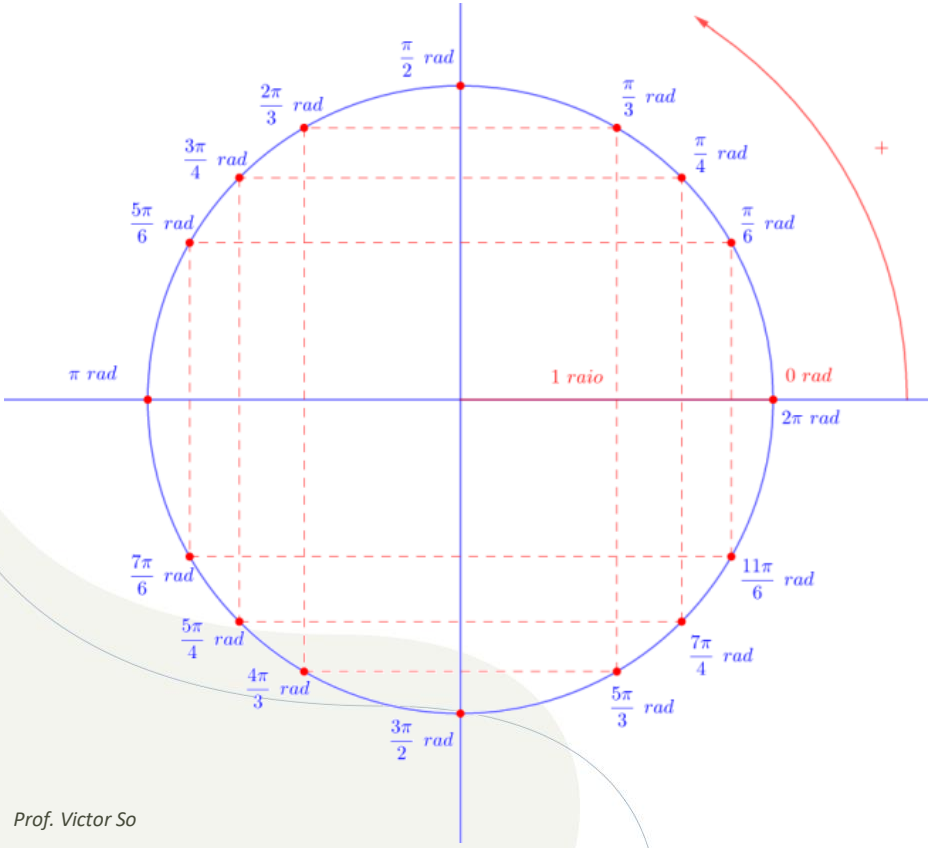
Definição



Ângulos notáveis



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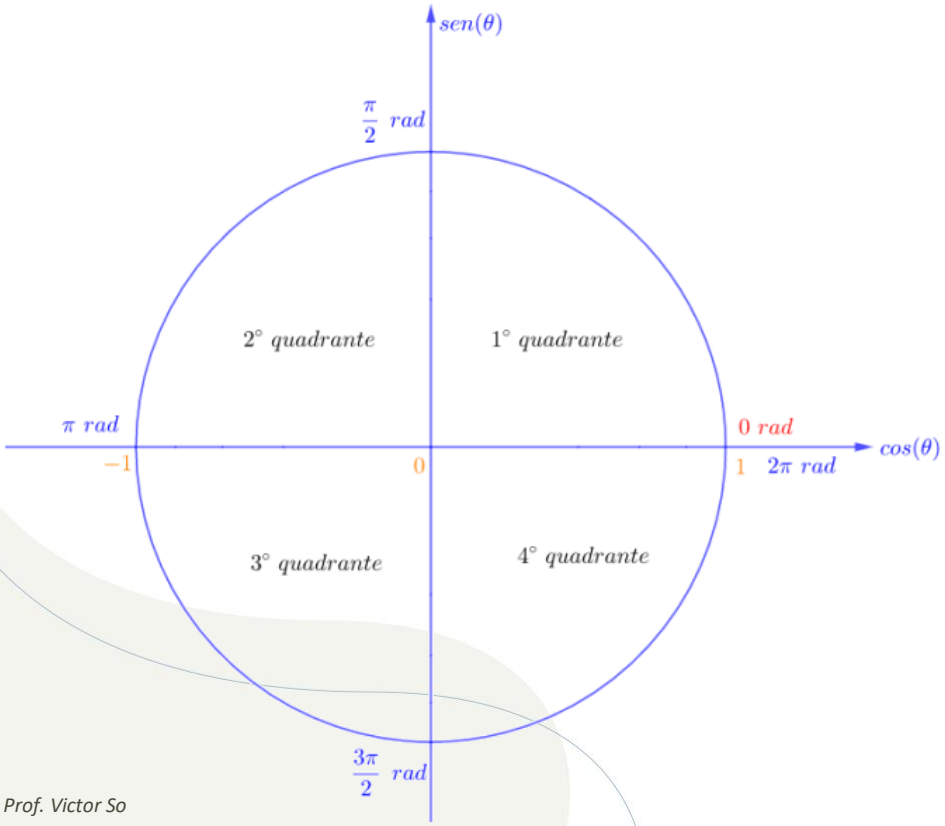
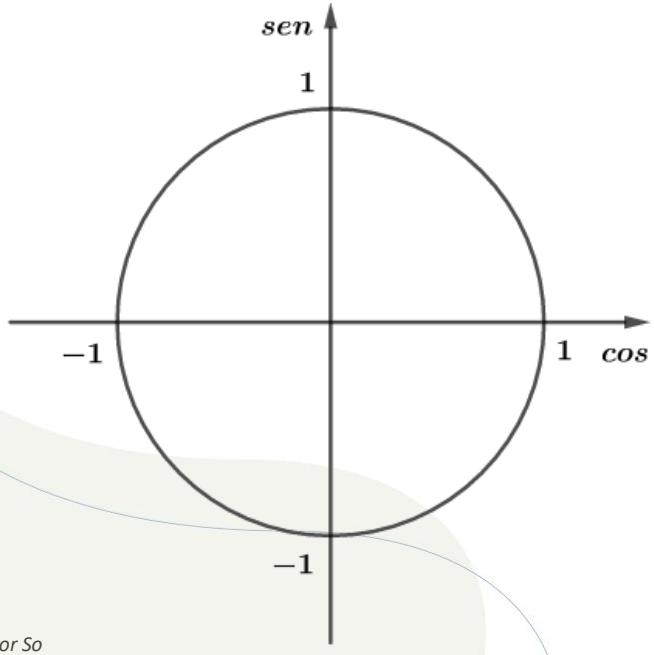
Quadrantes


$sen(\theta)$
$\frac{\pi}{2}$ rad
2° quadrante
1° quadrante
π rad
0 rad
−1
0
1
2π rad
$cos(\theta)$
3° quadrante
4° quadrante
$\frac{3\pi}{2}$ rad

Ângulos Congruentes

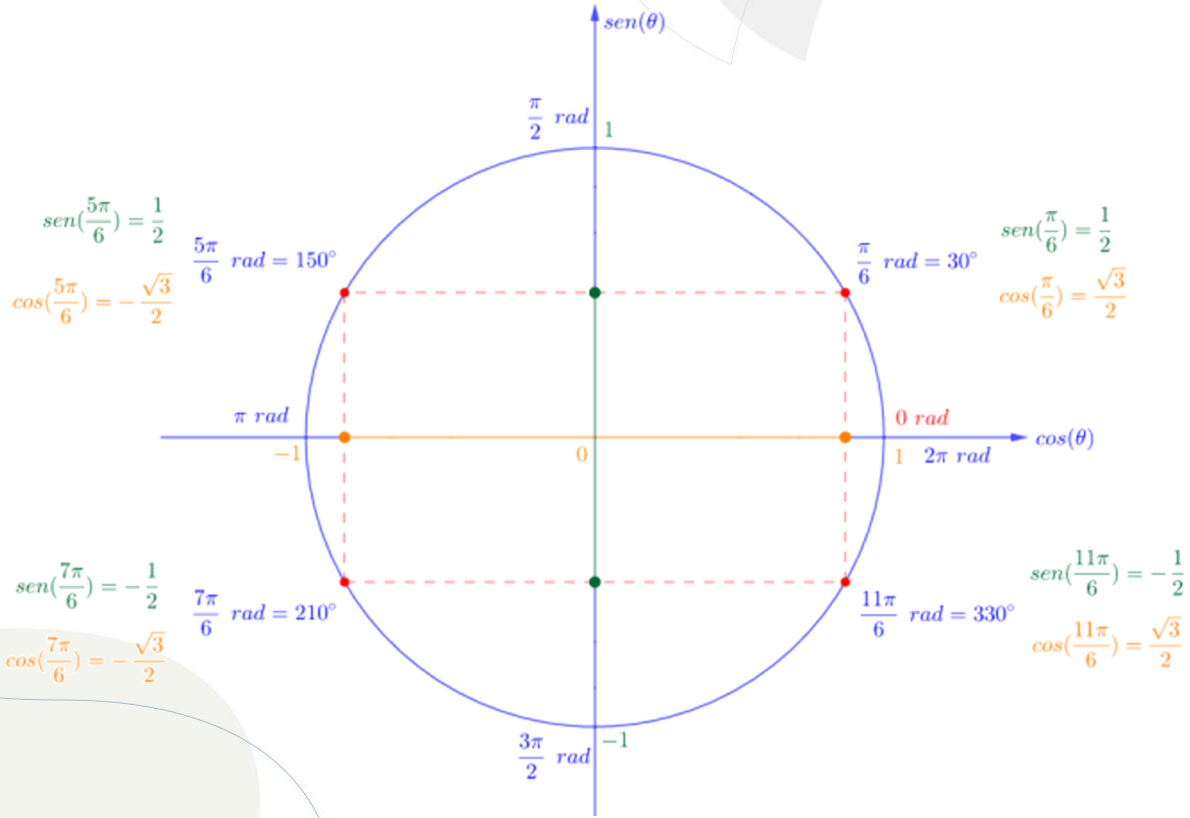
$$\boldsymbol{\alpha \equiv \beta \Leftrightarrow \alpha = \beta + 2k\pi, k \in \mathbb{Z}}$$



Exemplos

Determine os arcos positivos, menores do que 6π, congruentes a $\pi/3$.

