Lembrando:

$$A_{m\times p}\cdot B_{p\times n} = C_{m\times n}$$

Para a multiplição → $m=n$

1. Calcule AB e BA, sendo dadas:

$$A=\begin{pmatrix}3&4&2\\1&0&5\end{pmatrix} \text{ e } B=\begin{pmatrix}2&1\\4&3\\5&6\end{pmatrix}$$

$A_{2\times3}\cdot B_{3\times2} = AB_{2\times2}$

$$\begin{pmatrix}3&4&2\\1&0&5\end{pmatrix}\cdot\begin{pmatrix}2&1\\4&3\\5&6\end{pmatrix}$$

$E_{11}= 3\cdot2+4\cdot4+2\cdot5 = 32$

$E_{12}= 3\cdot1+4\cdot3+2\cdot6 = 27$

$E_{21}= 1\cdot2+0\cdot4+5\cdot5 = 27$

$E_{22}= 1\cdot1+0\cdot3+5\cdot6 = 31$

elementos

$$\begin{bmatrix}E_{11}&E_{12}\\E_{21}&E_{22}\end{bmatrix} \rightsquigarrow AB=\begin{bmatrix}32&27\\27&31\end{bmatrix}$$

$B_{3\times2}\cdot A_{2x} = BA_{3\times3}$

$$\begin{pmatrix}2&1\\4&3\\5&6\end{pmatrix}\cdot\begin{pmatrix}3&4&2\\1&0&5\end{pmatrix}$$

$E_{11}= 2\cdot3+1\cdot1=7$

$E_{12}= 2\cdot4+1\cdot0=8$

$E_{13}= 2\cdot2+1\cdot5=9$

$E_{21}= 4\cdot3+3\cdot1=15$

$E_{22}= 4\cdot4+3\cdot0=16$

$E_{23}= 4\cdot2+3\cdot5=23$

$E_{31}= 5\cdot3+6\cdot1=21$

$E_{32}= 5\cdot4+6\cdot0=20$

$E_{33}= 5\cdot2+5\cdot6=40$

$$B\times A=\begin{bmatrix}7&8&9\\15&16&23\\21&20&40\end{bmatrix}$$

2. Calcule AB e BA, sendo:

$$A=\begin{pmatrix}1&-1\\2&4\end{pmatrix} \text{ e } B=\begin{pmatrix}3&7\\10&2\end{pmatrix}$$

$A_{2\times2}\cdot B_{22} = AB_{2\times2}$

$$\begin{pmatrix}1&-1\\2&4\end{pmatrix}\begin{pmatrix}3&7\\10&2\end{pmatrix}$$

$E_{11}= 1\cdot3+(-1)\cdot10=-7$

$E_{12}= 1\cdot7+(-1)\cdot2=5$

$E_{21}= 2\cdot3+4\cdot10=46$

$E_{22}= 2\cdot7+4\cdot2=22$

$$AB=\begin{bmatrix}-7&5\\46&22\end{bmatrix}$$

$B_{2\times2}\cdot A_{2\times2} = BA_{2\times2}$

$$\begin{pmatrix}3&7\\10&2\end{pmatrix}\begin{pmatrix}1&-1\\2&4\end{pmatrix}$$

$E_{11}= 3\cdot1+7\cdot2=17$

$E_{12}= 3\cdot(-1)+7\cdot4=25$

$E_{21}= 10\cdot1+2\cdot2=14$

$E_{22}= 10\cdot(-1)+2\cdot4=-2$

$$BA=\begin{bmatrix}17&25\\14&-2\end{bmatrix}$$

3. Sendo $A=\begin{bmatrix}1&0&0\\1&2&0\\1&2&3\end{bmatrix}$, determine $A\cdot A^t$.

$$A=\begin{bmatrix}1&0&0\\1&2&0\\1&2&3\end{bmatrix}\cdot A^t=\begin{bmatrix}1&1&1\\0&2&2\\0&0&3\end{bmatrix}$$

$$A\cdot A^t=\begin{bmatrix}1&1&1\\1&5&5\\1&5&14\end{bmatrix}$$

4. Verifique se $A=\begin{pmatrix}3&1\\1&2\end{pmatrix}$ e $B=\begin{pmatrix}5&-1\\-1&6\end{pmatrix}$ são matrizes comutáveis.

