

Calcule o determinante de cada matriz:

1.  $\begin{pmatrix} 3 & 12 \\ 2 & 9 \end{pmatrix}$

$$D = \begin{vmatrix} 3 & 12 \\ 2 & 9 \end{vmatrix} \quad 27 - 24 \rightarrow \boxed{D = 3}$$

2.  $\begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{4} \end{pmatrix}$

$$D = \begin{vmatrix} 1 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{4} \end{vmatrix} \quad \frac{1}{4} - \frac{1}{6} \quad \text{MMC} = 12 \quad D = \frac{3 - 2}{12} \rightarrow \boxed{D = \frac{1}{12}}$$

3. Calcule o determinante da matriz  $A = (a_{ij})_{2 \times 2}$  definida por  $a_{ij} = i^2 + 2j$ .

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \quad D = \begin{vmatrix} (1^2 + 2 \cdot 1) & (1^2 + 2 \cdot 2) \\ (2^2 + 2 \cdot 1) & (2^2 + 2 \cdot 2) \end{vmatrix}$$

$$D = \begin{vmatrix} 3 + 2 & 1 + 4 \\ 4 + 2 & 4 + 4 \end{vmatrix} \quad D = \begin{vmatrix} 5 & 5 \\ 6 & 8 \end{vmatrix} \quad 24 - 30 \rightarrow \boxed{D = -6}$$

Calcule x em cada equação:

4.  $\begin{vmatrix} x & 1 \\ 1 & 2 \end{vmatrix} = 3$   $2x - 1 = 3 \rightarrow 2x = 3 + 1 \rightarrow 2x = 4$   
 $x = \frac{4}{2} \rightarrow \boxed{x = 2}$

5.  $\begin{vmatrix} 1 & x \\ 2x & x^2 + 1 \end{vmatrix} = 0$   
 $x^2 + 1 - 2x \cdot x = 0$   
 $x^2 + 1 - 2x^2 = 0$   
 $-x^2 = -1$   
 $x = \sqrt{1} \rightarrow \boxed{x = \pm 1}$

6. Para que valor de k a matriz  $\begin{bmatrix} 1 & 1-k \\ 1 & 2+k \end{bmatrix}$  tem determinante nulo?

$$\begin{vmatrix} 1 & 1-k \\ 1 & 2+k \end{vmatrix} \quad 2+k - (1-k) = 0 \quad 2 - 1 + k + k = 0$$

$$2k = -1 \rightarrow \boxed{k = -\frac{1}{2}}$$

Calcule os determinantes:

7.  $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 5 \end{pmatrix}$

$$D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 5 \end{vmatrix} \quad D = 30 + 1 + 1 - 3 - 2 - 5$$

$$\boxed{D = 22}$$

8.  $\begin{pmatrix} 4 & 2 & 1 \\ 9 & 3 & 1 \\ 1 & -1 & 1 \end{pmatrix}$

$$D = \begin{vmatrix} 4 & 2 & 1 \\ 9 & 3 & 1 \\ 1 & -1 & 1 \end{vmatrix} \quad D = 12 + 2 - 9 - 3 + 4 - 18$$

$$\boxed{D = -12}$$

9. Calcule m para que se verifique a igualdade

$$\begin{vmatrix} 1 & m & 2 \\ 0 & -1 & 3 \\ 2 & m & 4 \end{vmatrix} = 111$$

$$D = \begin{vmatrix} 1 & m & 2 \\ 0 & -1 & 3 \\ 2 & m & 4 \end{vmatrix} \quad D = -4 + 6m + 4 - 3m$$

$$6m - 3m = 111 \rightarrow 3m = 111$$

$$m = \frac{111}{3} \rightarrow \boxed{m = 37}$$

10. Resolva a equação:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & x & x \\ 1 & x & 4 \end{vmatrix} = 0$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & x & x \\ 1 & x & 4 \end{vmatrix} \quad D = 4x + x + x - x - x^2 - 4$$

$$0 = -x^2 + 5x - 4$$

$$x^2 - 5x + 4 = 0$$

$$\frac{1}{1} + \frac{4}{4} = -b/a = 5$$

$$\frac{1}{1} \cdot \frac{4}{4} = c/a = 4$$

$$\boxed{x = 1 \text{ ou } x = 4}$$

11. Calcule o determinante da matriz quadrada  $A = (a_{ij})$ , de ordem 3, em que:

$$a_{ij} = \begin{cases} 1, \text{ se } i < j \\ i + j, \text{ se } i \geq j \end{cases}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\begin{vmatrix} 1+1 & 1 & 1 \\ 2+1 & 2+2 & 1 \\ 3+1 & 3+2 & 3+3 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 1 & 1 \\ 3 & 4 & 1 \\ 4 & 5 & 6 \end{vmatrix}$$

$$D = 48 + 4 + 15 - 16 - 10 - 18$$

$$\boxed{D = 23}$$