Redução ao Primeiro Quadrante



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Função Seno

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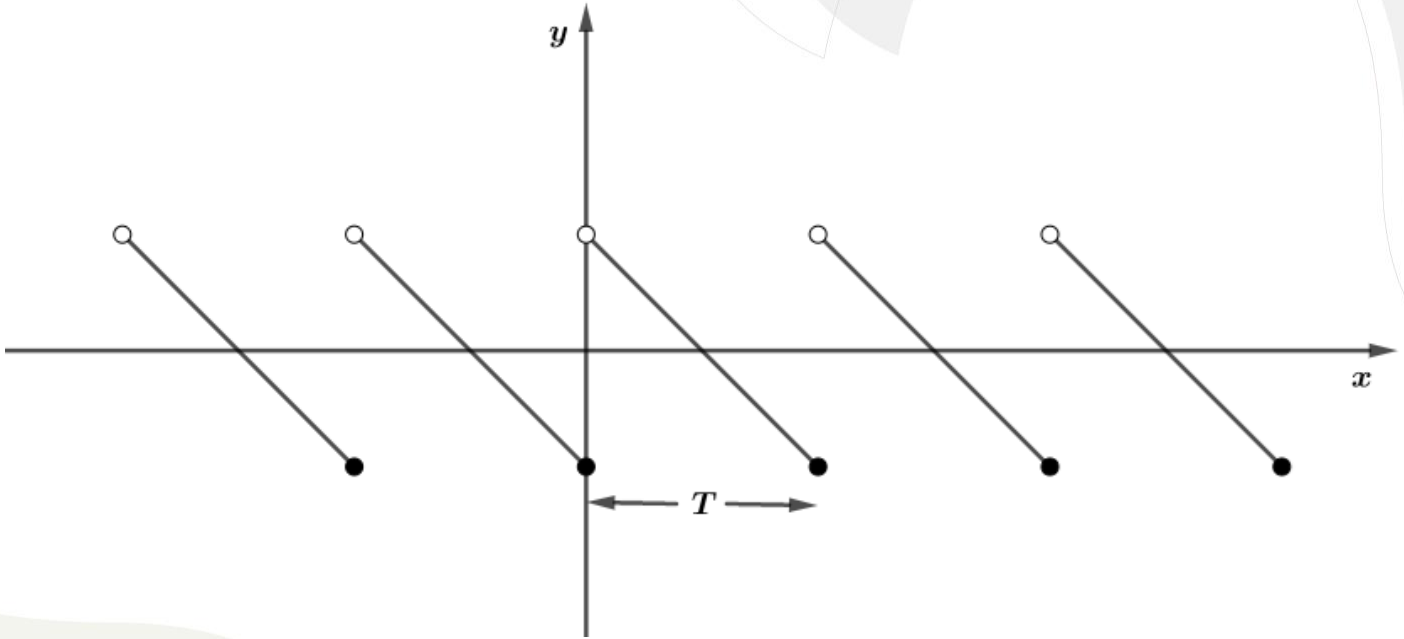
Definição de função periódica



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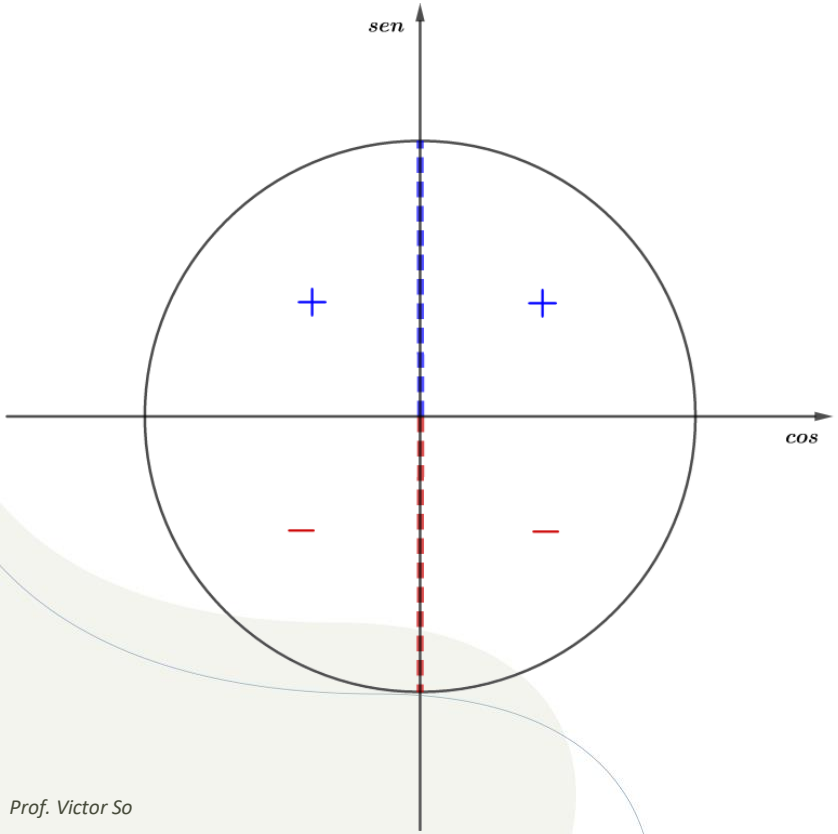
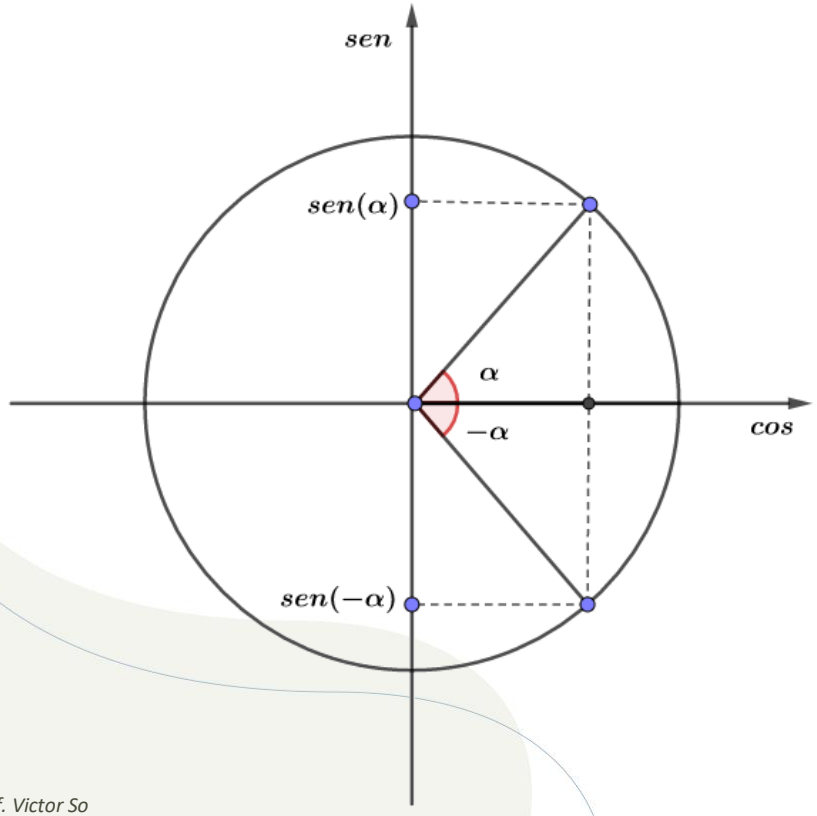
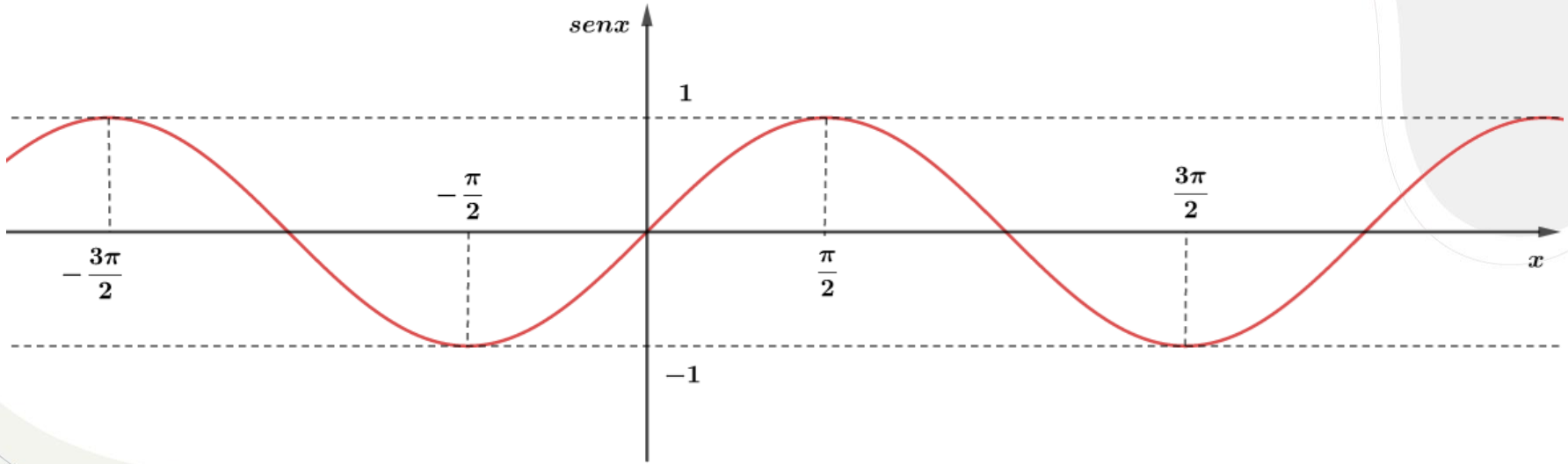
Estudo do sinal



