1. $\binom{6}{2} = C_6^2 = \frac{6!}{2!(6-2)!} = \frac{6!}{2!\,4!} = \frac{6.5.4!}{2.4!}$

$\boxed{\binom{6}{2} = 15}$

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2. $\binom{7}{3} = C_7^3 = \frac{7!}{3!(7-3)!} = \frac{7!}{3!\,4!} = \frac{7.6.5.4!}{3.2.4!}$

$\binom{7}{3} = \frac{7.6.5}{6} \Rightarrow \boxed{\binom{7}{3} = 35}$

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3. $\binom{6}{0} = C_6^0 = \frac{6!}{0!(6-0)!} = \frac{6!}{1.6!} = 1$

POR DEFINIÇÃO

$\boxed{\binom{6}{0} = 1}$

obs.: $\binom{x}{x} = 1$ e $\binom{x}{0} = 1$

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4. $\binom{m}{9} = \binom{m}{8} \Rightarrow 9 + 8 = m$

$m = 17$

$\binom{m}{17} = \binom{17}{17} = \frac{17!}{17!\,0!} = \frac{1}{0!} = \frac{1}{1} = 1$

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5. $\binom{20}{2x} = \binom{20}{x+1}$

$2x = x + 1$ ou $2x + x + 1 = 20$

$x = 1$ $\qquad 3x = 19$

$x = 19/3$

NÃO CONVÉM

Logo, $\boxed{x = 1}$

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6. $\frac{\binom{12}{4}}{\binom{12}{5}} = \frac{C_{12}^4}{C_{12}^5} = \frac{\frac{12!}{4!(12-4)!}}{\frac{12!}{5!(12-5)!}} =$

$= \frac{12!}{4!\,8!} \cdot \frac{5!\,7!}{12!} = \frac{5!\,7!}{4!\,8!} = \frac{5.4!\,7!}{4!\,8.7!} = \boxed{5/8}$

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7. $\frac{\binom{m}{p}}{1} = \frac{\binom{m}{p+1}}{2} = \frac{\binom{m}{p+2}}{3}$

I II

(I) MULTIPLICANDO CRUZADO:

$2\binom{m}{p} = \binom{m}{p+1} = 2 \cdot \frac{m!}{p!(m-p)!} = \frac{m!}{(p+1)!(m-p-1)!} =$

$= 2\frac{(p+1)!}{p!} = \frac{(m-p)!}{(m-p-1)!} = 2\frac{(p+1)p!}{p!} = \frac{(m-p)(m-p-1)!}{(m-p-1)!}$

$= 2p + 2 = m - p \Rightarrow \boxed{m = 3p + 2}$ ✱

(II) $3\binom{m}{p} = \binom{m}{p+2} \Rightarrow 3\frac{m!}{p!(m-p)!} = \frac{m!}{(p+2)!(m-p-2)!}$

$= 3 \cdot \frac{(p+2)!}{p!} = \frac{(m-p)!}{(m-p-2)!} =$

$= 3 \cdot \frac{(p+2)(p+1)p!}{p!} = \frac{(m-p)(m-p-1)(m-p-2)!}{(m-p-2)!} =$

✱

$3(p+2)(p+1) = (m-p)(m-p-1)$

$3(p^2 + 3p + 2) = (3p + 2 - p)(3p + 2 - p - 1)$

$3p^2 + 9p + 6 = (2p + 2)(2p + 1)$

$3p^2 + 9p + 6 = 4p^2 + 6p + 2$

$p^2 - 3p - 4 = 0$

$p' = 4$

$p'' = -1$ (NÃO CONVÉM)

Logo, $p = 4$ e $m = 3p + 2$

$m = 3.4 + 2$

$m = 14$