Calcule os determinantes aplicando a regra de Chió:

1. $\begin{vmatrix} 1 & 1 & 2 & 0 \\ 2 & 5 & 2 & 1 \\ -1 & 0 & 3 & 2 \\ 4 & 5 & 8 & -1 \end{vmatrix} = \begin{vmatrix} 5-2 & 2-4 & 1-0 \\ 0-(-1) & 3-(-2) & 2-0 \\ 5-4 & 8-8 & -1-0 \end{vmatrix}$

$\begin{vmatrix} 3 & -2 & 1 \\ 1 & 5 & 2 \\ 1 & 0 & -1 \end{vmatrix} \begin{matrix} 3 & -2 \\ 1 & 5 \\ 1 & 0 \end{matrix}$

$\overbrace{(-1)^{1+1}}^{a_{11}} \cdot -26$:

$\boxed{-26}$

$D = -15 - 4 - 5 - 2$

$D = -26$

2. $\begin{vmatrix} 1 & 0 & -1 & 2 \\ 3 & 5 & 2 & 8 \\ 4 & 1 & -1 & 0 \\ -1 & 3 & -2 & -3 \end{vmatrix} = \begin{vmatrix} 1-0 & -1-0 & 2-0 \\ 3-20 & 2-(-5) & 8-0 \\ -1-12 & -2-(-3) & -3-0 \end{vmatrix}$

$\begin{vmatrix} 1 & -1 & 2 \\ -17 & 7 & 8 \\ -13 & 1 & -3 \end{vmatrix} \begin{matrix} 1 & -1 \\ -17 & 7 \\ -13 & 1 \end{matrix}$

$(-1)^{3+2} \cdot 274$:

$\boxed{-274}$

$D = -21 + 104 - 34 + 182 - 8 + 51$

$D = 274$

3. Resolva a equação $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = 0$ $\begin{vmatrix} 1-x & 1-1 & 1-1 \\ 1-x & x-1 & 1-1 \\ 1-x & 1-1 & x-1 \end{vmatrix}$

$\begin{vmatrix} 1-x & 0 & 0 \\ 1-x & x-1 & 0 \\ 1-x & 0 & x-1 \end{vmatrix} \begin{matrix} 1-x & 0 \\ 1-x & x-1 \\ 1-x & 0 \end{matrix}$

$(x-1)^2 \cdot (1-x) = 0$

isolando o sinal

$(x^2-2x+1) \cdot (1-x) = 0 \rightarrow -(x-1) \cdot (x^2-2x+1)$

$(-1)^{2+1} \cdot (x-1).(x^2-2x+1) \rightarrow \underbrace{(x-1)}_{1º} \cdot \underbrace{(x^2-2x+1)}_{2º}$

+

Vamos igualar as duas partes à zero, para descobrir as raízes:

1º → $x-1=0$ $\quad x=1$

$x^2-2x+1=0$

2º → $\underline{1} + \underline{1} = -b/a = 2$

$\underline{1} \cdot \underline{1} = c/a = 1$

$x'=1;\ x''=1$

então: $\boxed{x=1}$

4. Calcule x sabendo

$\begin{vmatrix} 1 & a & a & a \\ 1 & x & a & a \\ 1 & a & x+a & a \\ 1 & a & a & x+2a \end{vmatrix} = 0$ $\begin{vmatrix} a-a & a-(x+a) & a-a \\ x-a & a-(x+a) & a-a \\ a-a & a-(x+a) & (x+2a)-a \end{vmatrix}$

$\begin{vmatrix} 0 & -x & 0 \\ x-a & -x & 0 \\ 0 & -x & x+a \end{vmatrix} \begin{matrix} 0 & -x \\ x-a & -x \\ 0 & -x \end{matrix}$

$-[(x+a).(x-a).(-x)] = 0$

$(x^2-a^2) \cdot (-x) = 0$ isolando o sinal

$-(x).(x^2+a^2) = 0$

+

$(-1)^{3+1} \cdot \underbrace{-(x)}_{1º} \cdot \underbrace{(x+a^2)}_{2º} = 0$

1º → $-x=0$

$\boxed{x=0}$

2º → $x^2-a^2=0$

$x^2=a^2$

$x=\sqrt{a^2}$

$\boxed{x=\pm a}$

5. Resolva a equação $\begin{vmatrix} x & a & a & a \\ x & x & a & a \\ x & x & x & a \\ x & x & x & x \end{vmatrix} = 0$ $\begin{vmatrix} 1 & a & a & a \\ 1 & x & a & a \\ 1 & x & x & a \\ 1 & x & x & x \end{vmatrix}$

vamos dividir esta coluna por x.

$\begin{vmatrix} x-a & a-a & a-a \\ x-a & x-a & a-a \\ x-a & x-a & x-a \end{vmatrix}$ $\begin{vmatrix} x-a & 0 & 0 \\ x-a & x-a & 0 \\ x-a & x-a & x-a \end{vmatrix}$

$(x-a).(x-a).(x-a)=0$

$(x-a)^3=0$

Como dividimos a coluna por x, precisamos considerar ela na resposta final. Vamos multiplicar a resposta por x.

$\underbrace{x}_{=0}\underbrace{(x-a)^3}_{=0}=0$

$\boxed{x=0}$

$(x-a)^3=0$

$x-a=\sqrt[3]{0}$

$x-a=0$

$\boxed{x=a}$

$\boxed{x=a \text{ ou } x=0}$