

Lembrete:

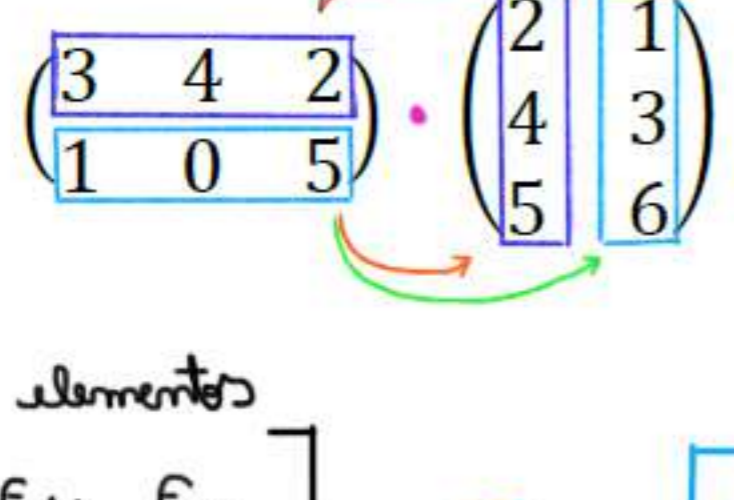
$$A_{m \times p} \cdot B_{p \times n} = C_{m \times n}$$

Para a multiplicação $\rightarrow m=n$

1. Calcule AB e BA , sendo dadas:

$$A = \begin{pmatrix} 3 & 4 & 2 \\ 1 & 0 & 5 \end{pmatrix} \text{ e } B = \begin{pmatrix} 2 & 1 \\ 4 & 3 \\ 5 & 6 \end{pmatrix}$$

$$A_{2 \times 3} \cdot B_{3 \times 2} = AB_{2 \times 2}$$

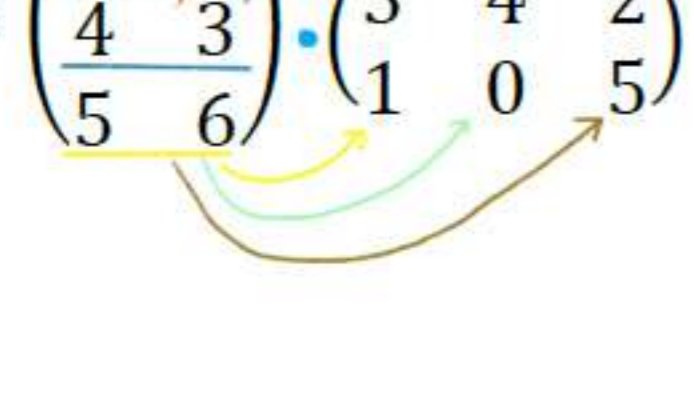


$$\begin{aligned} e_{11} &= 3 \cdot 2 + 4 \cdot 4 + 2 \cdot 5 = 32 \\ e_{12} &= 3 \cdot 1 + 4 \cdot 3 + 2 \cdot 6 = 27 \\ e_{21} &= 1 \cdot 2 + 0 \cdot 4 + 5 \cdot 5 = 27 \\ e_{22} &= 1 \cdot 1 + 0 \cdot 3 + 5 \cdot 6 = 31 \end{aligned}$$

Elementos

$$AB = \begin{bmatrix} 32 & 27 \\ 27 & 31 \end{bmatrix}$$

$$B_{3 \times 2} \cdot A_{2 \times 3} = BA_{3 \times 3}$$



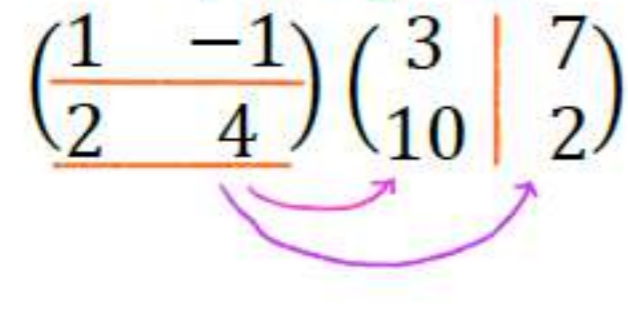
$$\begin{aligned} e_{11} &= 2 \cdot 3 + 1 \cdot 1 = 7 & e_{23} &= 4 \cdot 2 + 3 \cdot 5 = 23 \\ e_{12} &= 2 \cdot 4 + 1 \cdot 0 = 8 & e_{31} &= 5 \cdot 3 + 6 \cdot 1 = 21 \\ e_{13} &= 2 \cdot 2 + 1 \cdot 5 = 9 & e_{32} &= 5 \cdot 4 + 6 \cdot 0 = 20 \\ e_{21} &= 4 \cdot 3 + 3 \cdot 1 = 15 & e_{33} &= 5 \cdot 2 + 6 \cdot 6 = 40 \\ e_{22} &= 4 \cdot 4 + 3 \cdot 0 = 16 & & \end{aligned}$$

$$B \times A = \begin{bmatrix} 7 & 8 & 9 \\ 15 & 16 & 23 \\ 21 & 20 & 40 \end{bmatrix}$$