Paridade



Gráfico



Função Cosseno

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Função Cosseno

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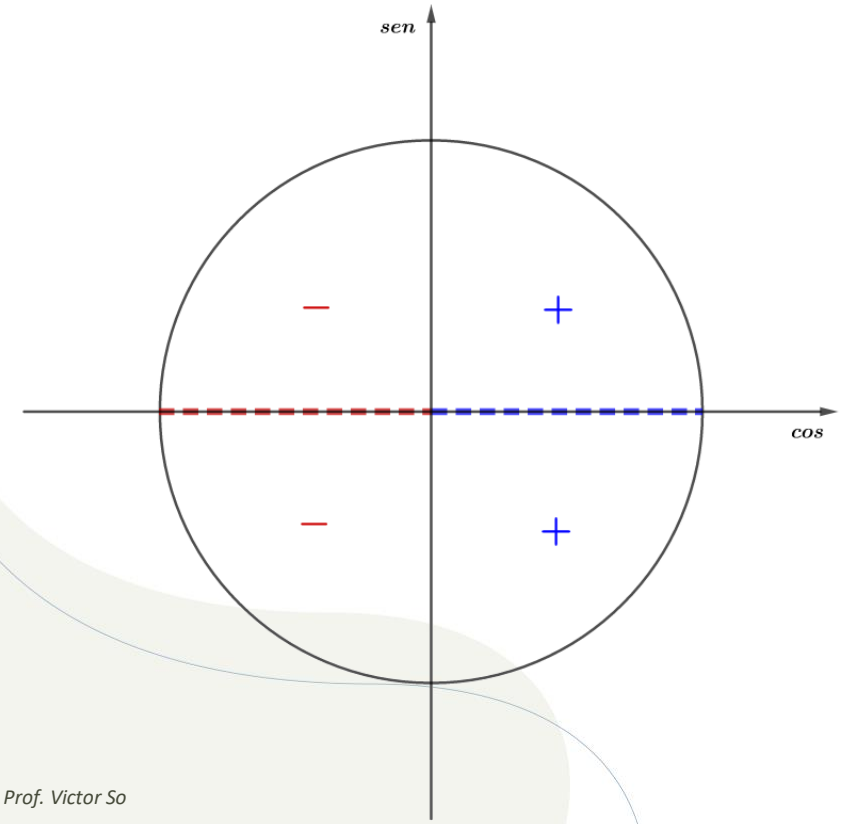
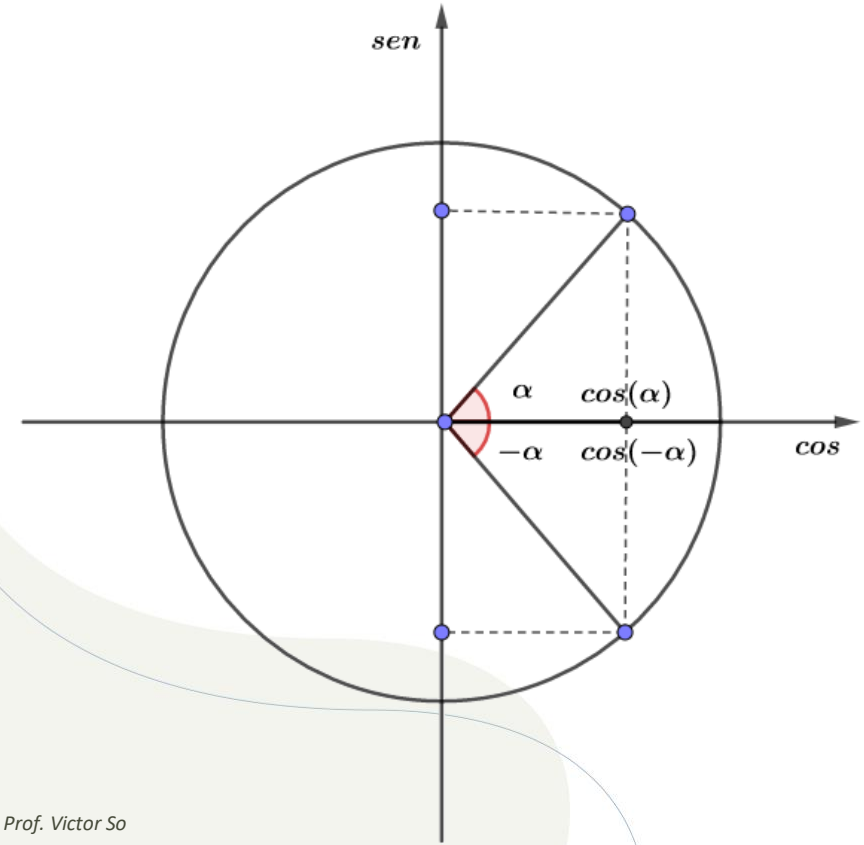
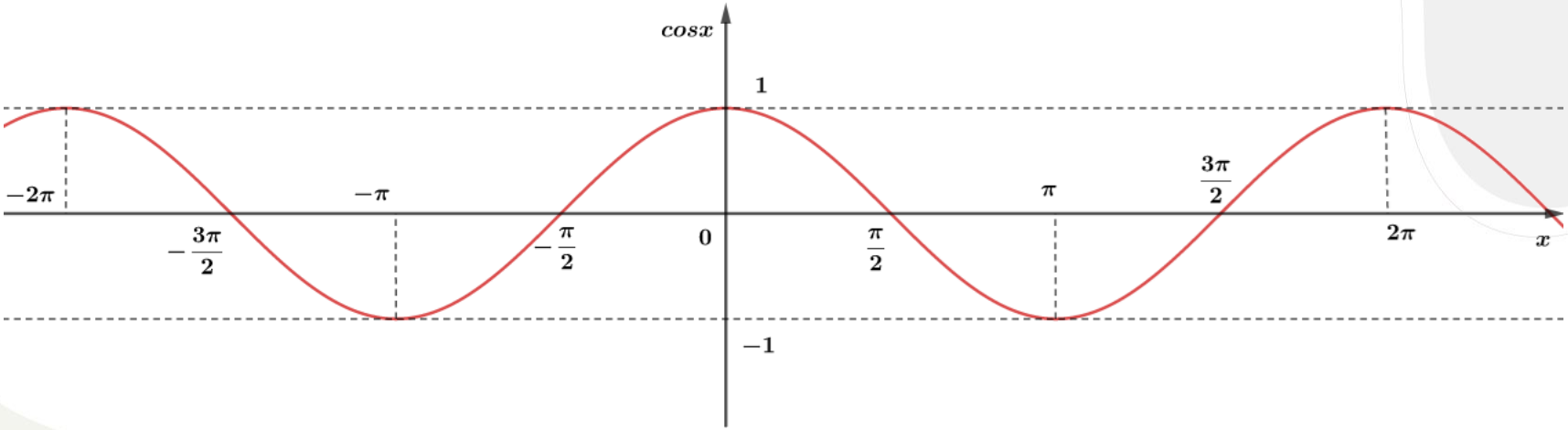


