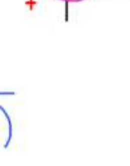
Calcule:

1. $cotg\ 165° =$

Sabendo que: $cotg\ x = \dfrac{1}{tg\ x}$

$180° - 165° = 15°$

$tg\ 165° = -(tg\ 15°)$



Assim:

$tg(15°) = tg(45° - 30°) = \dfrac{tg 45° - tg 30°}{1 + tg 45° \cdot tg 30°} = \dfrac{1 - \sqrt{3}/3}{1 + (1 \cdot \sqrt{3}/3)}$

$tg(15°) = \dfrac{\frac{(3-\sqrt{3})}{3}}{\frac{(3+\sqrt{3})}{3}} = \dfrac{3-\sqrt{3}}{3+\sqrt{3}}$

$cotg\ 165° = \dfrac{1}{-(tg\ 15°)} = \dfrac{1}{-\left(\frac{3-\sqrt{3}}{3+\sqrt{3}}\right)} = \dfrac{3+\sqrt{3}}{-3+\sqrt{3}} \cdot \dfrac{(-3-\sqrt{3})}{(-3-\sqrt{3})}$

$cotg\ 165° = \dfrac{-9 - 3\sqrt{3} - 3\sqrt{3} - 3}{3^2 - (\sqrt{3})^2} = \dfrac{-12 - 6\sqrt{3}}{6}$

$cotg\ 165° = -2 - \sqrt{3} = -(2 + \sqrt{3})$

$-(2+\sqrt{3})$

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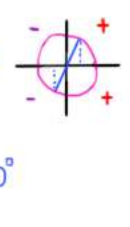
2. $sec\ 255° =$

Sabendo que: $sec\ x = \dfrac{1}{cos\ x}$

$255° - 180° = 75°$

$cos\ 225° = -(cos\ 75°)$



$cos\ 75° = cos(45° + 30°) = cos 45° \cdot cos 30° - sen 45° \cdot sen 30°$

$cos\ 75° = \dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2} = \dfrac{\sqrt{2}}{2} \cdot \left(\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}\right) = \dfrac{\sqrt{2}}{2} \cdot \left(\dfrac{\sqrt{3}-1}{2}\right)$

$cos\ 75° = \dfrac{\sqrt{6} - \sqrt{2}}{4}$

$sec\ 225° = \dfrac{1}{-\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)} = \dfrac{4}{\sqrt{2} - \sqrt{6}} \cdot \dfrac{\sqrt{2}+\sqrt{6}}{\sqrt{2}+\sqrt{6}} = \dfrac{4(\sqrt{2}+\sqrt{6})}{2-6} = \dfrac{4(\sqrt{2}+\sqrt{6})}{-4}$

$sec\ 225° = -(\sqrt{6} + \sqrt{2})$

$-(\sqrt{6}+\sqrt{2})$

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3. $cossec\ 15° =$

Sabendo que: $cossec\ x = \dfrac{1}{sen\ x}$

$sen\ 15° = sen(45° - 30°) = sen 45° cos 30° - sen 30° \cdot cos 45°$

$sen\ 15° = \dfrac{\sqrt{2}}{2} \cdot \dfrac{\sqrt{3}}{2} - \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{2}}{2} \cdot \left(\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}\right) = \dfrac{\sqrt{2}}{2} \cdot \left(\dfrac{\sqrt{3}-1}{2}\right)$

$sen\ 15° = \dfrac{\sqrt{6} - \sqrt{2}}{4}$

$cossec\ 15° = \dfrac{1}{\frac{\sqrt{6}-\sqrt{2}}{4}} = \dfrac{4}{\sqrt{6}-\sqrt{2}} \cdot \dfrac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}} = \dfrac{4(\sqrt{6}+\sqrt{2})}{6-2} = \dfrac{4(\sqrt{6}+\sqrt{2})}{4}$

$cossec\ 15° = \sqrt{6} + \sqrt{2}$

$\sqrt{6}+\sqrt{2}$

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4. Calcule o valor da expressão $sen\ 105° - cos\ 75°$.

Sabendo que:

$sen\ 105° = sen(60° + 45°)$

$cos\ 75° = cos(30° + 45°)$

$sen\ 105° - cos\ 75°$

$sen(60° + 45°) - cos(30° + 45°)$

$sen 60° cos 45° + sen 45° cos 60° - (cos 30° \cdot cos 45° - sen 30° sen 45°)$

$\dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2} - \left(\dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} - \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2}\right)$

$\dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2} \cdot \dfrac{1}{2} - \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} + \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{2}}{4} + \dfrac{\sqrt{2}}{4}$

$sen\ 105° - cos\ 75° = \dfrac{\sqrt{2}}{2}$

$\sqrt{2}/2$

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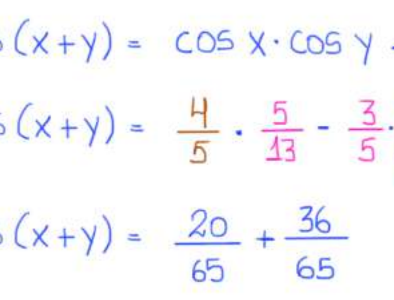
5. Dados: $sen\ x = \frac{3}{5}$ e $cos\ y = \frac{5}{13}$, calcule o $cos\ (x + y)$, sabendo que $0 < x < \frac{\pi}{2}$ e $\frac{3\pi}{2} < y < 2\pi$.



