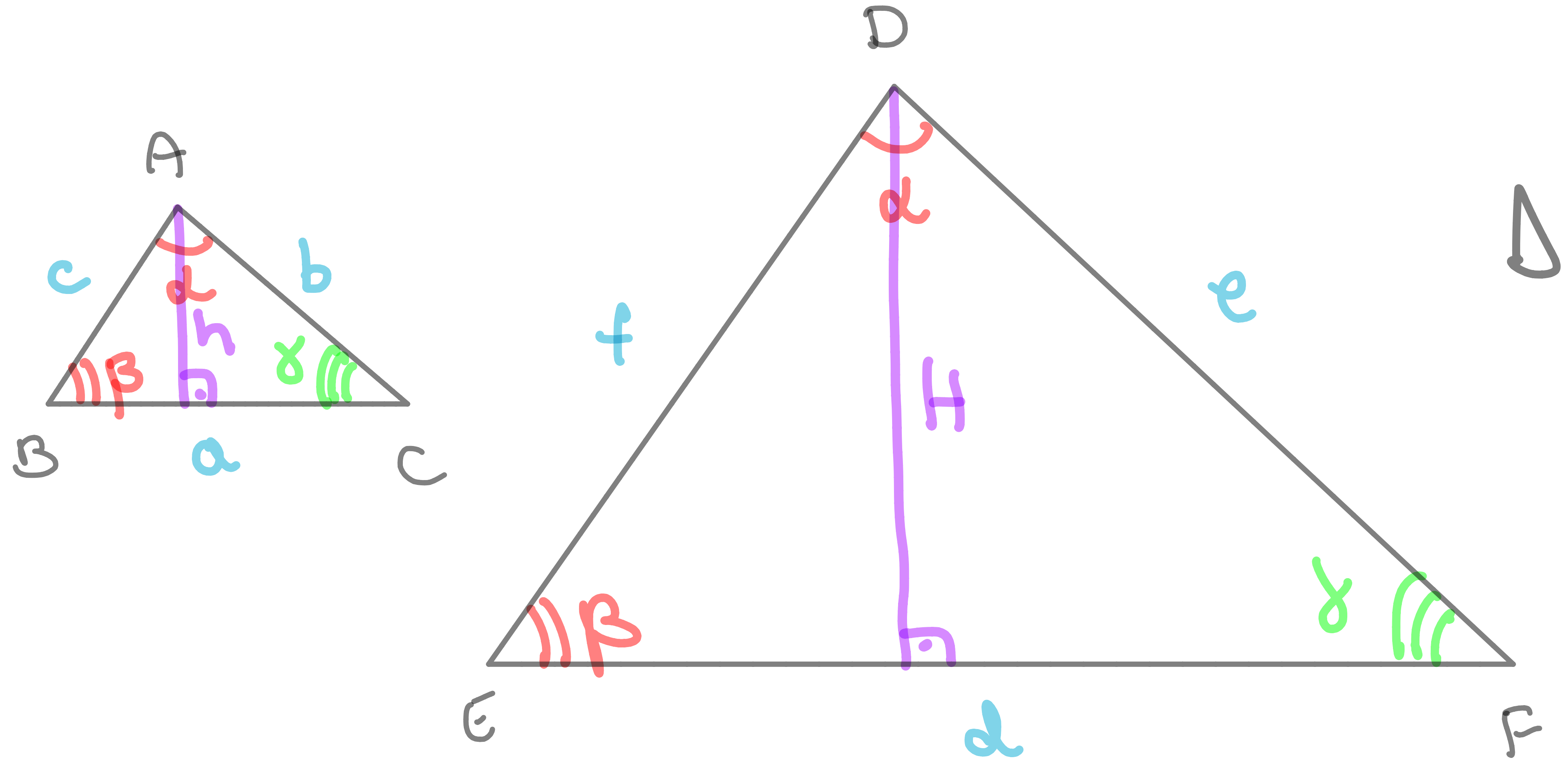
Semelhança de Δs (Importantíssimo %) !!



$$\triangle ABC \sim \triangle DEF \Rightarrow \begin{cases} \underbrace{\hat{A} = \hat{D} \text{ e } \hat{B} = \hat{E}}_{\text{A.A (ângulo-ângulo)}} \Rightarrow \hat{C} = \hat{F} \\ \text{e} \\ \dfrac{a}{d} = \dfrac{b}{e} = \dfrac{c}{f} = K = \dfrac{h}{H} = \dfrac{2p_1}{2p_2} \end{cases}$$

$K \rightarrow$ Razão de semelhança!

OBS: E as áreas?

$$\boxed{\frac{A_{\triangle ABC}}{A_{\triangle DEF}} = K^2}$$

Por quê?

$$\frac{\frac{a \cdot h}{\cancel{2}}}{\frac{d \cdot H}{\cancel{2}}} = \underbrace{\frac{a}{d}}_{K} \cdot \underbrace{\frac{h}{H}}_{K} = K^2 \;!!$$

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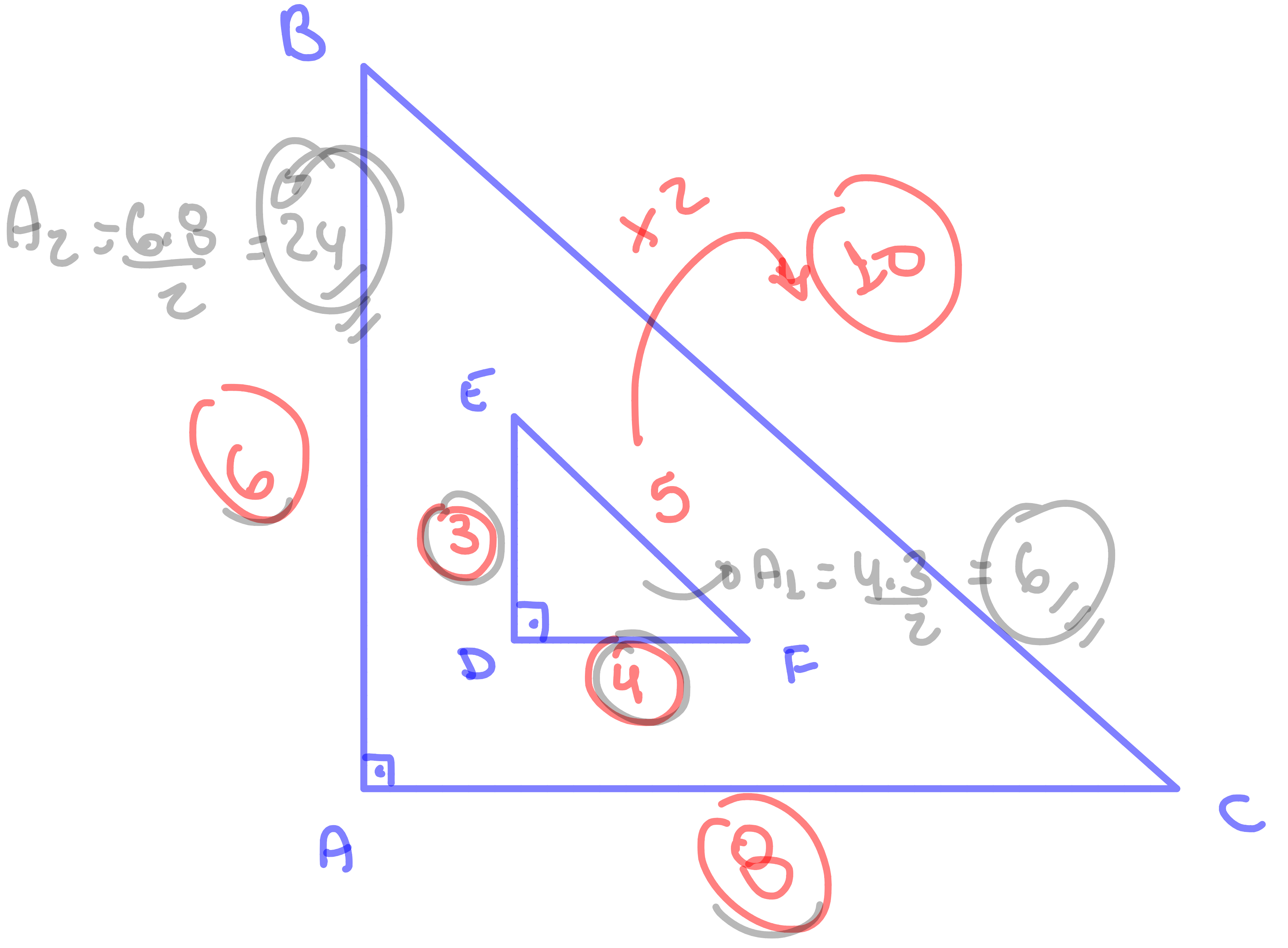
OBS: Na prática temos:



$\Delta ABC \sim \Delta DEF$

$\Downarrow$

$$\frac{5}{10} = \frac{4}{8} = \frac{3}{6} = K = \frac{1}{2}$$

OBS: E as áreas?

$$\frac{A_{\Delta 1}}{A_{\Delta 2}} = K^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

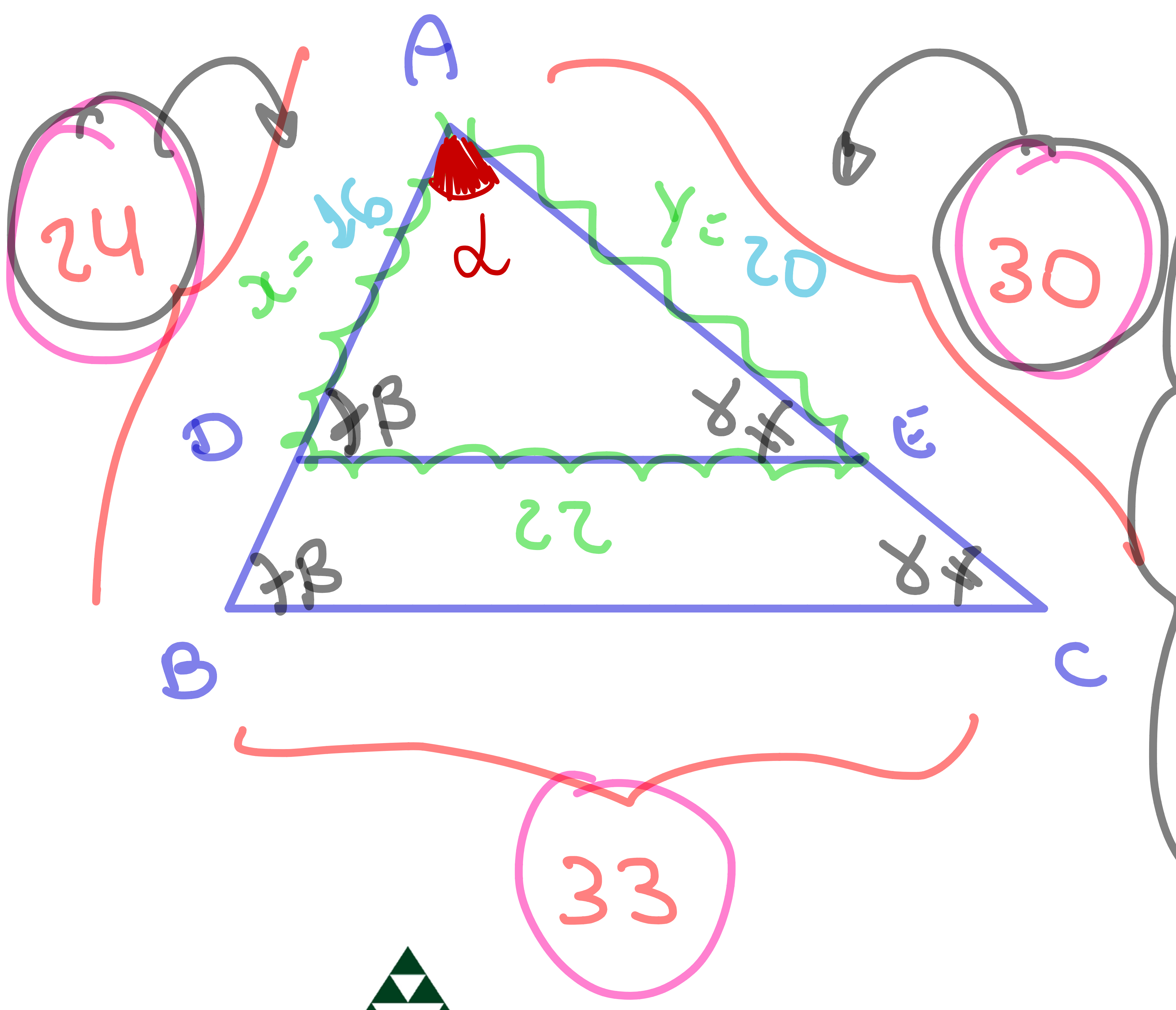
$$\boxed{A_{\Delta 2} = 4 \cdot A_{\Delta 1}}$$