Paridade



Gráfico



Função Tangente

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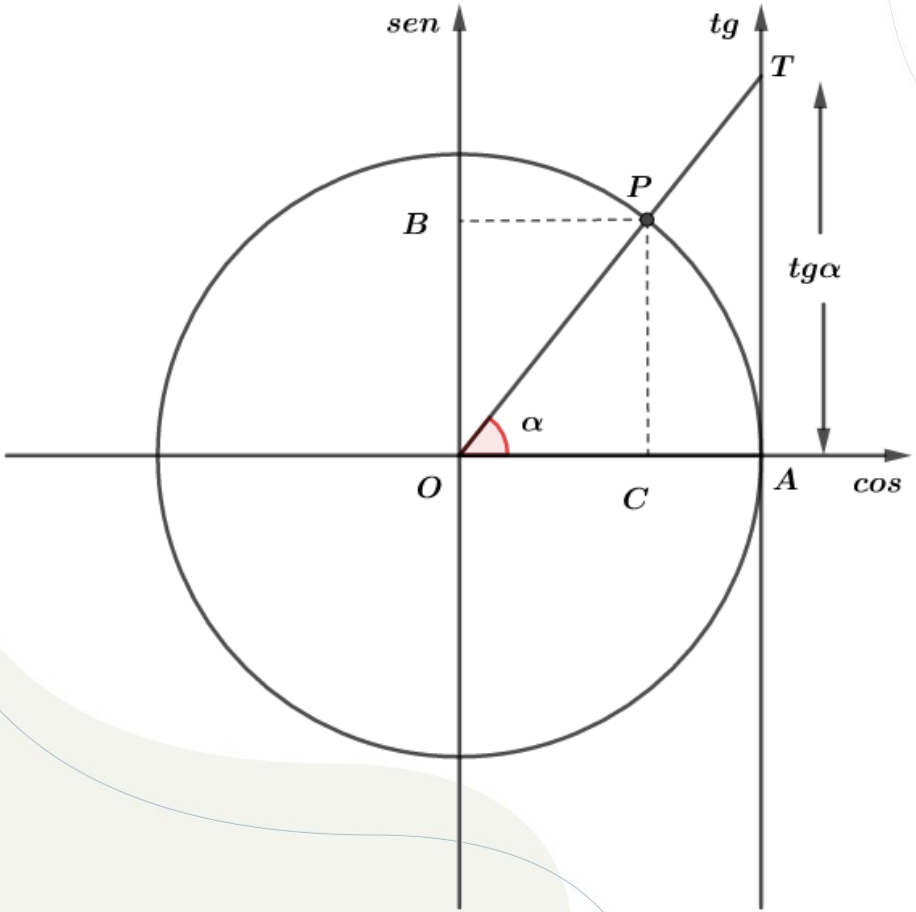
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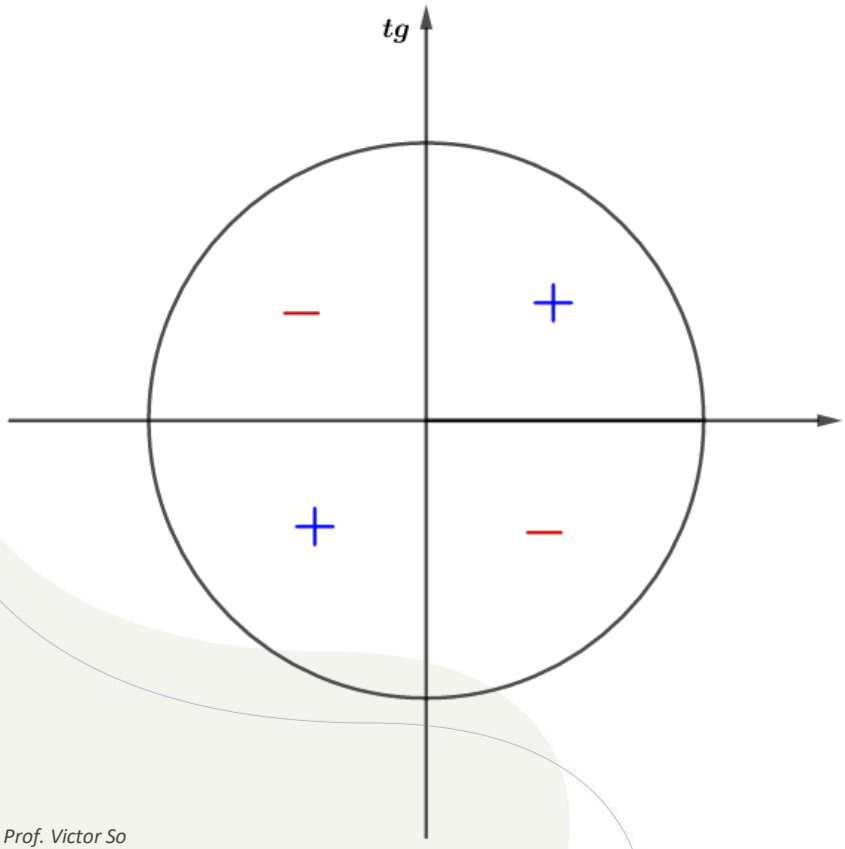
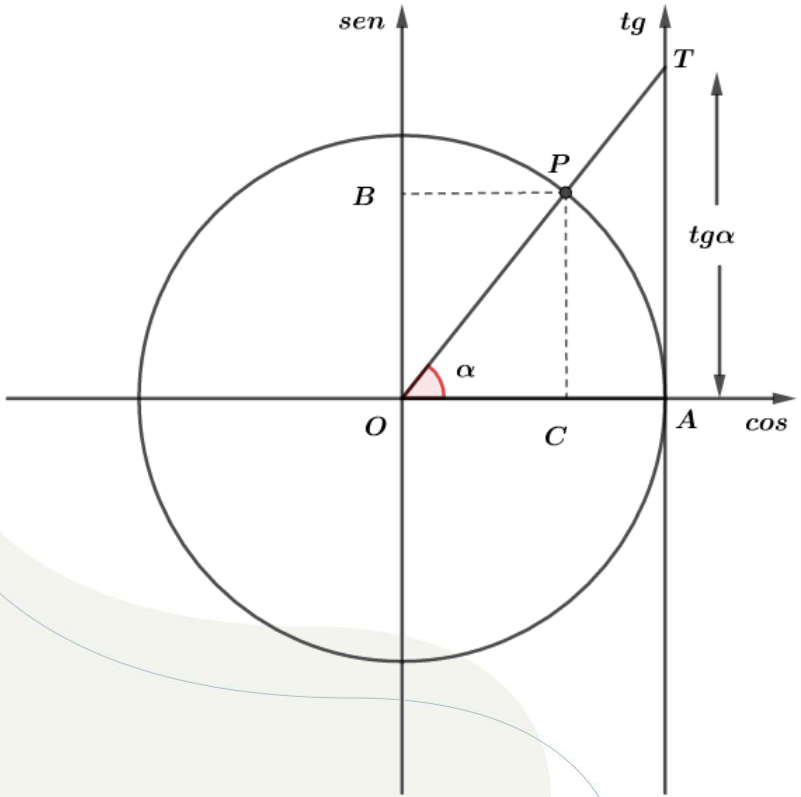
Estudo do sinal



Intervalo de valores



Paridade

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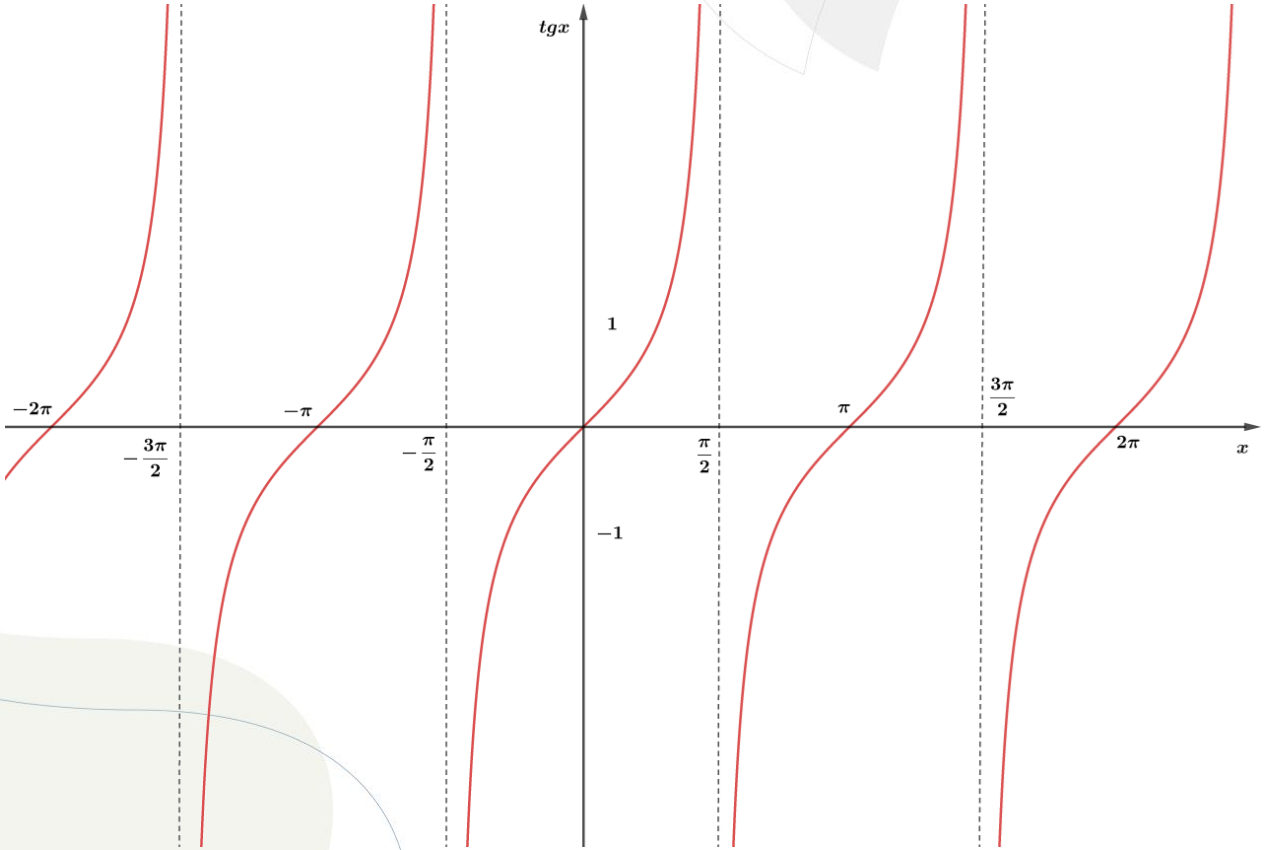
Gráfico