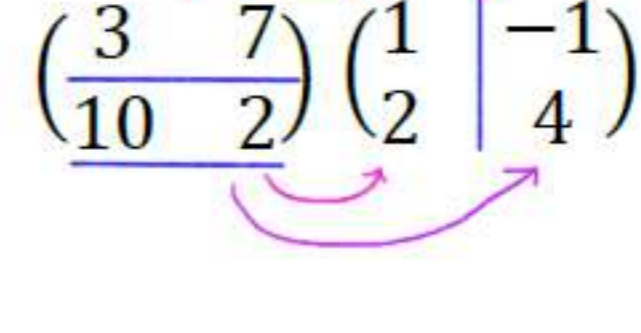
2. Calcule AB e BA , sendo:

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} \text{ e } B = \begin{pmatrix} 3 & 7 \\ 10 & 2 \end{pmatrix}$$

$$A_{2 \times 2} \cdot B_{2 \times 2} = AB_{2 \times 2}$$



$$B_{2 \times 2} \cdot A_{2 \times 2} = BA_{2 \times 2}$$



$$\begin{aligned} e_{11} &= 1 \cdot 3 + (-1) \cdot 10 = -7 & e_{11} &= 3 \cdot 1 + 7 \cdot 2 = 17 \\ e_{12} &= 1 \cdot 7 + (-1) \cdot 2 = 5 & e_{12} &= 3 \cdot (-1) + 7 \cdot 4 = 25 \\ e_{21} &= 2 \cdot 3 + 4 \cdot 10 = 46 & e_{21} &= 10 \cdot 1 + 2 \cdot 2 = 14 \\ e_{22} &= 2 \cdot 7 + 4 \cdot 2 = 22 & e_{22} &= 10 \cdot (-1) + 2 \cdot 4 = -2 \end{aligned}$$

$$AB = \begin{bmatrix} -7 & 5 \\ 46 & 22 \end{bmatrix}$$

$$BA = \begin{bmatrix} 17 & 25 \\ 14 & -2 \end{bmatrix}$$

3. Sendo $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix}$, determine $A \cdot A^t$.

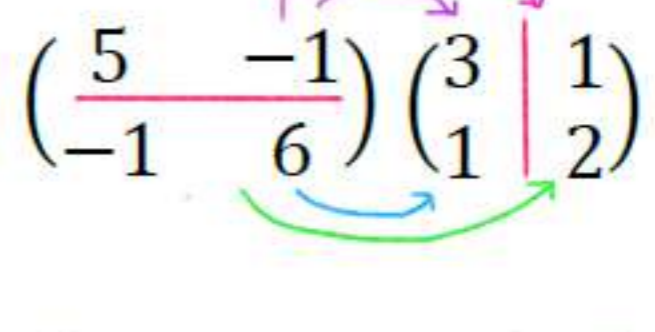
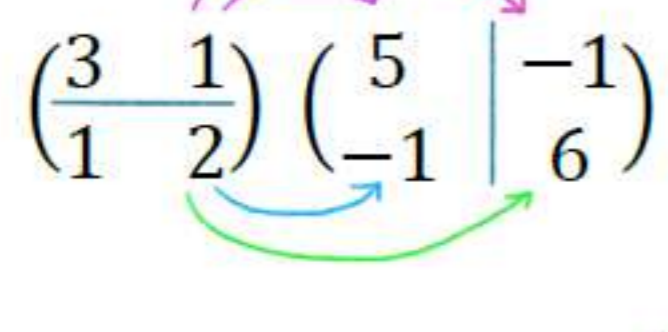
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix} \cdot A^t = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$A \cdot A^t = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 5 & 5 \\ 1 & 5 & 14 \end{bmatrix}$$

4. Verifique se $A = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$ e $B = \begin{pmatrix} 5 & -1 \\ -1 & 6 \end{pmatrix}$ são matrizes comutáveis.

$$A_{2 \times 2} \cdot B_{2 \times 2} = AB_{2 \times 2}$$

$$B_{2 \times 2} \cdot A_{2 \times 2} = BA_{2 \times 2}$$



$$\begin{aligned} e_{11} &= 3 \cdot 5 + 1 \cdot (-1) = 14 & e_{11} &= 5 \cdot 3 + (-1) \cdot 1 = 14 \\ e_{12} &= 3 \cdot (-1) + 1 \cdot 6 = 3 & e_{12} &= 5 \cdot 1 + (-1) \cdot 2 = 3 \\ e_{21} &= 1 \cdot 5 + 2 \cdot (-1) = 3 & e_{21} &= -1 \cdot 3 + 6 \cdot 1 = 3 \\ e_{22} &= 1 \cdot (-1) + 2 \cdot 6 = 11 & e_{22} &= -1 \cdot 1 + 6 \cdot 2 = 11 \end{aligned}$$