Exemplo 1

(BÁSICÃO)

- $\overline{DE} \parallel \overline{BC}$
- $\overline{DE} = 22$
- $2p_{\Delta ADE} = ?$

$$2p_{\Delta ADE} = 16 + 20 + 22 = 58$$



$\Delta ADE \sim \Delta ABC$

$(\overline{DE} \parallel \overline{BC})$

$$\frac{22}{33} = \frac{x}{24} = \frac{y}{30} = K$$

$$\frac{22}{33} = \frac{2}{3}$$

$$\frac{x}{24} = \frac{2}{3} \Rightarrow \boxed{x = 16}$$

$$\frac{y}{30} = \frac{2}{3} \Rightarrow \boxed{y = 20}$$

ou (Progressivo!)

$$\frac{2p_{\Delta ADE}}{2p_{\Delta ABC}} = K = \frac{2}{3}$$

$$\frac{2p_{\Delta ADE}}{87} = \frac{2}{3}$$

$$\boxed{2p_{\Delta ADE} = 58}$$

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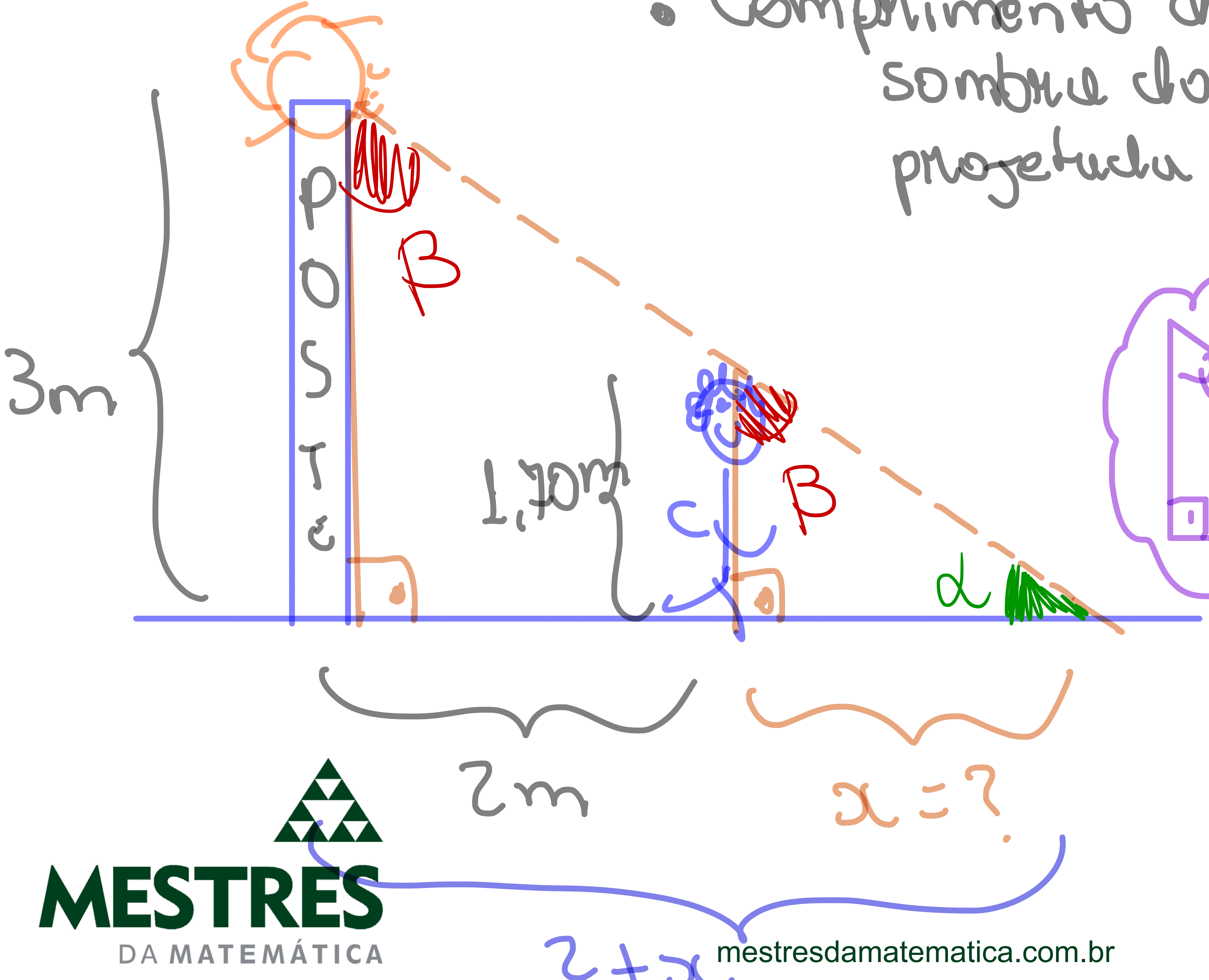
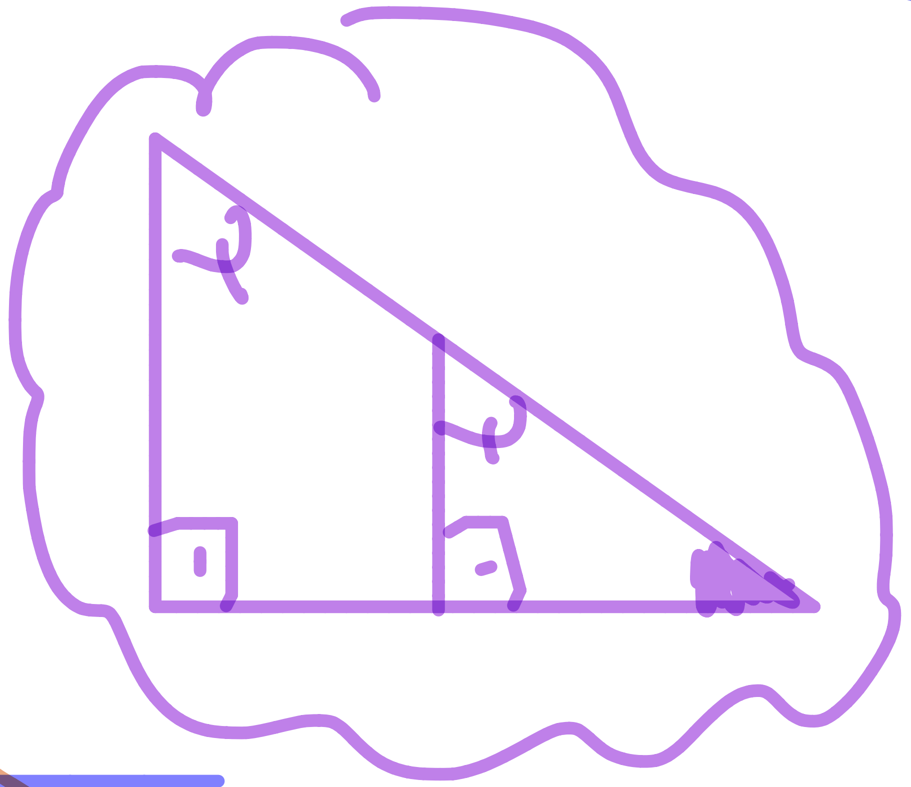
Exemplo 2

(CLÁSSICA)

- $H_{poste} = 3m$
- $h_{criança} = 1,70m$ (aproximadamente)
- Comprimento da sombra do cidadão projetada no solo = ?
- $d_{\text{poste ao cidadão}} = 2,0m$