Função Cotangente

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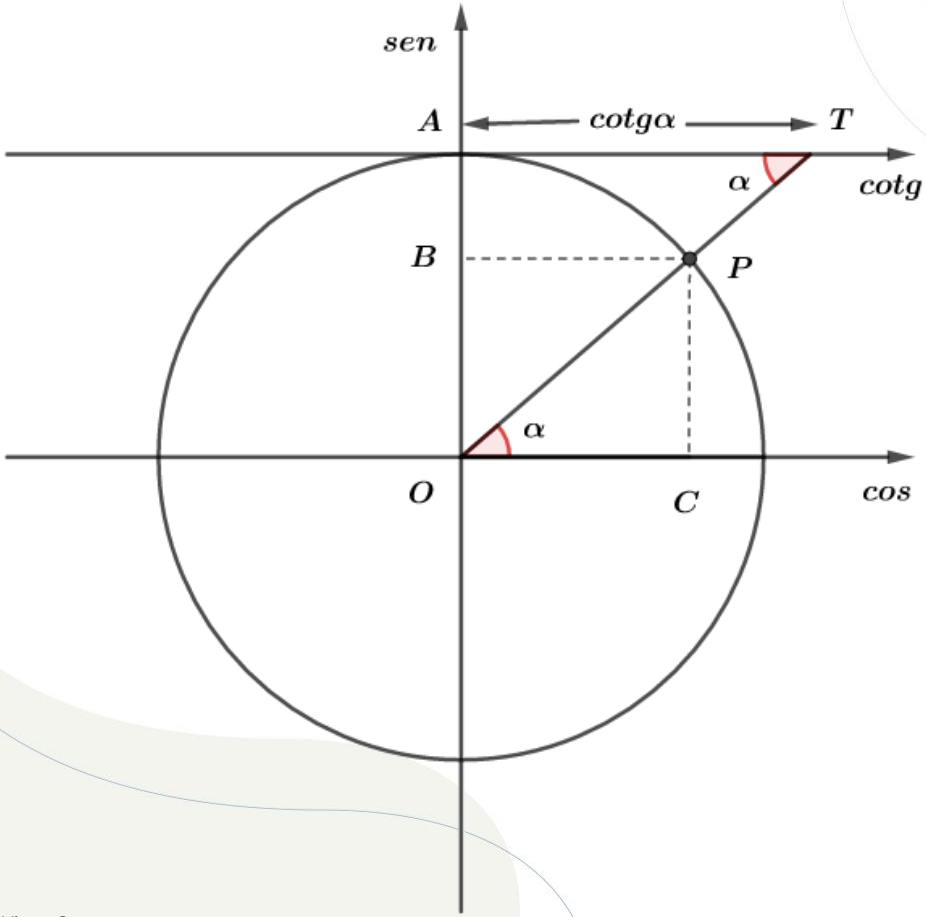
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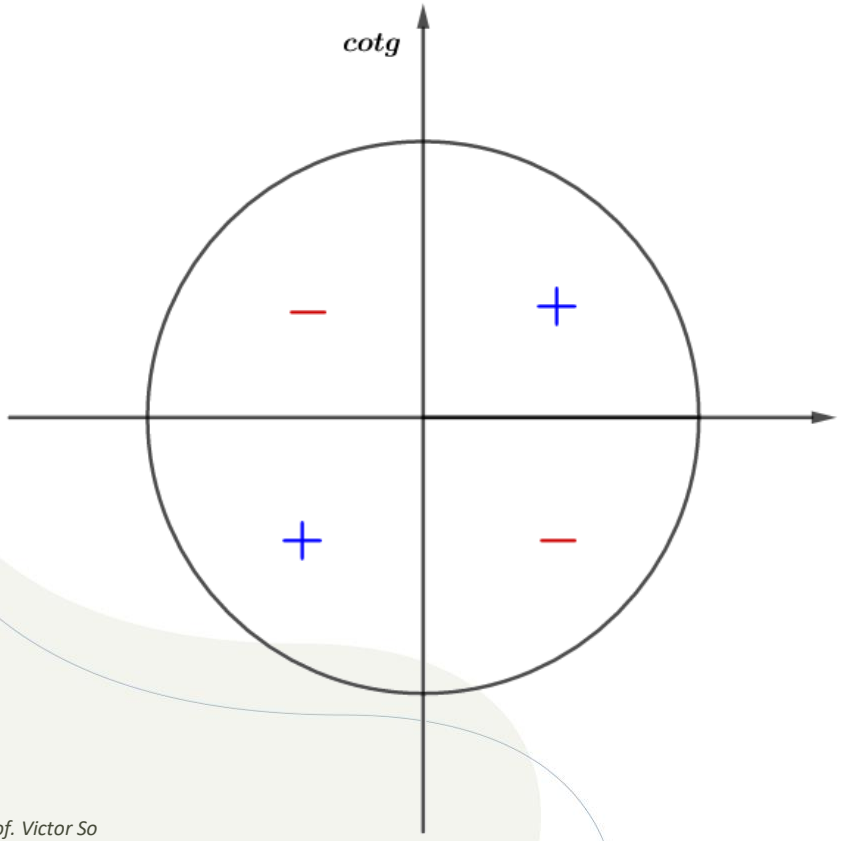
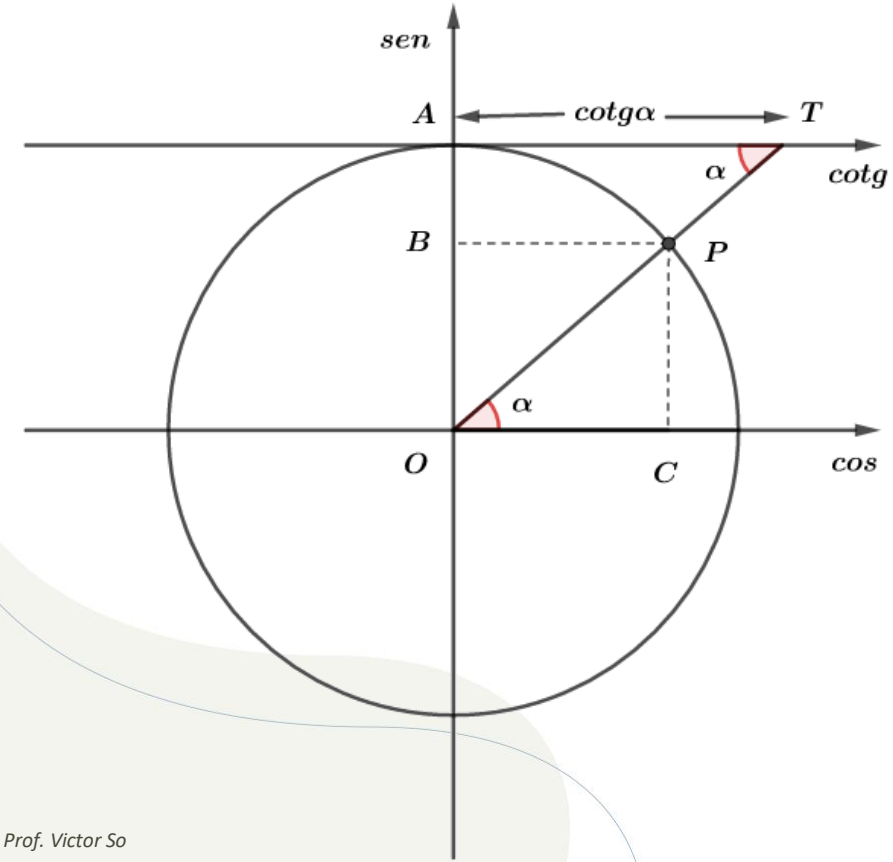
Estudo do sinal



Intervalo de valores



Funções Secante e Cossecante

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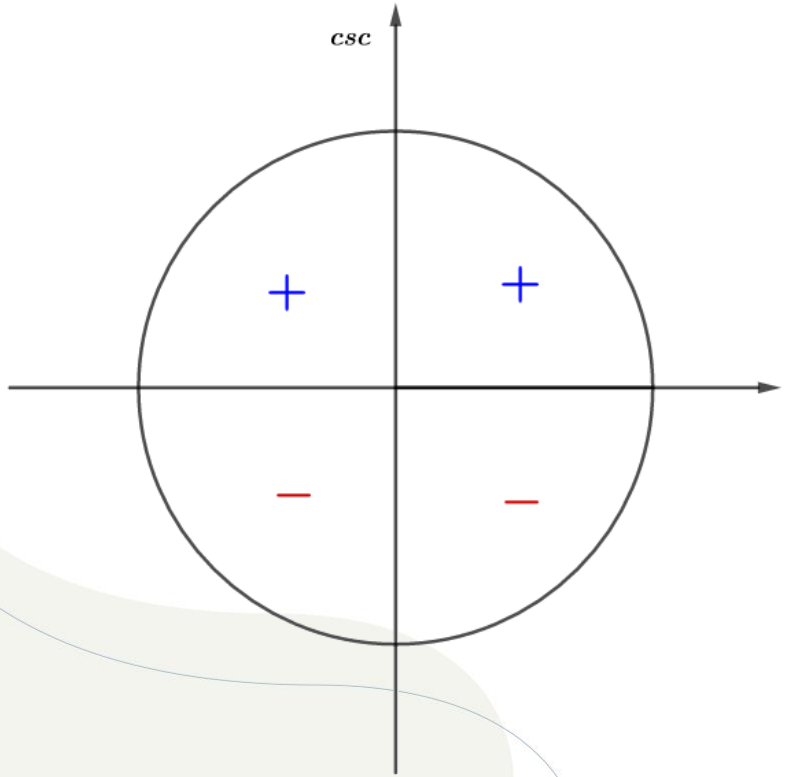
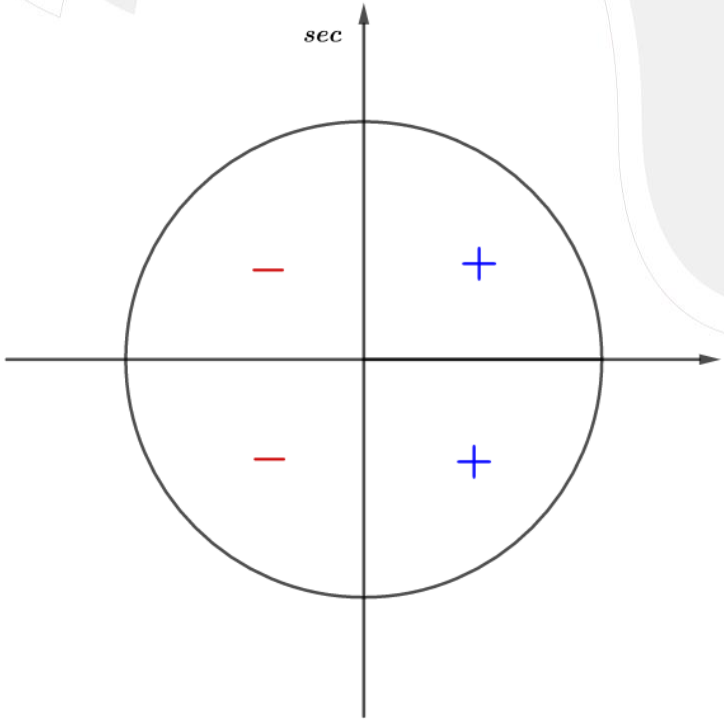
Estudo do sinal