$sen^2 x + cos^2 x = 1$

$\left(\dfrac{3}{5}\right)^2 + cos^2 x = 1$

$cos^2 x = 1 - \dfrac{9}{25}$

$cos\ x = \pm\sqrt{\dfrac{16}{25}}$

$cos\ x = +\dfrac{4}{5}$

$sen^2 y + cos^2 y = 1$

$sen^2 y + \left(\dfrac{5}{13}\right)^2 = 1$

$sen^2 y = 1 - \dfrac{25}{169}$

$sen\ y = \pm\sqrt{\dfrac{144}{169}}$

$sen\ y = -\dfrac{12}{13}$

$cos(x+y) = cos\ x \cdot cos\ y - sen\ x \cdot sen\ y$

$cos(x+y) = \dfrac{4}{5} \cdot \dfrac{5}{13} - \dfrac{3}{5} \cdot \left(-\dfrac{12}{13}\right)$

$cos(x+y) = \dfrac{20}{65} + \dfrac{36}{65}$

$cos(x+y) = \dfrac{56}{65}$

56/65

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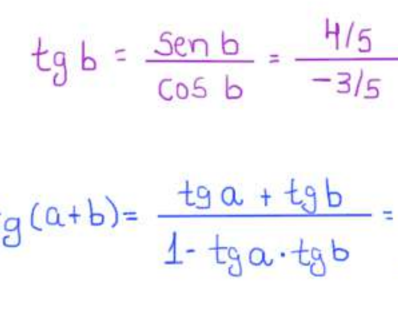
6. Sabendo que $tg\ a = \frac{2}{3}$ e $sen\ b = \frac{4}{5}$ com $\frac{\pi}{2} < b < \pi$, calcule $tg\ (a + b)$.



$sen^2 + cos^2 = 1$

$\left(\dfrac{4}{5}\right)^2 + cos^2 x = 1$

$cos^2 x = 1 - \dfrac{16}{25}$

$cos\ x = \pm\sqrt{\dfrac{9}{25}}$

$cos\ x = -\dfrac{3}{5}$

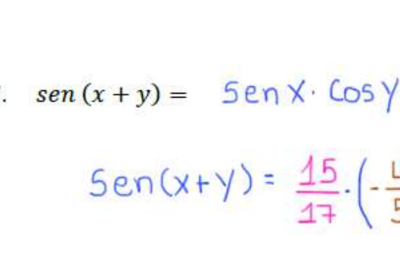
$tg\ b = \dfrac{sen\ b}{cos\ b} = \dfrac{4/5}{-3/5} = -\dfrac{4}{3}$

$tg(a+b) = \dfrac{tg\ a + tg\ b}{1 - tg\ a \cdot tg\ b} = \dfrac{2/3 + (-4/3)}{1 - (2/3 \cdot -4/3)} = \dfrac{-2/3}{1 + 8/9} = \dfrac{-2/3}{17/9} = \dfrac{-2}{3} \cdot \dfrac{9}{17}$

$tg(a+b) = -6/17$

$-6/17$

Sabendo que $sen\ x = \frac{15}{17}, sen\ y = -\frac{3}{5}, 0 < x < \frac{\pi}{2}\ e\ \pi < y < \frac{3\pi}{2}$, calcule:



$sen^2 x + cos^2 x = 1$

$\left(\dfrac{15}{17}\right)^2 + cos^2 x = 1$

$cos^2 x = 1 - \dfrac{225}{289}$

$cos\ x = \pm\sqrt{\dfrac{64}{289}}$

$cos\ x = +\dfrac{8}{17}$

$sen^2 y + cos^2 y = 1$

$\left(\dfrac{-3}{5}\right)^2 + cos^2 y = 1$

$cos^2 y = 1 - \dfrac{9}{25}$

$cos\ y = \pm\sqrt{\dfrac{16}{25}}$

$cos\ y = -\dfrac{4}{5}$

7. $sen\ (x + y) = sen\ x \cdot cos\ y + sen\ y \cdot cos\ x$

$sen(x+y) = \dfrac{15}{17} \cdot \left(-\dfrac{4}{5}\right) + \left(-\dfrac{3}{5}\right) \cdot \dfrac{8}{17}$

$sen(x+y) = -\dfrac{60}{85} - \dfrac{24}{85} = -\dfrac{84}{85}$

-84/85

8. $cos\ (x + y) = cos\ x \cdot cos\ y - sen\ x \cdot sen\ y$

$cos(x+y) = \dfrac{8}{17} \cdot \left(-\dfrac{4}{5}\right) - \dfrac{15}{17} \cdot \left(-\dfrac{3}{5}\right)$

$cos(x+y) = -\dfrac{32}{85} + \dfrac{45}{85} = \dfrac{13}{85}$

13/85

$tg\ x = \dfrac{sen\ x}{cos\ x} = \dfrac{15/17}{8/17} = \dfrac{15}{8}$

$tg\ y = \dfrac{sen\ y}{cos\ y} = \dfrac{-3/5}{-4/5} = \dfrac{3}{4}$

9. $tg\ (x + y) = \dfrac{tg\ x + tg\ y}{1 - tg\ x \cdot tg\ y}$

$tg(x+y) = \dfrac{15/8 + 3/4}{1 - (15/8 \cdot 3/4)} = \dfrac{\frac{60+24}{32}}{\frac{32-45}{32}} = \dfrac{84}{-13} = -\dfrac{84}{13}$

-84/13