manjado

$$3,4 \div 1,3 = 2,61\ldots$$

$$\begin{array}{r} 3,4 \\ -2\,6 \\ \hline 80 \\ -78 \\ \hline 20 \end{array}$$

$\Delta menor \sim \Delta maior$

$$\frac{1,70}{3} = \frac{x}{2+x}$$

$$3x = 1,7(2+x)$$

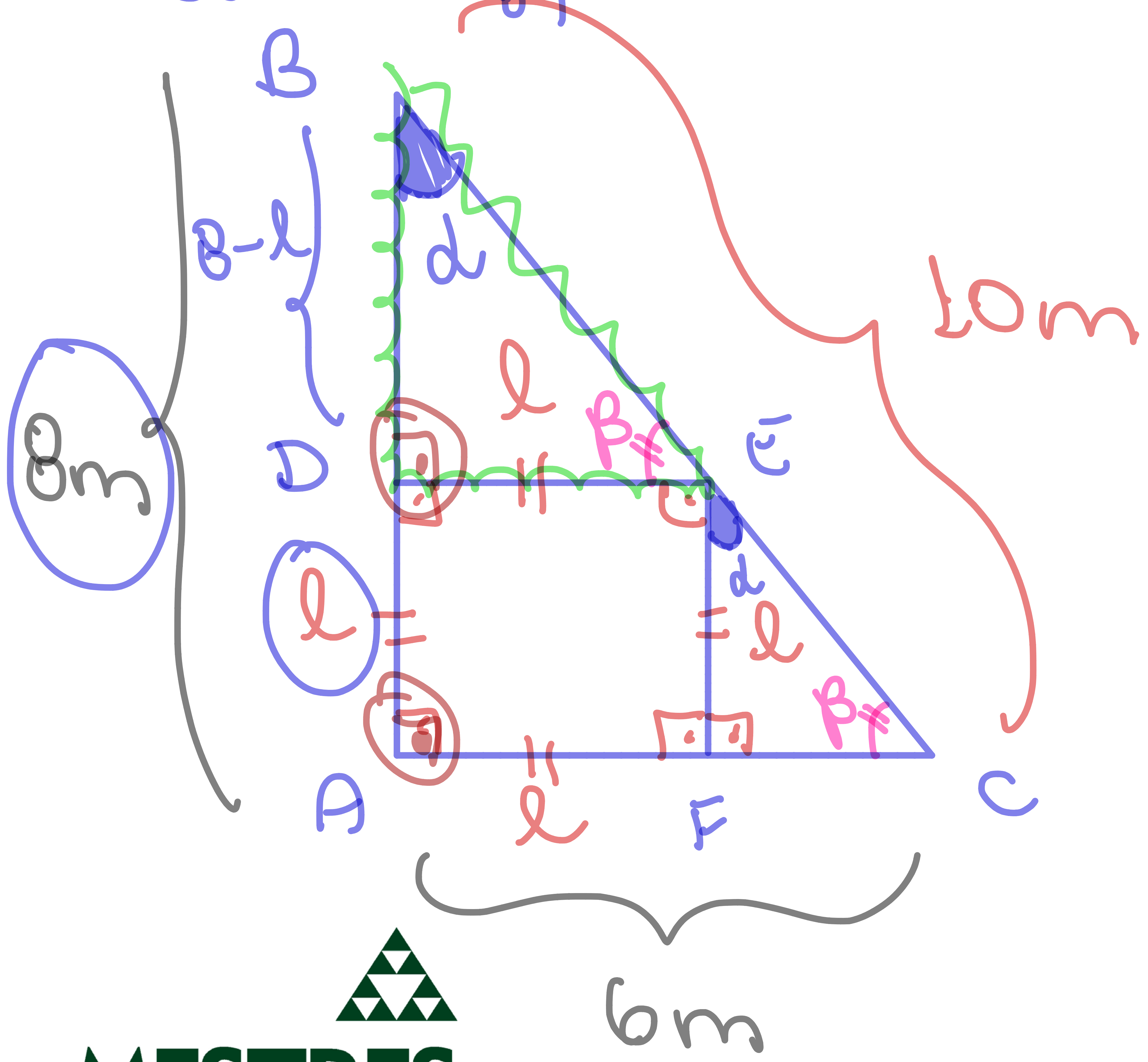
$$3x = 3,4 + 1,7x$$

$$1,3x = 3,4$$

$$x = \frac{3,4}{1,3} \simeq 2,61\,m$$

Exemplo 3 (clássico)

• $2p_{\text{quadrado } ADEF} = ?$



R: $2p_{\square} = 4\ell$

$= 4 \cdot \frac{24}{7}$

$= \frac{96}{7}$ m

$\Delta BDE \sim \Delta BAC$

$\overline{DE} \parallel \overline{AC}$

$$\frac{\ell}{\cancel{6}\,3} = \frac{8-\ell}{\cancel{8}\,4}$$

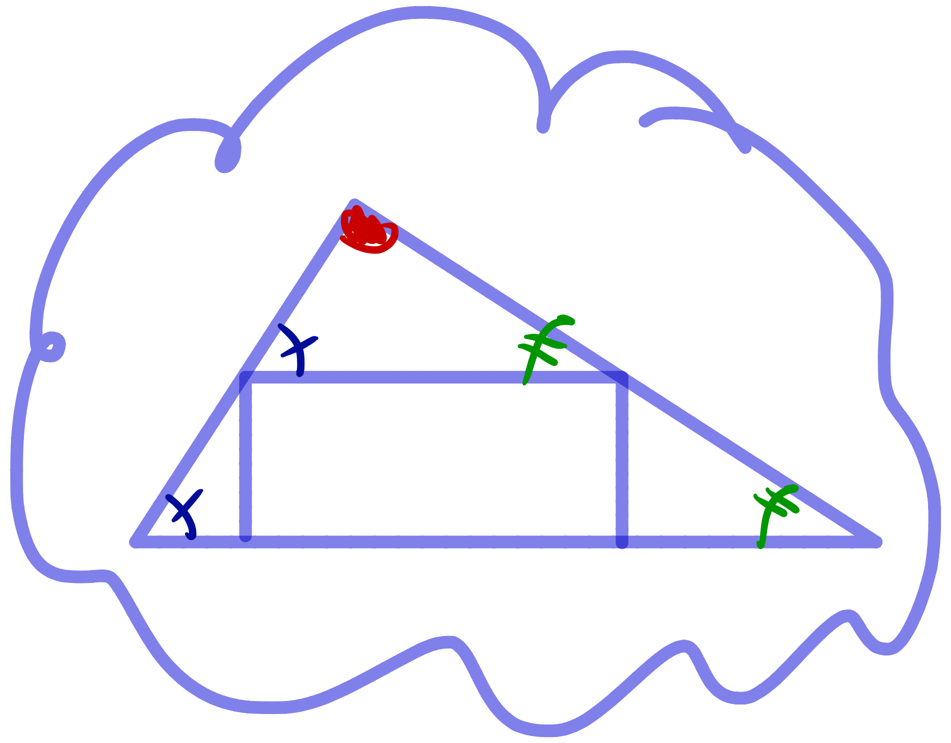
$4\ell = 24 - 3\ell$

$7\ell = 24$

$\ell = 24/7$ m

Exemplo 4

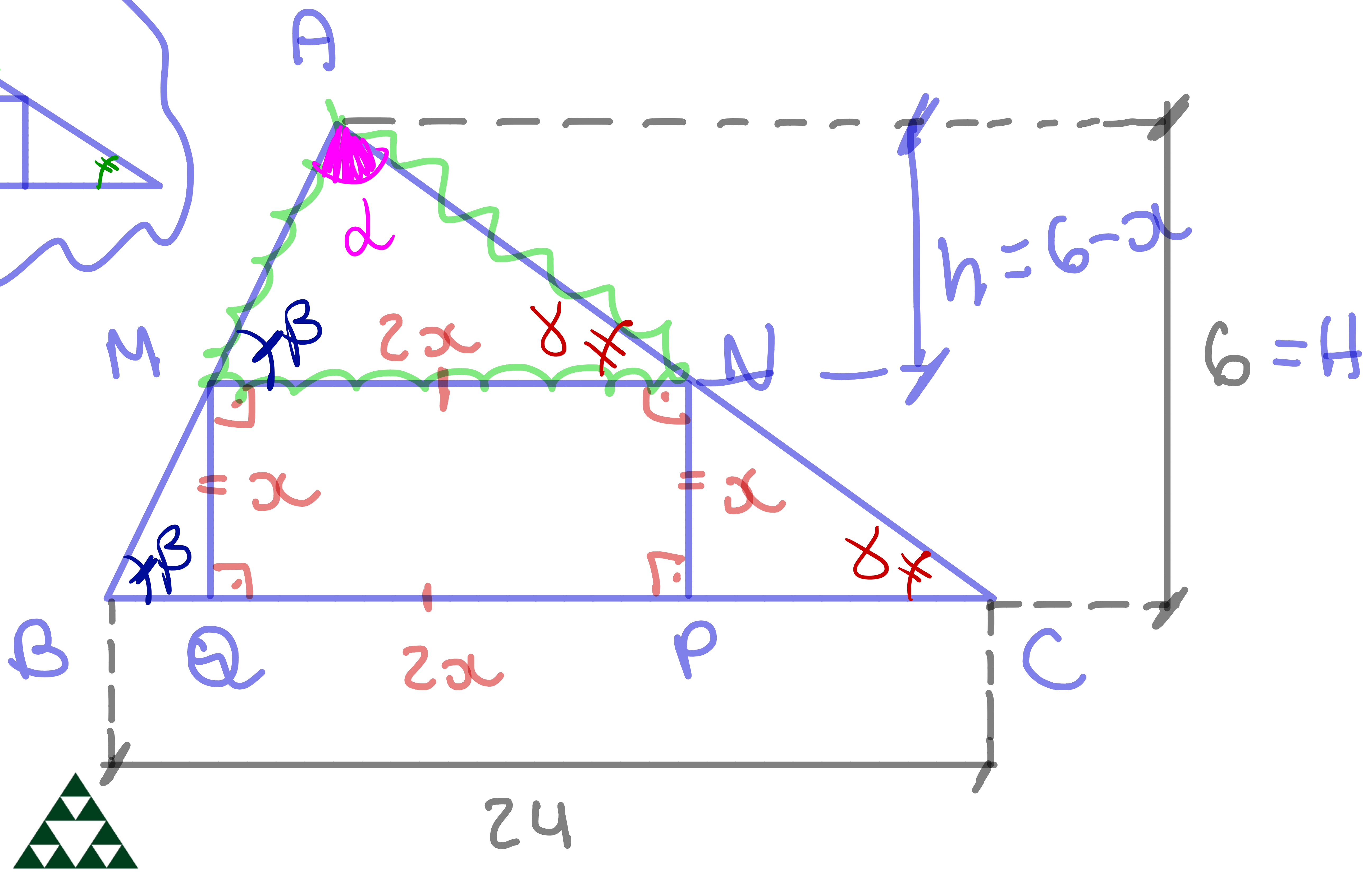
(CLÁSSICA)//



- $\overline{MN} = 2\overline{PN}$
- $2p_{retângulo\ MNPQ} = ?$

$$\boxed{2p_{\square} = 6x}$$

$2p_{\square} = 6 \cdot 4 = 24$ ////



$\Delta AMN \sim \Delta ABC$

$(\overline{MN} // \overline{BC})$

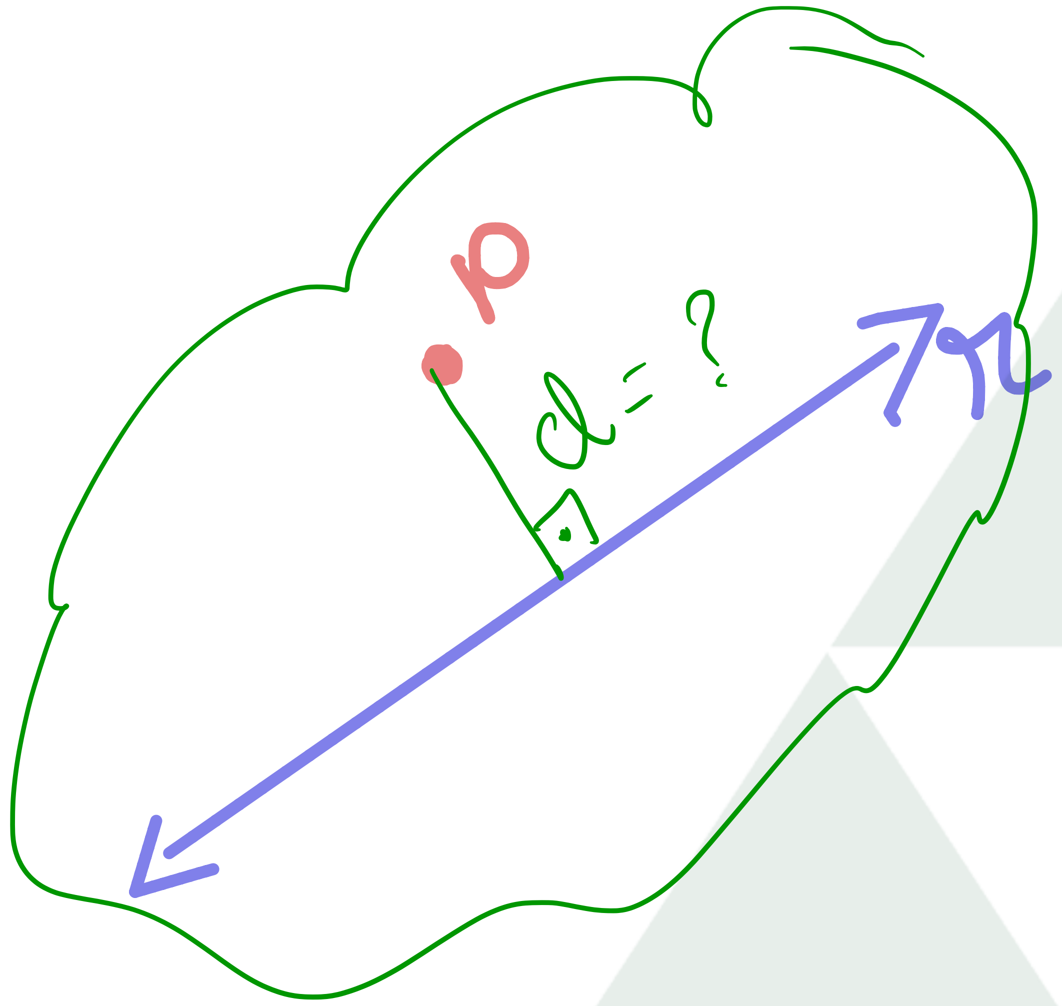
$$\frac{\cancel{2x}^{1}}{\cancel{24}_{12}} = \frac{h}{H} = \frac{6-x}{6}$$