Intervalo de valores

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Funções Inversas

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Função Arco-Seno

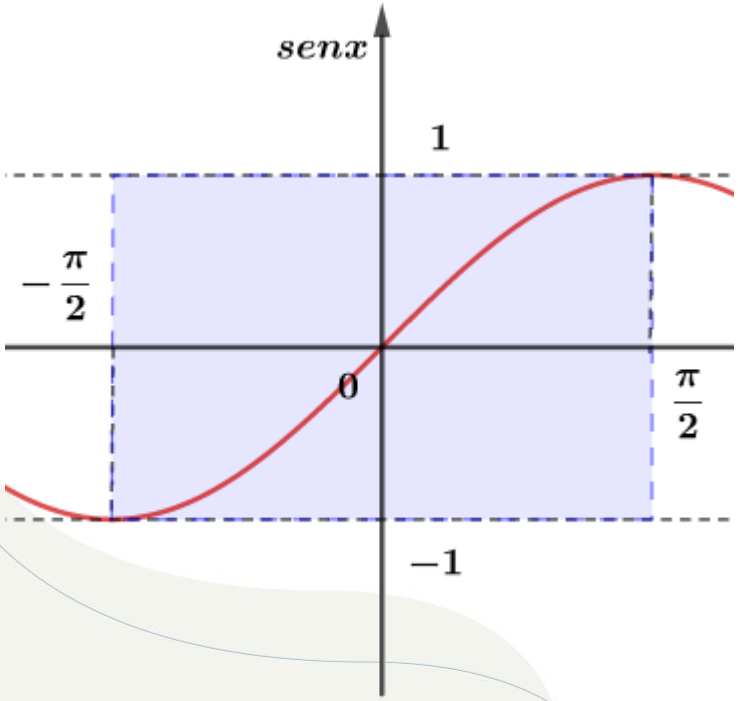
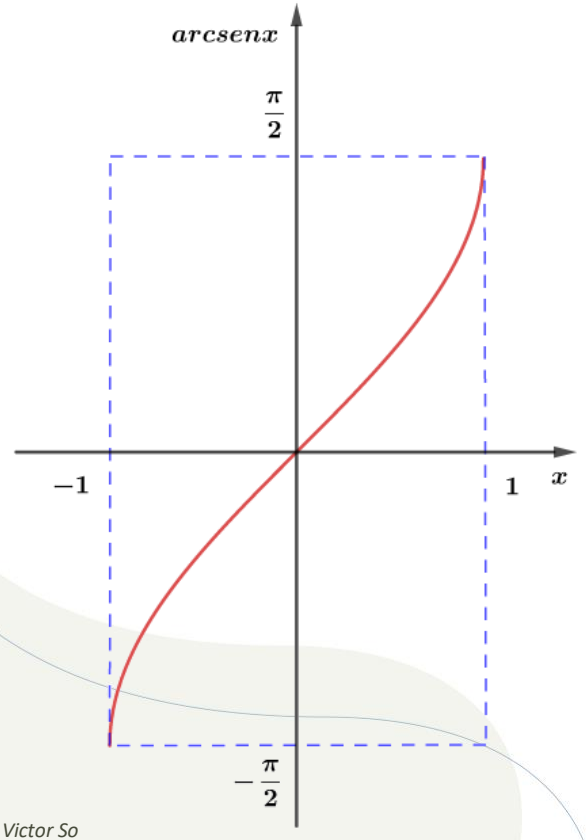
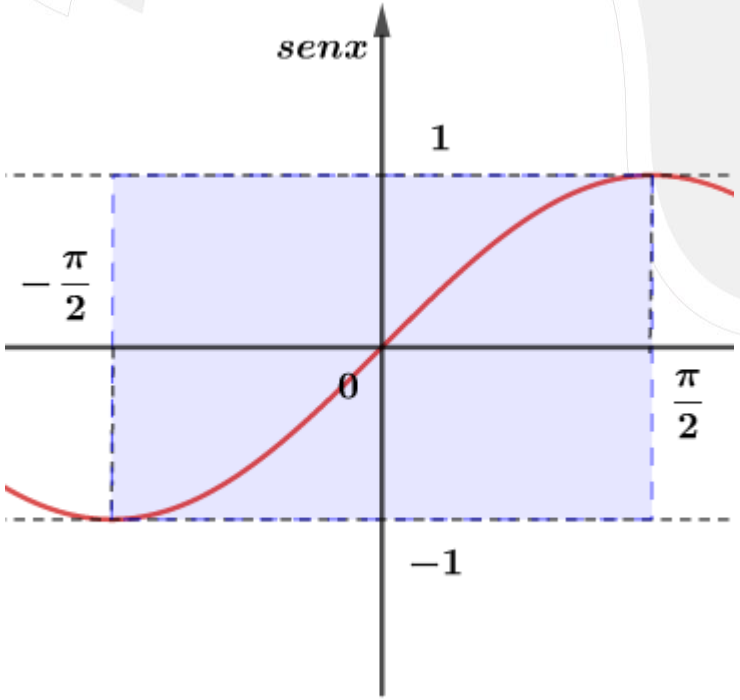


Gráfico da Função Arco-Seno





Função Arco-Cosseno

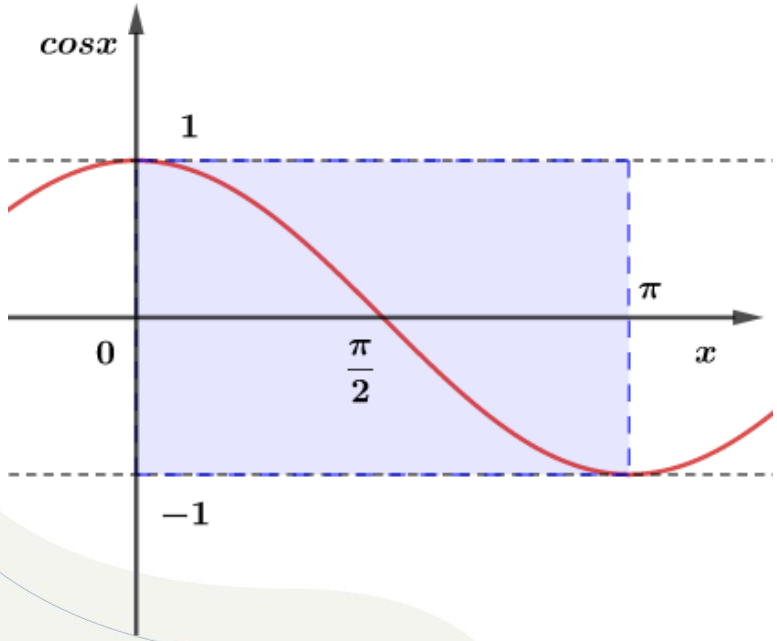
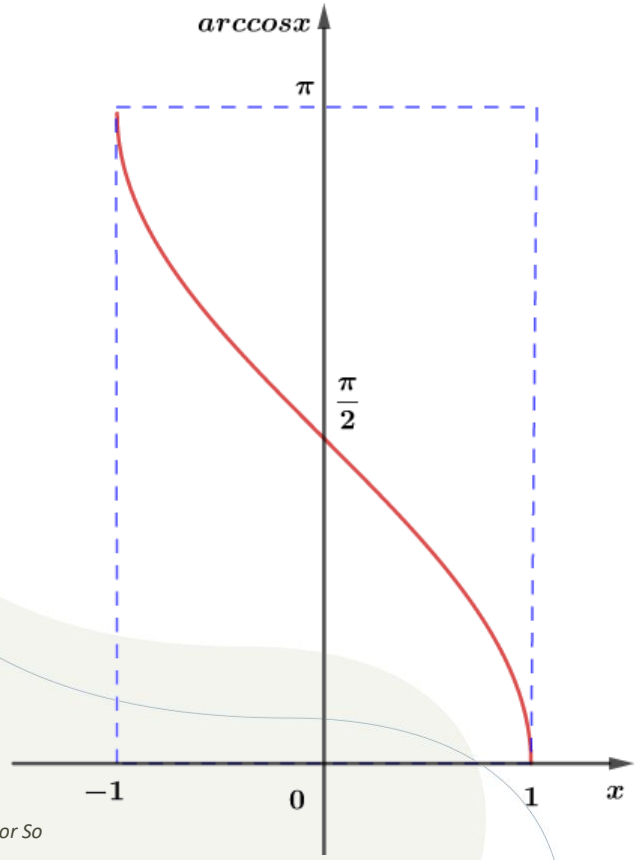
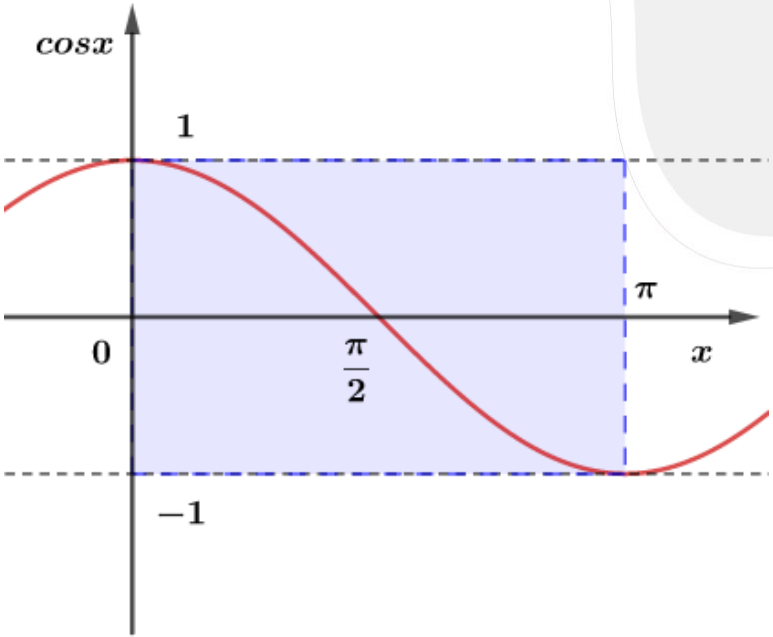