$A_{2\times2}\cdot B_{2\times2} = AB_{2\times2}$

$$\begin{pmatrix}3&1\\1&2\end{pmatrix}\begin{pmatrix}5&-1\\-1&6\end{pmatrix}$$

$E_{11}= 3\cdot5+1\cdot(-1)=14$

$E_{12}= 3\cdot(-1)+1\cdot6=3$

$E_{21}= 1\cdot5+2\cdot(-1)=3$

$E_{22}= 1\cdot(-1)+2\cdot6=11$

$$AB=\begin{bmatrix}14&3\\3&11\end{bmatrix}$$

$B_{2\times2}\cdot A_{2\times2} = BA_{2\times2}$

$$\begin{pmatrix}5&-1\\-1&6\end{pmatrix}\begin{pmatrix}3&1\\1&2\end{pmatrix}$$

$E_{11}= 5\cdot3+(-1)\cdot1=14$

$E_{12}= 5\cdot1+(-1)\cdot2=3$

$E_{21}= -1\cdot(3)+6\cdot1=3$

$E_{22}= -1(1)+6\cdot2=11$

$$BA=\begin{bmatrix}14&3\\3&11\end{bmatrix}$$

• Elas são comutáveis

5. Para que valores de x e y as matrizes $A=\begin{bmatrix}1&1\\0&3\end{bmatrix}$ e $B=\begin{bmatrix}-1&2\\x&y\end{bmatrix}$ são comutáveis?

$A_{2\times2}\cdot B_{2\times2} = AB_{2\times2}$

$$\begin{bmatrix}1&1\\0&3\end{bmatrix}\begin{bmatrix}-1&2\\x&y\end{bmatrix}$$

$E_{11}= 1\cdot(-1)+1\cdot x = x-1$

$E_{12}= 1\cdot2+1\cdot y = 2+y$

$E_{21}= 0\cdot-1+3x = 3x$

$E_{22}= 0\cdot2+3y = 3y$

$$AB=\begin{bmatrix}x-1&2+y\\3x&3y\end{bmatrix}$$

$B_{2\times2}\cdot A_{2\times2} = BA_{2\times2}$

$$\begin{bmatrix}-1&2\\x&y\end{bmatrix}\begin{bmatrix}1&1\\0&3\end{bmatrix}$$

$E_{11}= -1\cdot1+2\cdot0=-1$

$E_{12}= -1\cdot1+2\cdot3=5$

$E_{21}= x\cdot1+y\cdot0=x$

$E_{22}= x\cdot1+y\cdot3=x+3y$

$$BA=\begin{bmatrix}-1&5\\x&x+3y\end{bmatrix}$$

Para que elas sejam comutáveis:

$x-1=-1$

$x=-1+1$

$x=0$

$2+y=5$

$y=5-2$

$y=3$

6. Calcule x e y para que se verifique a igualdade:

$$\begin{pmatrix}1&-1\\-3&3\end{pmatrix}\begin{pmatrix}x&y\\4&5\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$$

$E_{11}= 1\cdot x+(-1)\cdot4 = x-4$

$E_{12}= 1\cdot y+(-1)\cdot5 = y-5$

$E_{21}= -3\cdot x+3\cdot4 = 12-3x$

$E_{22}= -3\cdot y+3\cdot5 = 15-3y$

$$\begin{bmatrix}x-4&y-5\\12-3x&15-3y\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$x-4=0$

$x=4$

$y-5=0$

$y=5$

Satisfazem as duas igualdades.

7. Dadas $A=\begin{pmatrix}1&0\\3&2\end{pmatrix}$ e $B=\begin{pmatrix}7&6\\12&-4\end{pmatrix}$, determine a matriz X que satisfaz a equação $A\cdot X=B$.

$$\begin{pmatrix}1&0\\3&2\end{pmatrix}\cdot\begin{pmatrix}x_{11}&x_{12}\\x_{21}&x_{22}\end{pmatrix}=\begin{pmatrix}7&6\\12&-4\end{pmatrix}$$

- $a_{11}= 1.x_{11}+0.x_{21}=x_{11}$
- $a_{12}= 1.x_{12}+0.x_{22}=x_{12}$
- $a_{21}= 3.x_{11}+2.x_{21}=$
- $a_{22}= 3.x_{12}+2x_{22}=$

$$\begin{bmatrix}x_{11}&x_{12}\\3x_{11}+2x_{21}&3x_{12}+2x_{22}\end{bmatrix}=\begin{bmatrix}7&6\\12&-4\end{bmatrix}$$

$x_{11}=7$

$x_{12}=6$

$3.x_{11}+2x_{21}=12$

$3.7+2x_{21}=12$

$2x_{21}=12-21$

$x_{21}=-9/2$

$3x_{12}+2x_{22}=-4$

$3.6+2x_{22}=-4$

$2x_{22}=-4-18$

$x_{22}=-11$

Logo:

$$X=\begin{pmatrix}7&6\\-9/2&-11\end{pmatrix}$$