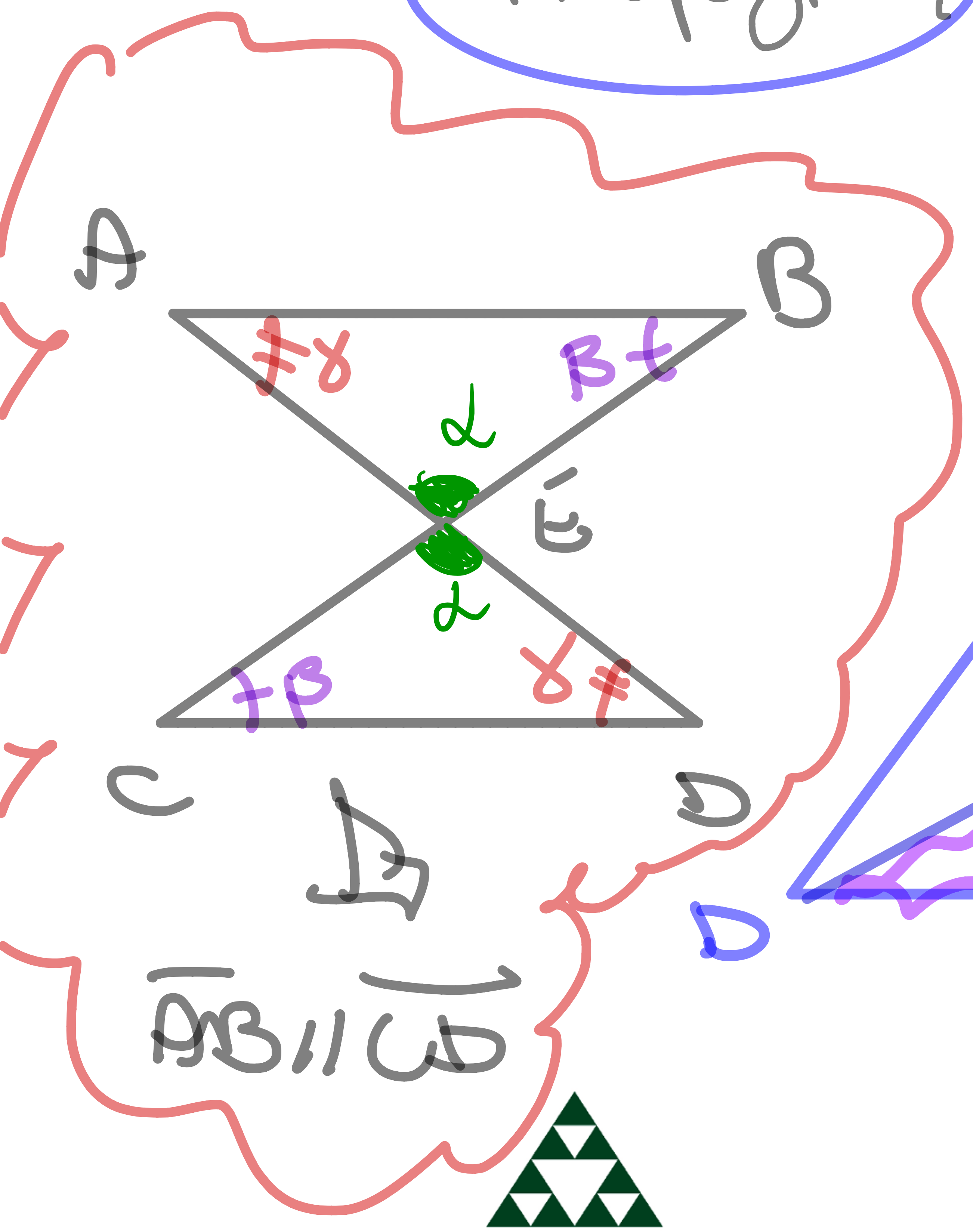
$$\frac{x}{\cancel{12}_{2}} = \frac{6-x}{\cancel{6}_{1}}$$

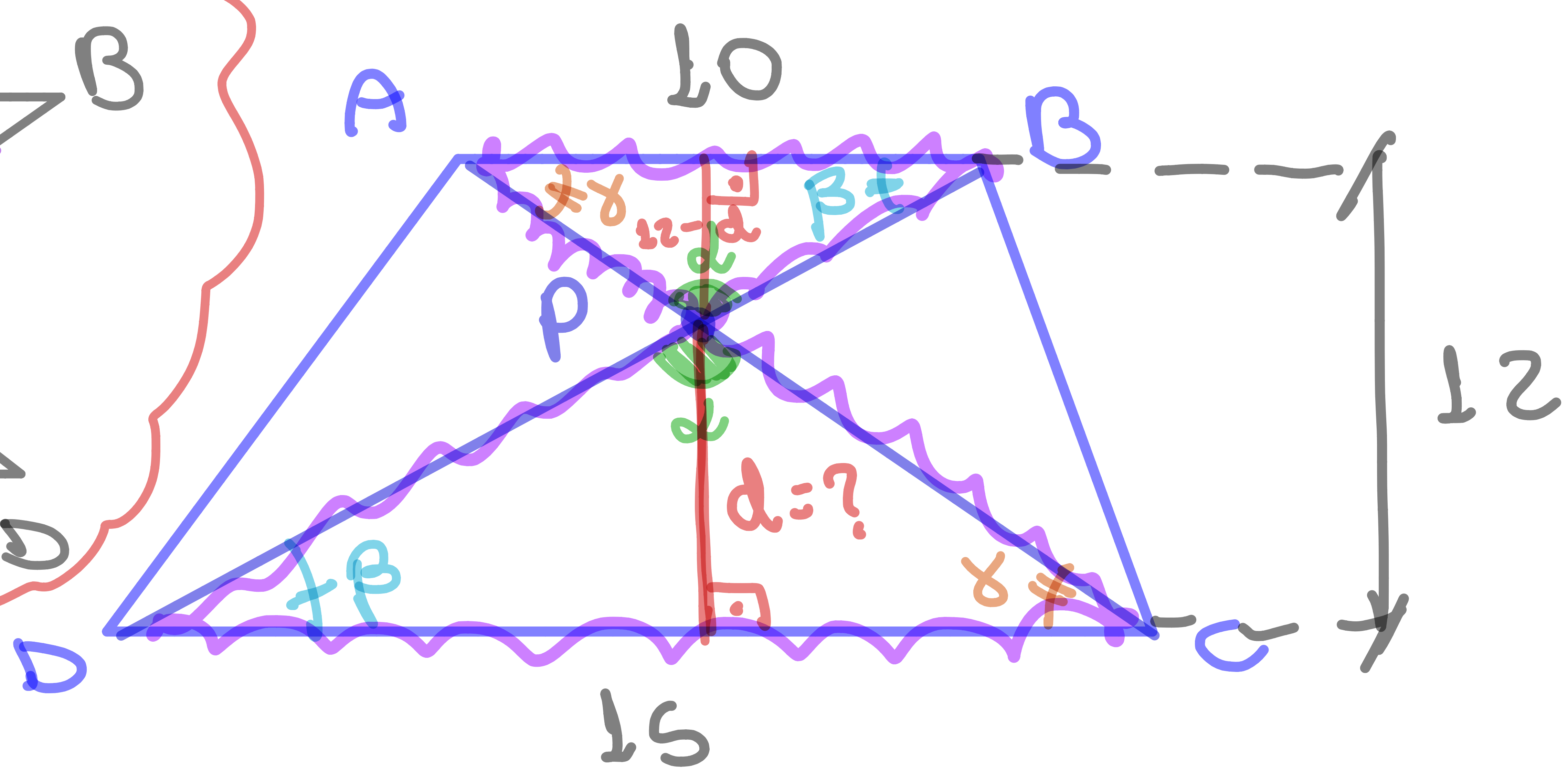
$x = 12 - 2x$

$3x = 12 \Rightarrow x = 4$

Exemplo 5) As bases de um trapézio medem 10cm e 15cm e a altura 12cm. A que $\boxed{\text{DISTÂNCIA}}$ da base MAIOR cortam-se as diagonais desse trapézio?







$\Delta PAB \sim \Delta PCD$

$(\overline{AB} // \overline{DC})$

$\Downarrow$

$$\frac{\cancel{15}^{\,3}}{\cancel{10}_{\,2}} = \frac{d}{12-d} = \frac{h_1}{h_2}$$