Gráfico da Função Arco-Cosseno





Função Arco-Tangente

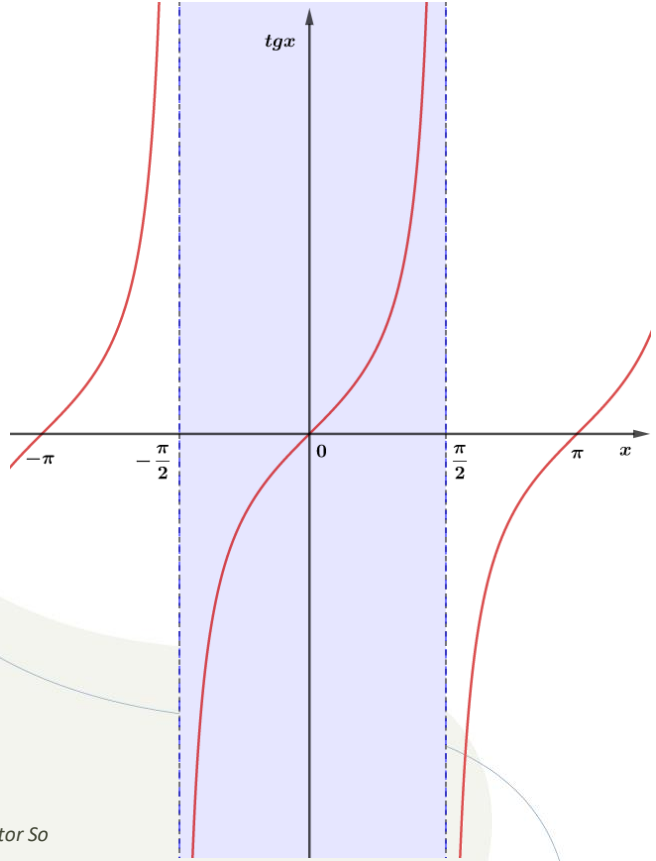
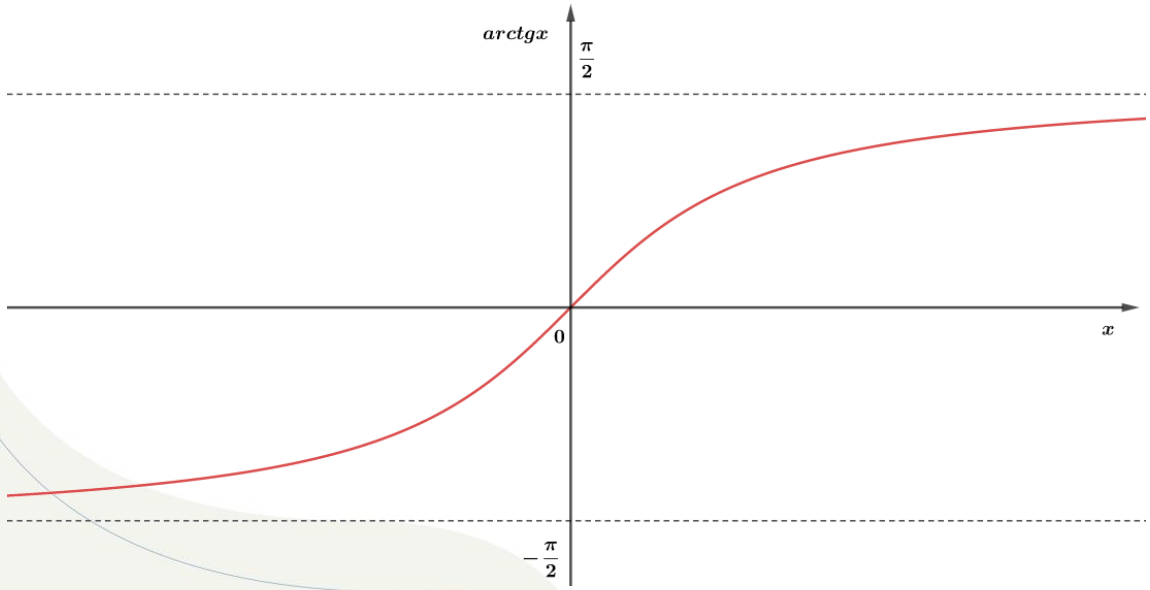
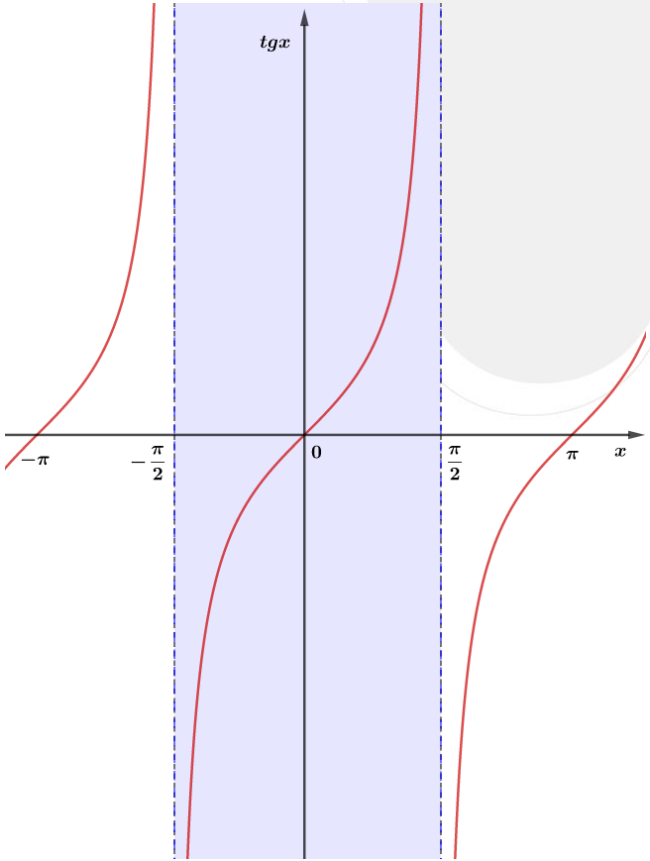


Gráfico da Função Arco-Tangente





Calcule:

a) $sen\left(arctg(1)\right)$

b) $\cos\left(arcsen\left(\frac{3}{5}\right)\right)$

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Transformações
Soma e diferença de arcos

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$$\cos(A+B) = \cos(A)\cos(B) - sen(A)sen(B)$$

$$\cos(A-B) = \cos(A)\cos(B) + sen(A)sen(B)$$

$$sen(A+B) = \text{sen}(\text{A})\cos(\text{B}) + \text{sen}(\text{B})\cos(\text{A})$$

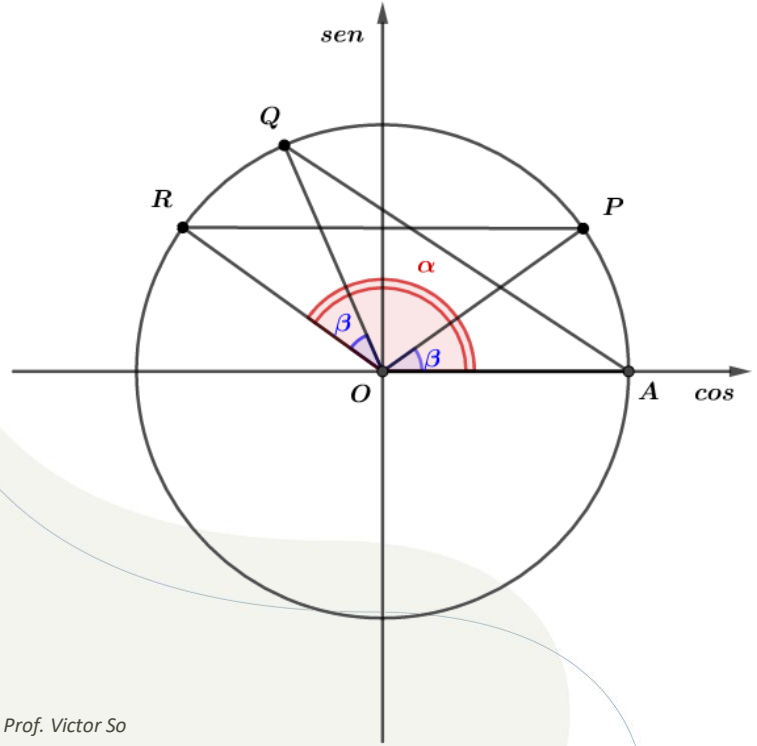
$$sen(A-B) = \text{sen}(\text{A})\cos(\text{B}) - \text{sen}(\text{B})\cos(\text{A})$$

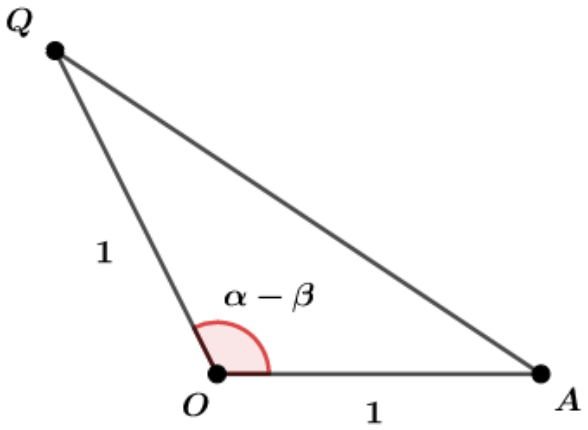
$$tg(A+B) = \frac{tg(A) + tg(B)}{1 - tg(A)tg(B)}$$