$$AB = \begin{bmatrix} 14 & 3 \\ 3 & 11 \end{bmatrix}$$

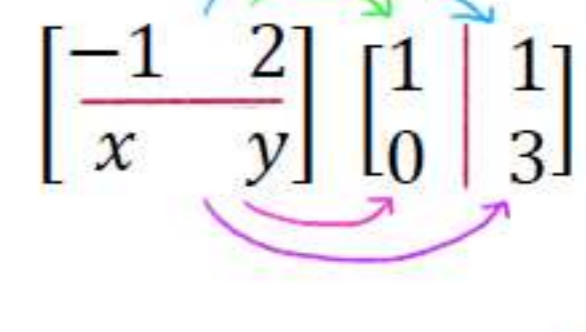
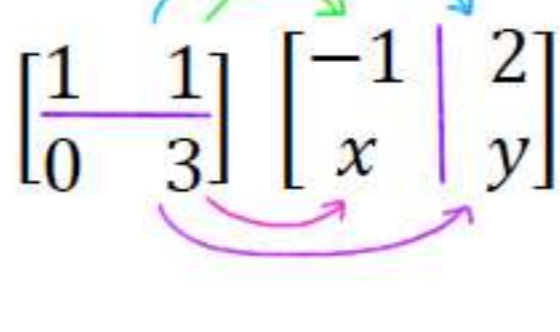
$$BA = \begin{bmatrix} 14 & 3 \\ 3 & 11 \end{bmatrix}$$

• Ambos são comutáveis

5. Para que valores de x e y as matrizes $A = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}$ e $B = \begin{bmatrix} -1 & 2 \\ x & y \end{bmatrix}$ são comutáveis?

$$A_{2 \times 2} \cdot B_{2 \times 2} = AB_{2 \times 2}$$

$$B_{2 \times 2} \cdot A_{2 \times 2} = BA_{2 \times 2}$$



$$\begin{aligned} e_{11} &= 1 \cdot (-1) + 1 \cdot x = x - 1 & e_{11} &= -1 \cdot 1 + 2 \cdot 0 = -1 \\ e_{12} &= 1 \cdot 2 + 1 \cdot y = 2 + y & e_{12} &= -1 \cdot 1 + 2 \cdot 3 = 5 \\ e_{21} &= 0 \cdot (-1) + 3 \cdot x = 3x & e_{21} &= x \cdot 1 + y \cdot 0 = x \\ e_{22} &= 0 \cdot 2 + 3 \cdot y = 3y & e_{22} &= x \cdot 1 + y \cdot 3 = x + 3y \end{aligned}$$

$$AB = \begin{bmatrix} x-1 & 2+y \\ 3x & 3y \end{bmatrix}$$

$$BA = \begin{bmatrix} -1 & 5 \\ x & x+3y \end{bmatrix}$$

Para que elas sejam comutáveis:

$$\begin{aligned} x-1 &= -1 & 2+y &= 5 \\ x &= -1+1 & y &= 5-2 \\ \boxed{x=0} & & \boxed{y=3} & \end{aligned}$$

6. Calcule x e y para que se verifique a igualdade:

$$\begin{pmatrix} 1 & -1 \\ -3 & 3 \end{pmatrix} \begin{pmatrix} x & y \\ 4 & 5 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{aligned} e_{11} &= 1 \cdot x + (-1) \cdot 4 = x - 4 & \begin{bmatrix} x-4 & y-5 \\ 12-3x & 15-3y \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ e_{12} &= 1 \cdot y + (-1) \cdot 5 = y - 5 \\ e_{21} &= -3 \cdot x + 3 \cdot 4 = 12 - 3x \\ e_{22} &= -3 \cdot y + 3 \cdot 5 = 15 - 3y \end{aligned}$$

$$\begin{aligned} x-4 &= 0 & y-5 &= 0 \\ \boxed{x=4} & & \boxed{y=5} & \end{aligned}$$

Satisfazem os dois equações.

7. Dadas $A = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$ e $B = \begin{pmatrix} 7 & 6 \\ 12 & -4 \end{pmatrix}$, determine a matriz X que satisfaz a equação $A \cdot X = B$.

$$\begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix} \cdot \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 12 & -4 \end{pmatrix}$$

- $a_{11} = 1 \cdot x_{11} + 0 \cdot x_{21} = x_{11}$
- $a_{12} = 1 \cdot x_{12} + 0 \cdot x_{22} = x_{12}$
- $a_{21} = 3 \cdot x_{11} + 2 \cdot x_{21}$
- $a_{22} = 3 \cdot x_{12} + 2 \cdot x_{22}$

$$\begin{bmatrix} x_{11} & x_{12} \\ 3x_{11} + 2x_{21} & 3x_{12} + 2x_{22} \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 12 & -4 \end{bmatrix}$$

$$\begin{aligned} x_{11} &= 7 & 3x_{11} + 2x_{21} &= 12 & 3x_{12} + 2x_{22} &= -4 \\ x_{12} &= 6 & 3 \cdot 7 + 2x_{21} &= 12 & 3 \cdot 6 + 2x_{22} &= -4 \\ & & 2x_{21} &= 12 - 21 & 2x_{22} &= -4 - 18 \\ & & x_{21} &= -9/2 & x_{22} &= -11 \end{aligned}$$

Logo:
$$X = \begin{pmatrix} 7 & 6 \\ -9/2 & -11 \end{pmatrix}$$