$$2d = 36 - 3d$$

$$5d = 36 \Rightarrow d = 7{,}2\,m$$

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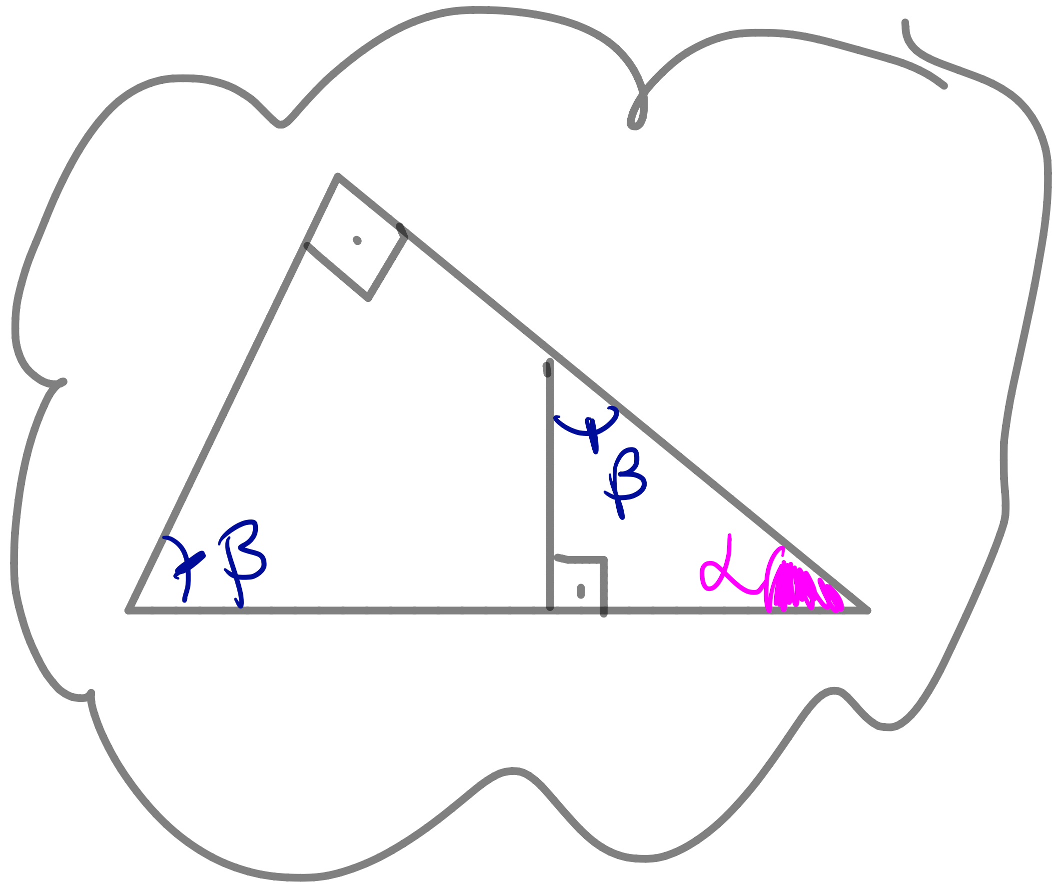
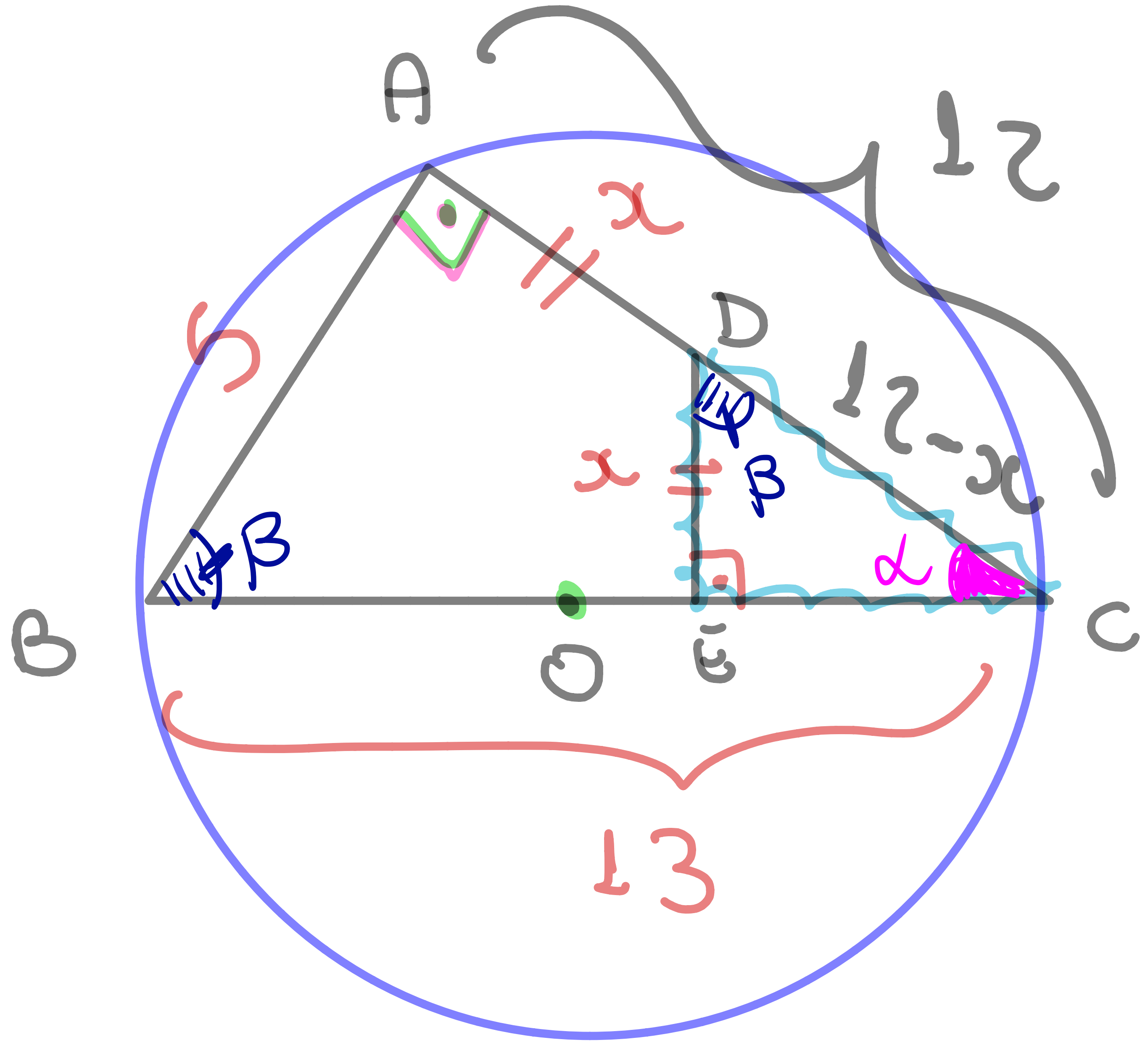
Exemplo 6





- $\overline{AB} = 5cm$
- $\overline{BC} = 13cm$
- $\overline{DE} \perp \overline{BC}$
- $\overline{AD} = \overline{DE} = ?$

$\Delta CED \sim CAB$

(A.A.)

$$\frac{x}{5} = \frac{12-x}{13}$$

$$13x = 60 - 5x$$

$$18x = 60$$

$$x = \frac{60}{18} = \frac{10}{3} cm$$

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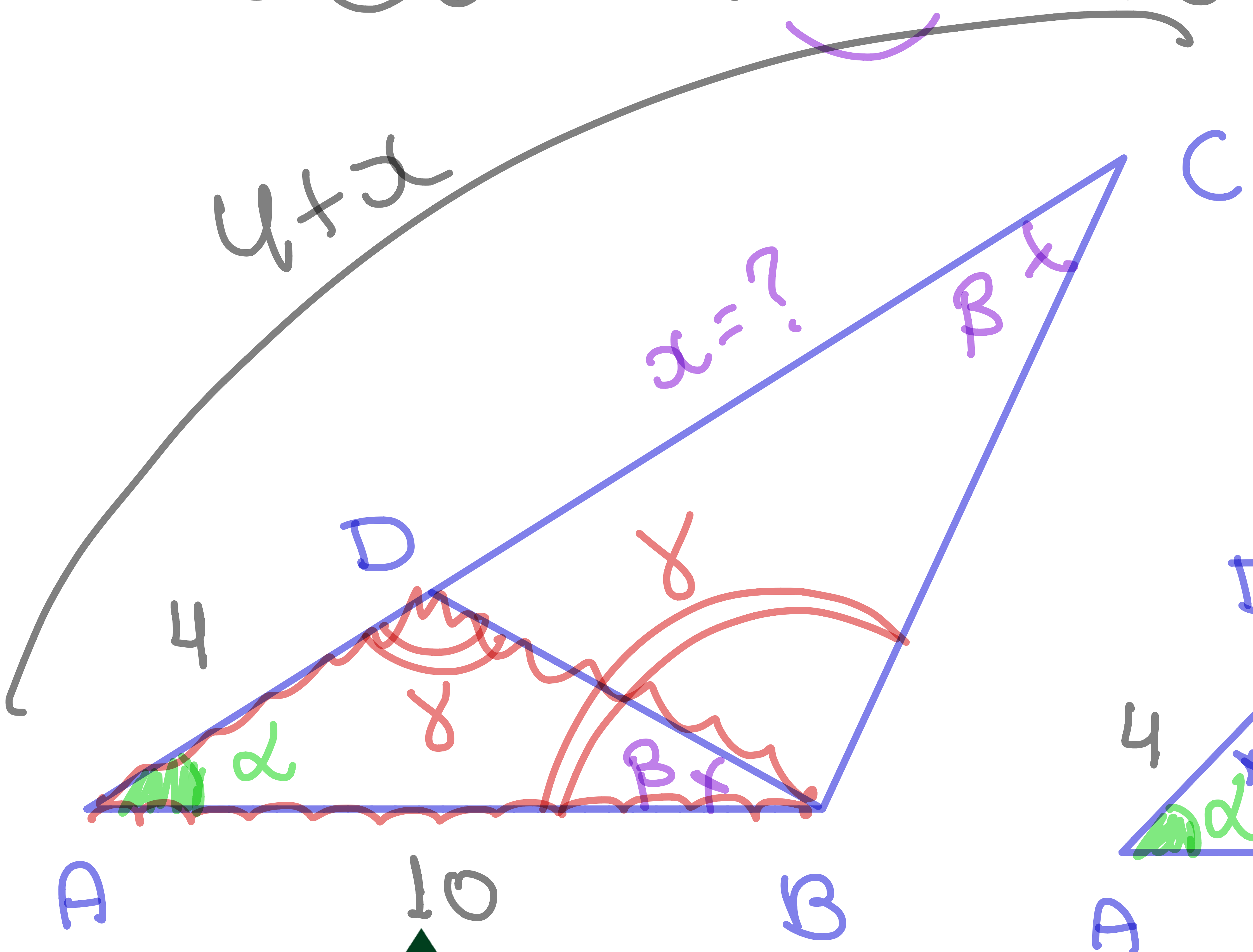
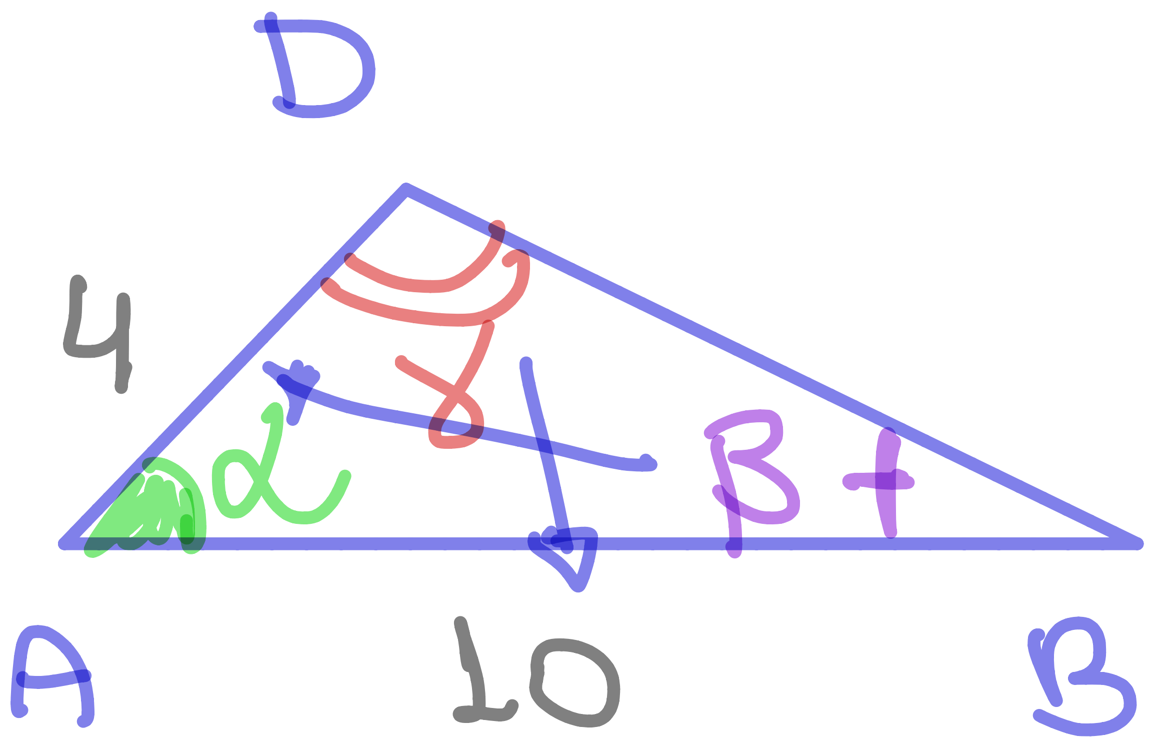
EXEMPLOS + SOFISTICADOS!