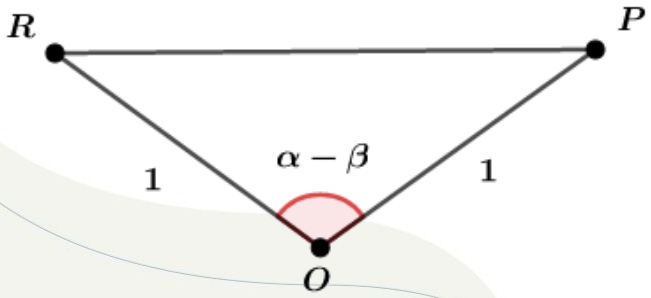
$$tg(A-B) = \frac{tg(A) - tg(B)}{1 + tg(A)tg(B)}$$

Demonstração

1) $\cos(A+B) = \cos(A)\cos(B) - sen(A)sen(B)$




Q
1
α − β
O
1
A


R
P
1
α − β
1
O

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2) $\cos(A + B) = cosAcosB - senAsenB$

3) $sen(A + B) = \text{sen(A)}\cos(\text{B}) + \text{sen(B)}\cos(\text{A})$

4) $sen(A - B) = \text{sen(A)}\cos(\text{B}) - \text{sen(B)}\cos(\text{A})$

5) $tg(A+B) = \dfrac{tg(A) + tg(B)}{1 - tg(A)tg(B)}$

6) $tg(A-B)=\dfrac{tg(A)-tg(B)}{1+tg(A)tg(B)}$

Transformações
Arco duplo

@profvictorso

Fórmulas de arco duplo

$$\boxed{\cos(2A) = \cos^2 A - sen^2 A}$$

$$\boxed{sen(2A) = 2senA\cos A}$$

$$\boxed{tg(2A) = \frac{2tgA}{1 - tg^2 A}}$$

Demonstração

1) $\cos(2A) = \cos^2 A - sen^2 A$

Demonstração

2) $sen(2A) = 2senAcosA$

Demonstração

3) $tg(2A) = \dfrac{2tgA}{1 - tg^2A}$

Transformações
Arco metade

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Fórmulas de arco metade

$$\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+cosA}{2}}$$

$$\text{sen}\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-cosA}{2}}$$

$$tg\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-cosA}{1+cosA}}$$

$$tg\left(\frac{A}{2}\right) = \frac{1-cosA}{senA} = \frac{senA}{1+cosA}$$

$$senA = \frac{2tg\left(\frac{A}{2}\right)}{1+tg^2\left(\frac{A}{2}\right)}$$

$$\cos(A) = \frac{1-tg^2\left(\frac{A}{2}\right)}{1+tg^2\left(\frac{A}{2}\right)}$$

$$tgA = \frac{2tg\left(\frac{A}{2}\right)}{1-tg^2\left(\frac{A}{2}\right)}$$

Demonstração

1) $\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+cosA}{2}}$

Demonstração

2) $\text{sen}\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-cosA}{2}}$

Demonstração

3) $tg\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-cosA}{1+cosA}}$

Demonstração

4) $tg\left(\frac{A}{2}\right) = \frac{1-cosA}{senA} = \frac{senA}{1+cosA}$

Demonstração

5) $sen A = \frac{2tg\left(\frac{A}{2}\right)}{1+tg^2\left(\frac{A}{2}\right)}$

Demonstração

6) $tgA = \frac{2tg\left(\frac{A}{2}\right)}{1-tg^2\left(\frac{A}{2}\right)}$

Demonstração

7) $\cos(A) = \frac{1-tg^2\left(\frac{A}{2}\right)}{1+tg^2\left(\frac{A}{2}\right)}$

Transformações
Arco triplo

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Fórmulas de arco triplo

$$\cos(3A) = 4\cos^3 A - 3\cos A$$

$$sen(3A) = 3senA - 4sen^3 A$$

$$tg(3A) = \frac{3tgA - tg^3 A}{1 - 3tg^2 A}$$

Demonstração

1) $\cos(3A) = 4\cos^3 A - 3cosA$

Demonstração

2) $sen(3A) = 3senA - 4sen^3A$

Demonstração

3) $tg(3A) = \dfrac{3tgA - tg^3A}{1 - 3tg^2A}$

Fórmulas de Werner

@profvictorso

Transformação Produto-Soma

$$\boxed{2cosAcosB = \cos(A+B) + \cos(A-B)}$$

$$\boxed{-2senAsenB = \cos(A+B) - \cos(A-B)}$$

$$\boxed{2senAcosB = sen(A+B) + sen(A-B)}$$

$$\boxed{2senBcosA = sen(A+B) - sen(A-B)}$$

Demonstração

1) $2cosAcosB = \cos(A + B) + \cos(A - B)$

Demonstração

2) $-2senAsenB = \cos(A+B) - \cos(A-B)$

Demonstração

3) $2senAcosB = sen(A + B) + sen(A - B)$

Demonstração

4) $2senBcosA = sen(A + B) - sen(A - B)$

Fórmulas de Prostaférese

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Transformação Soma-Produto

$$sen(p) + sen(q) = 2sen\left(\frac{p+q}{2}\right)cos\left(\frac{p-q}{2}\right)$$

$$sen(p) - sen(q) = 2sen\left(\frac{p-q}{2}\right)cos\left(\frac{p+q}{2}\right)$$

$$\cos(p) + \cos(q) = 2\cos\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$$

$$\cos(p) - \cos(q) = -2sen\left(\frac{p+q}{2}\right)sen\left(\frac{p-q}{2}\right)$$

Demonstração

1) $sen(p) + sen(q) = 2sen\left(\frac{p+q}{2}\right) cos\left(\frac{p-q}{2}\right)$

Demonstração

2) $sen(p) - sen(q) = 2sen\left(\frac{p-q}{2}\right) cos\left(\frac{p+q}{2}\right)$

Demonstração

3) $\cos(p) + \cos(q) = 2\cos\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$

Demonstração

4) $\cos(p) - \cos(q) = -2sen\left(\frac{p+q}{2}\right) sen\left(\frac{p-q}{2}\right)$

Transformando-se $sen40° + cos10°$ em produto, obtemos:

a) $\frac{\sqrt{3}}{2} sen40°$

b) $\sqrt{3} sen20°$

c) $\sqrt{3} cos20°$

d) $\sqrt{2} sen20°$

Questões ESA/EEAR

@profvictorso

(ESA/2016)

Sabendo que x pertence ao 4° quadrante e que $\cos x = 0{,}8$, pode-se afirmar que o valor de $sen\ 2x$ é igual a:

a) $0{,}28$

b) $-0{,}96$

c) $-0{,}28$

d) $0{,}96$

e) 1

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(ESA/2016)

Sabendo que x pertence ao 4° quadrante e que $\cos x = 0{,}8$, pode-se afirmar que o valor de $sen\ 2x$ é igual a:

a) $0{,}28$

b) $-0{,}96$

c) $-0{,}28$

d) $0{,}96$

e) 1

(ESA/2012)

A soma dos valores m que satisfazem as igualdades $\operatorname{sen} x = \frac{m+1}{m}$ e $\cos x = \frac{m+2}{m}$ é:

a) 5

b) 6

c) 4

d) -4

e) -6

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(ESA/2012)

A soma dos valores m que satisfazem as igualdades $\operatorname{sen} x = \frac{m+1}{m}$ e $\cos x = \frac{m+2}{m}$ é:

a) 5

b) 6

c) 4

d) -4

e) -6

(EEAR/2019)

Simplificando a expressão $\text{sen}(2\pi - x) + \text{sen}(3\pi + x)$, obtém-se

a) $\text{sen}\, x$

b) $-\text{sen}\, x$

c) $2\,\text{sen}\, x$

d) $-2\,\text{sen}\, x$

(EEAR/2019)

Gabriel verificou que a medida de um ângulo é $\frac{3\pi}{10}$ rad. Essa medida é igual a

a) $48°$

b) $54°$

c) $66°$

d) $72°$

(EEAR/2018)

O valor de $\operatorname{sen} 1270°$ é igual a

a) $-\cos 10°$

b) $-\operatorname{sen} 30°$

c) $-\operatorname{sen} 10°$

d) $-\cos 30°$

(EEAR/2016)

O valor correspondente ao $\cos 15°$ é

a) $\frac{\sqrt{2}+\sqrt{6}}{4}$

b) $\frac{\sqrt{2}+\sqrt{3}}{2}$

c) $\frac{\sqrt{3}}{4}$

d) 1

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Questões ITA/IME

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(ITA/2019)

Seja $f: [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ a função definida por $f(x) = arcsen(x)$. Então, a soma $\sum_{n=0}^{4} f\left(\cos\left(\frac{2\pi}{3^n}\right)\right)$ é igual a

a) $\frac{253}{162}\pi$

b) $\frac{245}{162}\pi$

c) $-\frac{152}{81}\pi$

d) $-\frac{82}{81}\pi$

e) $-\frac{79}{162}\pi$

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(ITA/2019)

Seja $f: [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ a função definida por $f(x) = arcsen(x)$. Então, a soma $\sum_{n=0}^{4} f\left(\cos\left(\frac{2\pi}{3^n}\right)\right)$ é igual a

a) $\frac{253}{162}\pi$

b) $\frac{245}{162}\pi$

c) $-\frac{152}{81}\pi$

d) $-\frac{82}{81}\pi$

e) $-\frac{79}{162}\pi$

(ITA/2019)

Sejam a, b e c três números reais em progressão aritmética crescente, satisfazendo

$$\cos a + \cos b + \cos c = 0 \text{ e } \operatorname{sen} a + \operatorname{sen} b + \operatorname{sen} c = 0.$$

Encontre a menor razão possível para essa progressão aritmética.

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(IME/2020)

Seja $\frac{1}{b} = sen\frac{\pi}{14} \cdot sen\frac{3\pi}{14} \cdot sen\frac{5\pi}{14}$. Determine b, onde b pertence ao conjunto dos números inteiros não nulos.

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Obrigado

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