Exemplo 9

- $CD = ?$
- $A\hat{B}D = B\hat{C}D$





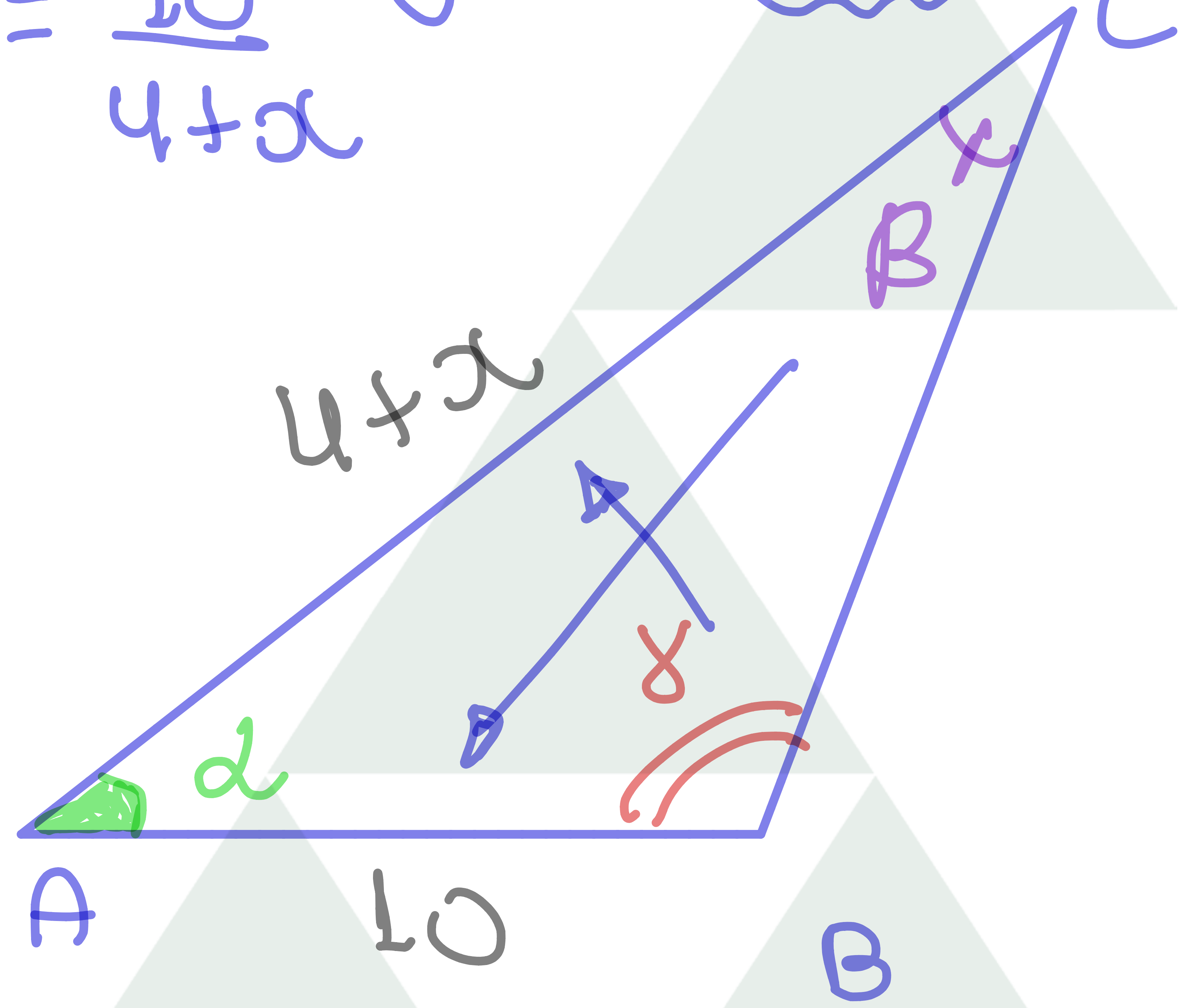
$\Delta ABD \sim \Delta ABC$
(A.A.)

$$\frac{\cancel{4}^{\,2}}{\cancel{10}_{\,5}} = \frac{10}{4+x}$$

$$8 + 2x = 50$$

$$2x = 42$$

$$\boxed{x = 21}$$



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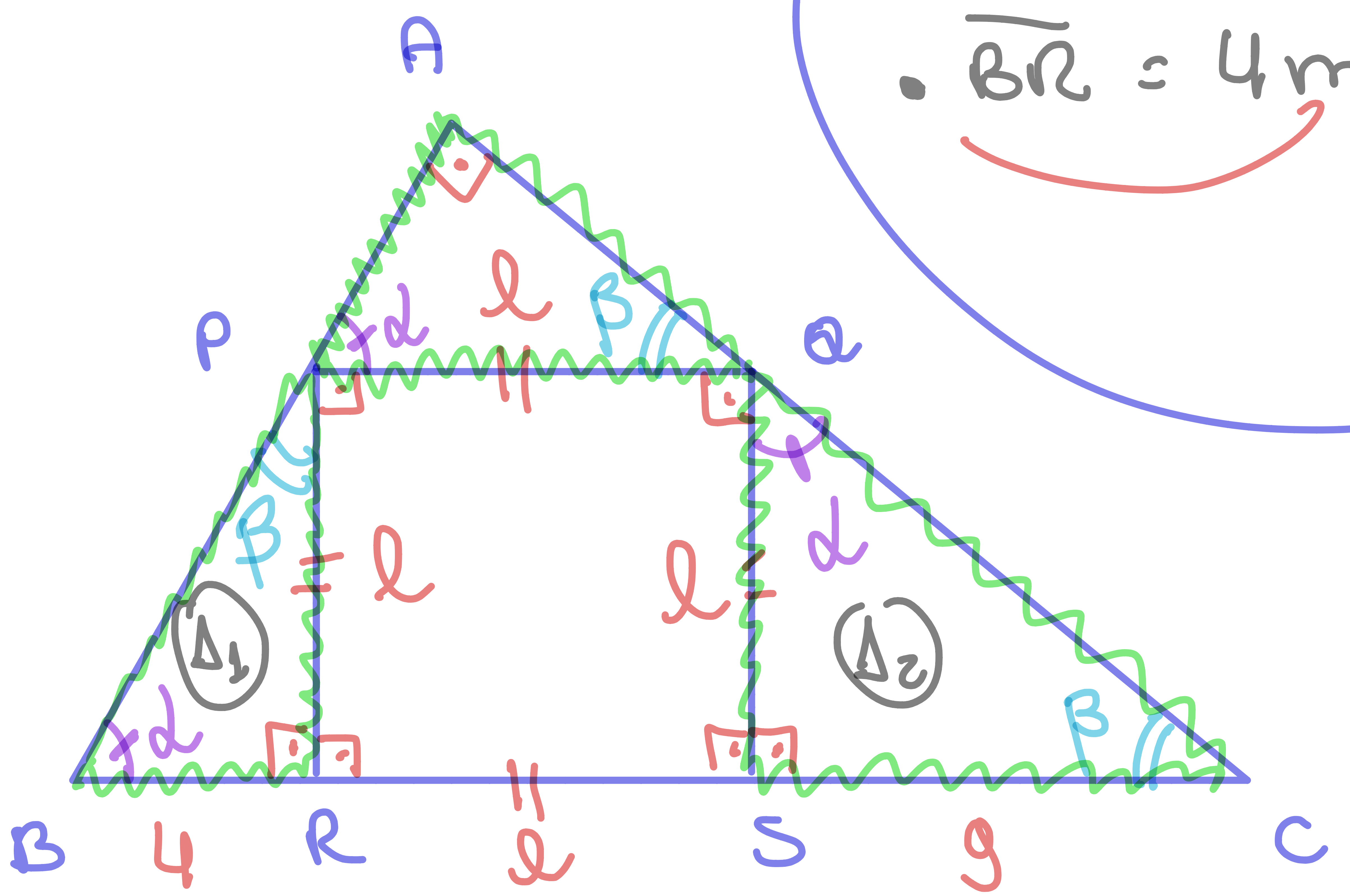
Exemplo 8

- $\overline{AB} \perp \overline{AC}$ → Perpendicular
- $A_{\square}$ pans = ?
- $\overline{BR} = 4m$
- $\overline{SC} = 9m$



$A_{\square} = \ell^2 = 6^2 = 36m^2$

$\Delta_1 \sim \Delta_2$
(A,A)

$\Rightarrow$

$\dfrac{\ell}{9} = \dfrac{4}{\ell} \Rightarrow \ell^2 = 36$

$\ell = \sqrt{36}$

$\ell = 6m$

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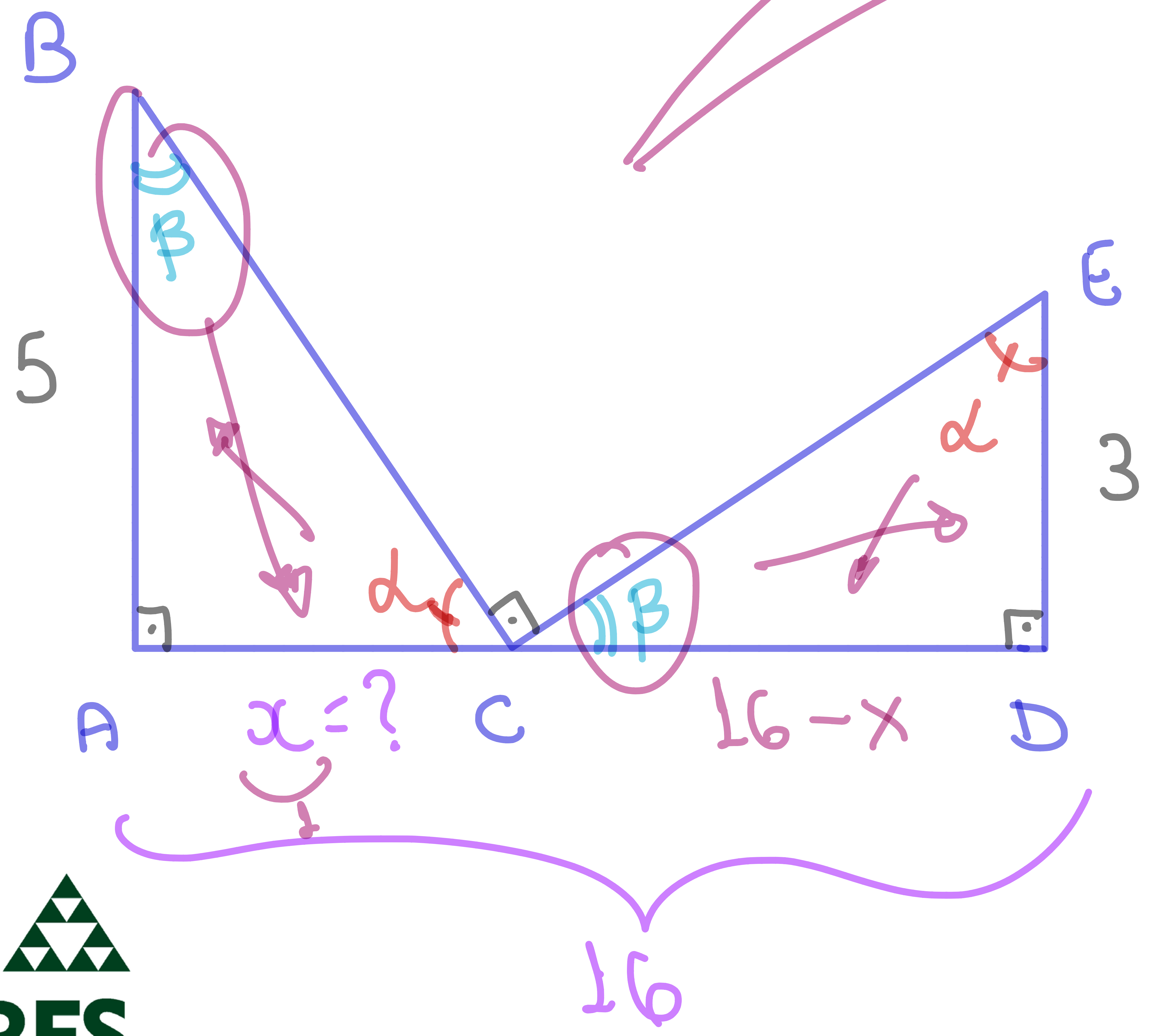
Exemplo 9

- $\overline{AD} = 16$
- $\overline{AC} = ?$



$\Delta ACB \sim \Delta DEC$
(A.A.)

$$\frac{5}{16-x} = \frac{x}{3}$$

$$15 = 16x - x^2$$

$$x^2 - 16x + 15 = 0$$

$x' = 1$ e $x'' = 15$

$$\boxed{S = \{1, 15\}}$$

Exemplo 10

(Desafio %)

Determine a área de um trapézio retângulo de bases iguais a 16 cm e 25 cm cujas diagonais são perpendiculares entre si.

Para casa

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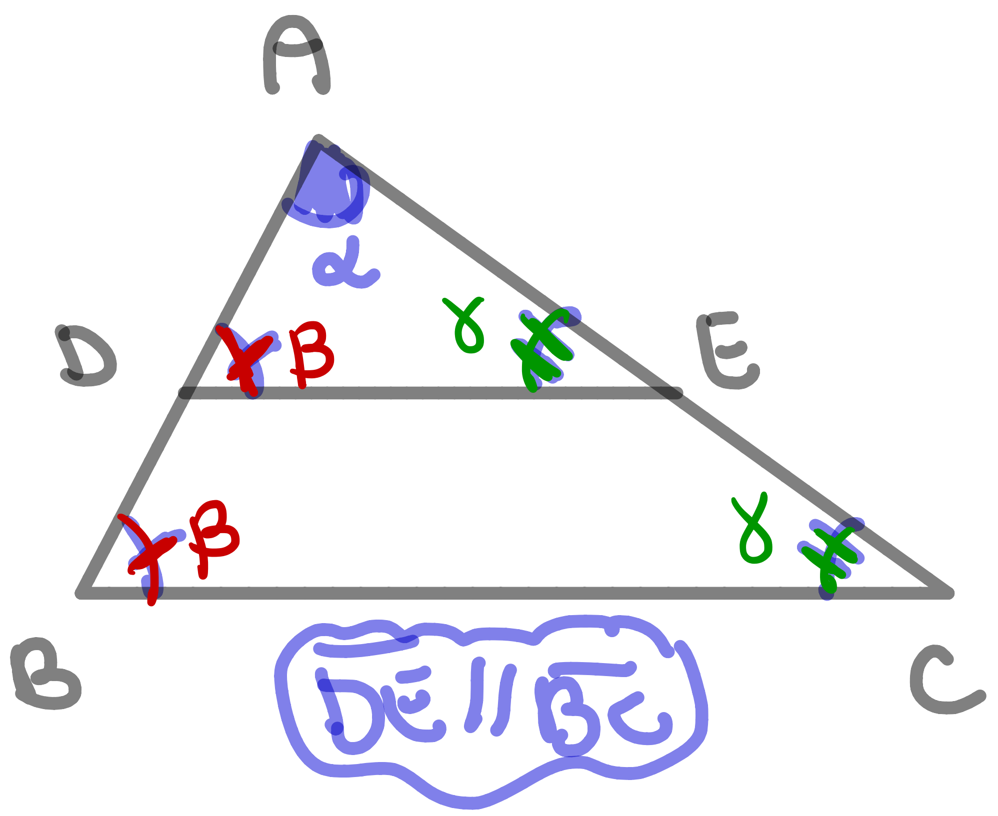
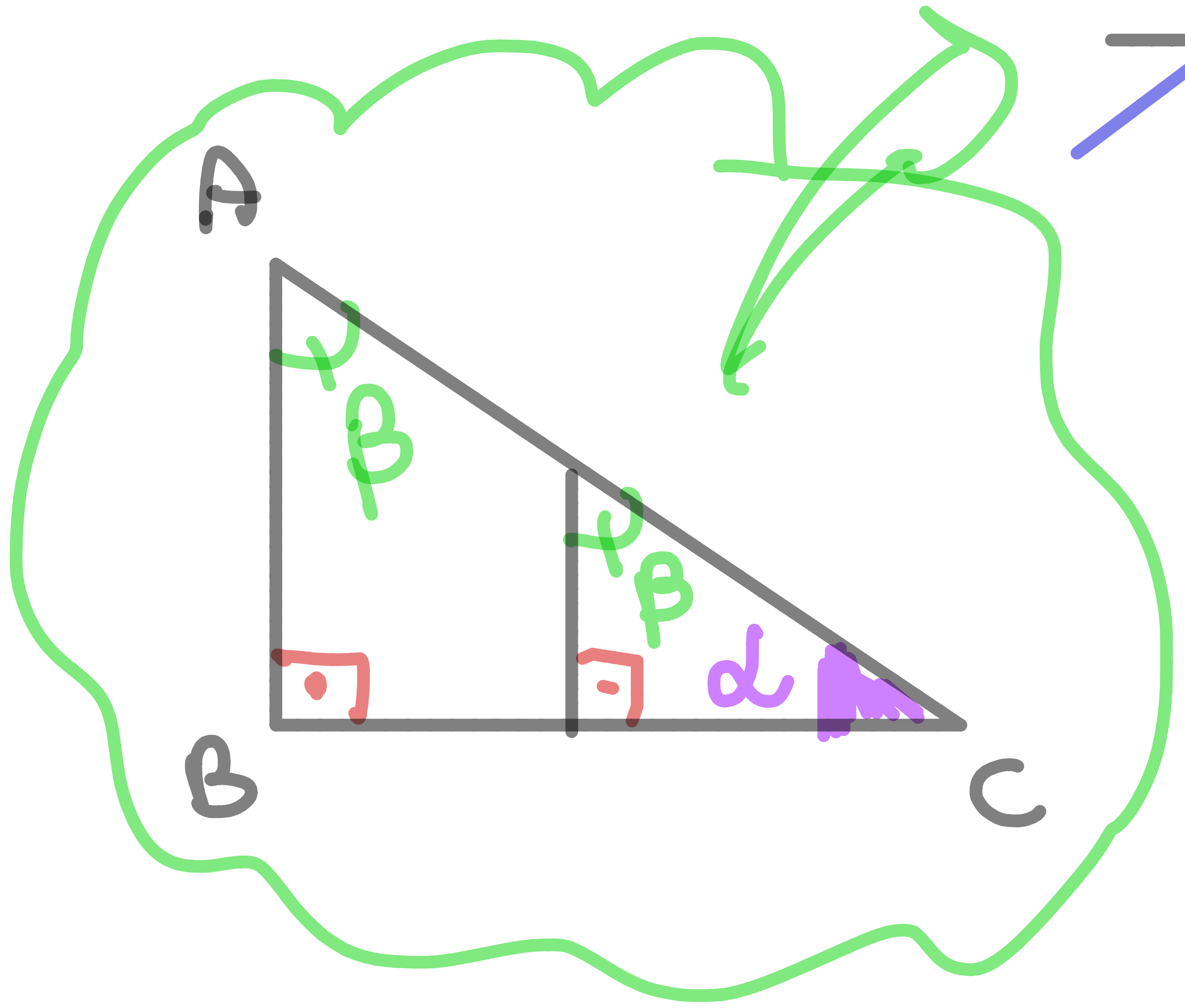
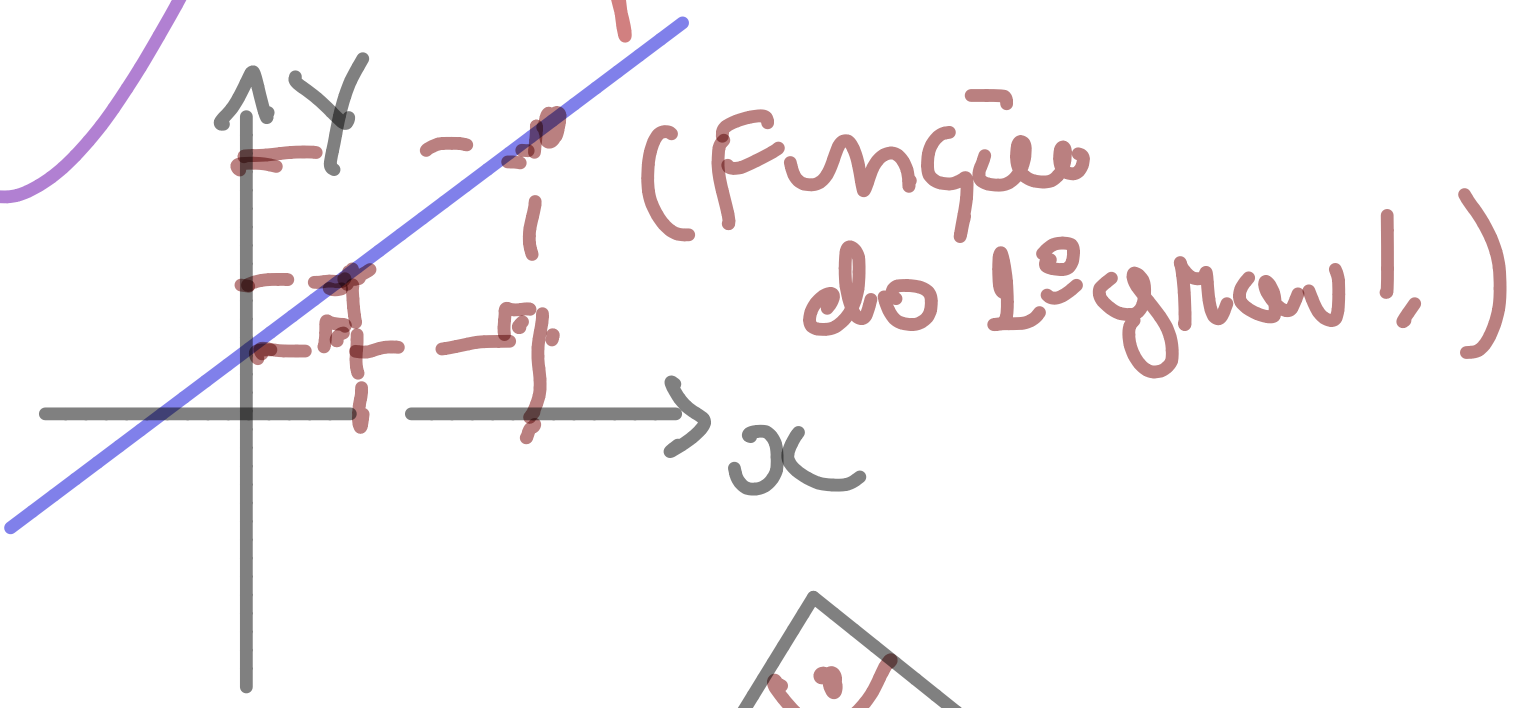
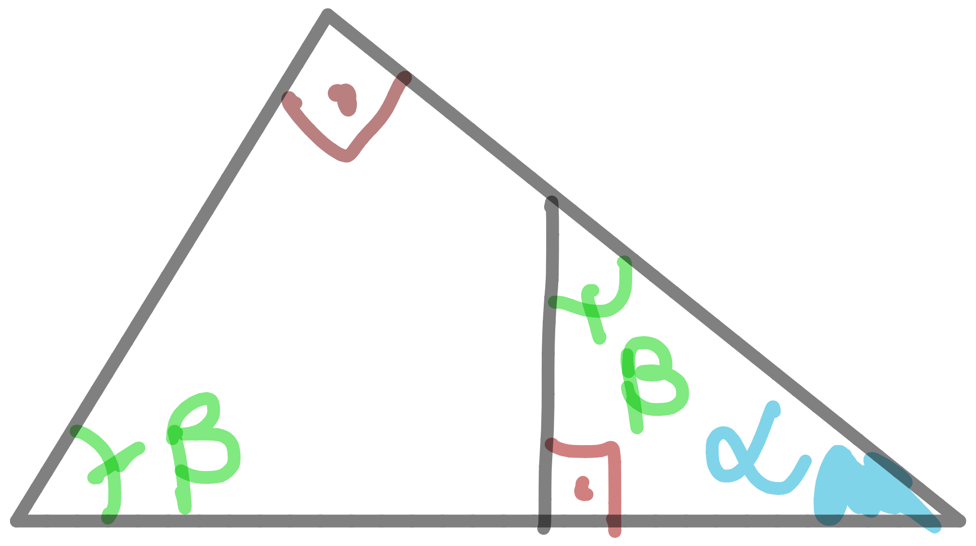
OBS: (Figuras clássicas de provas diversas!)

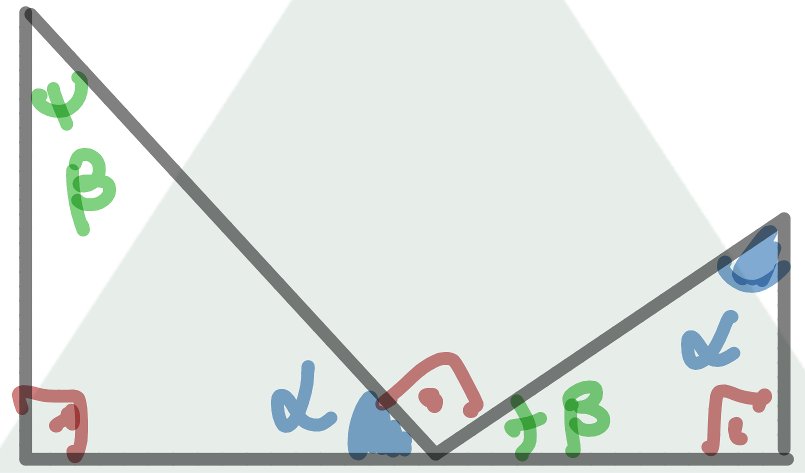
(importantíssimo!)

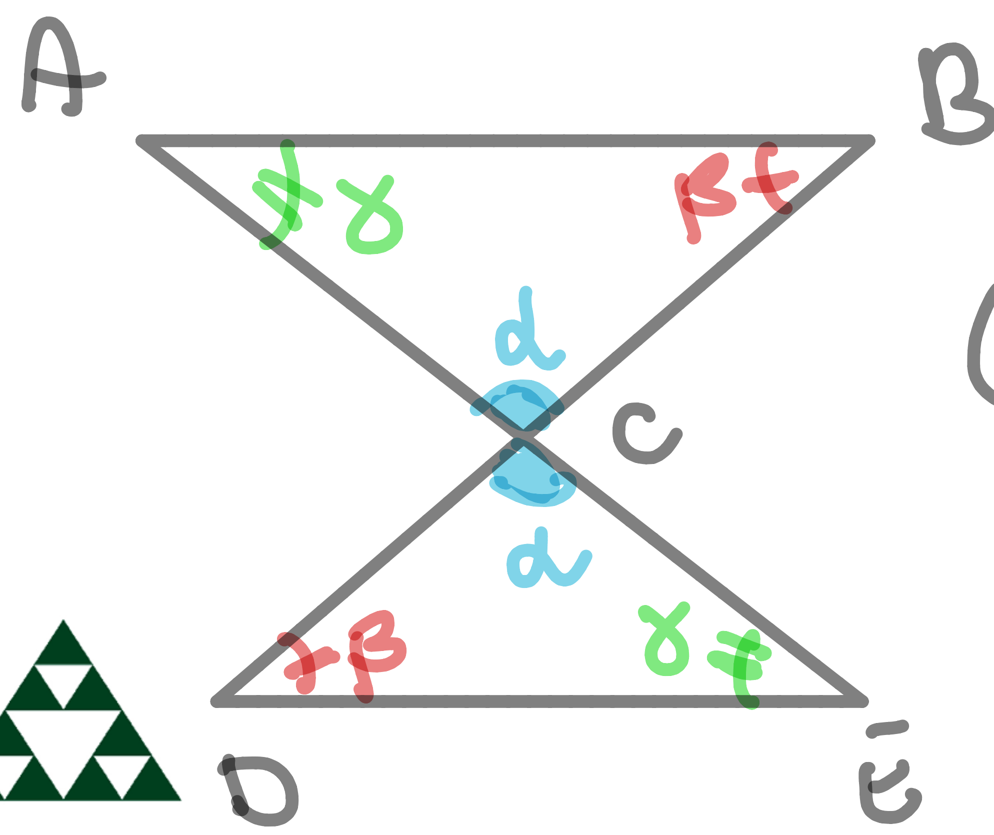




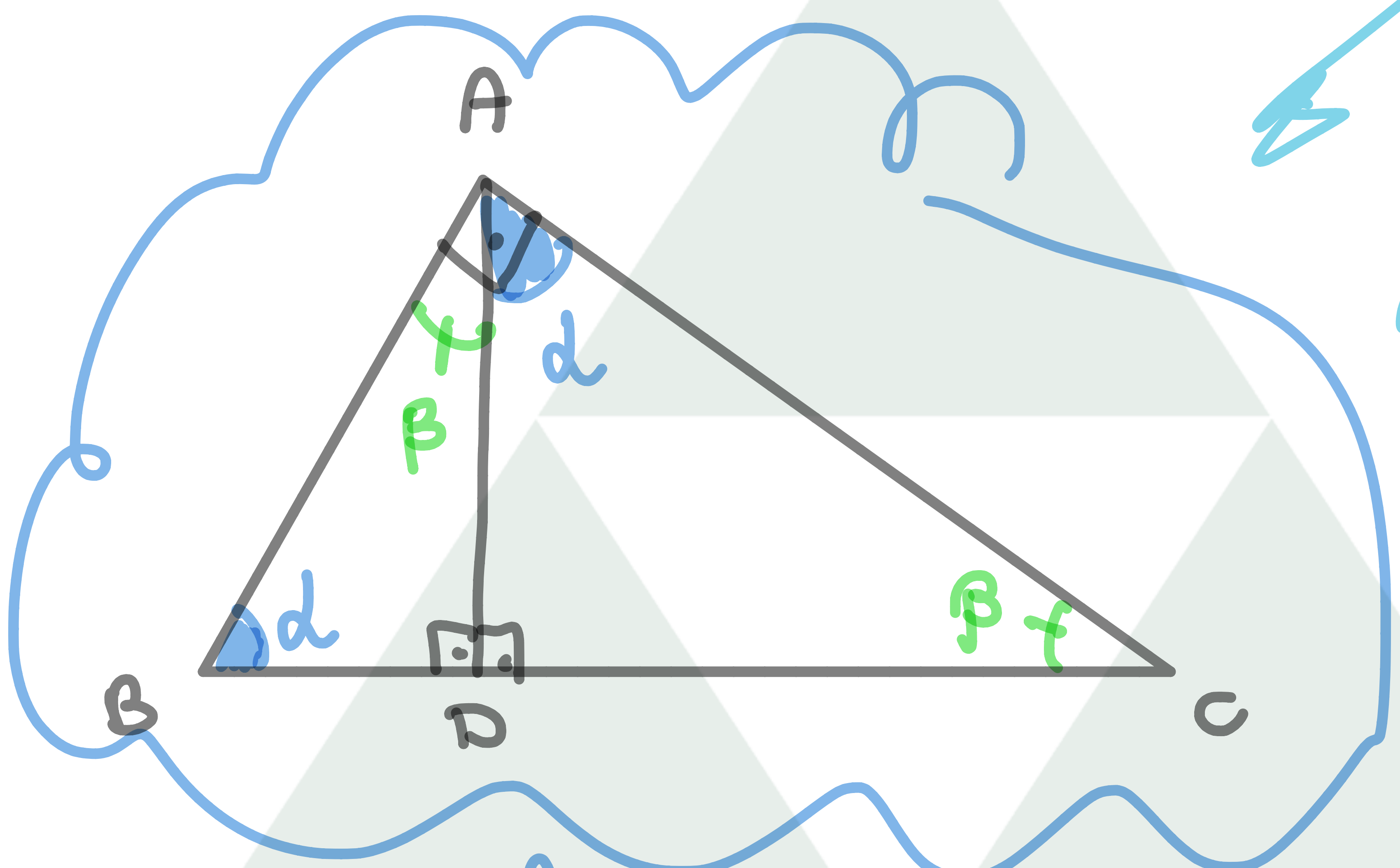








$\overline{AB} \parallel \overline{DE}$



Rel. métricas no △ retângulo!