FUNÇÃO MODULAR

1) MÓDULO DE UM NÚMERO REAL

$$\sqrt{x^2} = |x| = \begin{cases} x \text{ SE } x \geq 0 \\ -x \text{ SE } x < 0 \end{cases}$$

$$\sqrt{5^2} = |5| = 5$$

$$\sqrt{(-5)^2} = |-5| = -(-5) = 5$$

EX: $|\underbrace{2-\sqrt{3}}_{+}| = 2-\sqrt{3}$

EX: $|\underbrace{2-\sqrt{5}}_{-}| = -(2-\sqrt{5}) = -2+\sqrt{5}$

$= \sqrt{5}-2$

OBS: $|x-2| = \begin{cases} x-2 \text{ SE } x-2 \geq 0 \\ -(x-2) \text{ SE } x-2 < 0 \end{cases}$

$f(x) = |x-2| = \begin{cases} x-2 \text{ SE } x \geq 2 \\ -x+2 \text{ SE } x < 2 \end{cases}$

OBS: $|\underbrace{\pi - e}_{+}| = \pi - e$

$\pi = 3,14\ldots$

$e = 2,71\ldots$

OBS: $|\underbrace{\sqrt{3}-\sqrt{5}}_{-}| = -(\sqrt{3}-\sqrt{5}) = -\sqrt{3}+\sqrt{5}$

$= \sqrt{5}-\sqrt{3}$

MESTRES DA MATEMÁTICA

2) EQUAÇÕES MODULARES

a) $|x| = 3$

$x = 3$ ou $x = -3$

$S = \{\pm 3\}$

b) $|x| = 0$

$x = 0$ $S = \{0\}$

c) $|x| = -4$

$\nexists\, x$ $S = \{\,\}$

↓ NÃO EXISTE

d) $|2x - 7| = 5$

$2x - 7 = 5$ ou $2x - 7 = -5$

$2x = 12$ $2x = 2$

$\boxed{x = 6}$ $\boxed{x = 1}$

e) $|3x + 7| = x - 3$

CE: $x - 3 \geq 0 \therefore x \geq 3$

$3x + 7 = x - 3$ ou $3x + 7 = -(x - 3)$

$2x = -10$ $3x + 7 = -x + 3$

$\cancel{x = -5}$ $4x = -4$

$\cancel{x = -1}$

$S = \{\,\}$

f) $|2x - 5| = x - 2$

CE: $x - 2 \geq 0 \therefore x \geq 2$

$2x - 5 = x - 2$

$\boxed{x = 3}$

ou

$2x - 5 = -(x - 2)$

$2x - 5 = -x + 2$

$3x = 7$

$\boxed{x = 7/3}$

g) $|x|^2 - 3\cdot|x| - 10 = 0$

FAZENDO $|x| = y$, TEMOS:

$y^2 - 3y - 10 = 0$

$\Delta = 49$

$y = \dfrac{3 \pm 7}{2} \begin{cases} y = 5 \\ y = -2 \end{cases}$

SE $y = 5 \Rightarrow |x| = 5 \Rightarrow x = \pm 5$

SE $y = -2 \Rightarrow |x| = -2 \Rightarrow \nexists\ x$

$S = \{\pm 5\}$

DESAFIO: $\big||x-3| - 7\big| = 5$

$|x-3| - 7 = 5$ ou $|x-3| - 7 = -5$

$|x-3| = 12$ ou $|x-3| = 2$

$x - 3 = 12$ ou $x - 3 = -12$ ou $x - 3 = 2$ ou $x - 3 = -2$

$x = 15$ | $x = -9$ | $x = 5$ | $x = 1$

MESTRES DA MATEMÁTICA

h) $|x^2+5x-6| = |x^2+2x-3|$

$x^2+5x-6 = x^2+2x-3$

$3x = 3 \Rightarrow \boxed{x = 1}$

ou

$x^2+5x-6 = -(x^2+2x-3)$

$x^2+5x-6 = -x^2-2x+3$

$2x^2+7x-9 = 0$

$\Delta = 121$

$x = \dfrac{-7 \pm 11}{4}$ $\begin{cases} \boxed{x = 1} \\ \boxed{x = -\frac{9}{2}} \end{cases}$

$S = \{1, -\frac{9}{2}\}$

OBS: DETERMINE TODOS OS NÚMEROS REAIS X TAIS QUE:

$\sqrt{x^2+12x} + 2 = x+6$

$\sqrt{x^2+12x} = x+4$ ⟶ CE: $x+4 \geq 0$

$x \geq -4$

$\left(\sqrt{x^2+12x}\right)^2 = (x+4)^2$

$x^2+12x = x^2+8x+16$

$12x = 8x+16$

$4x = 16$

$\boxed{x = 4}$

DESAFIO: DETERMINE TODOS OS NÚMEROS REAIS X SATISFAZENDO A EQUAÇÃO:

$$\sqrt{x^2-2x+1} + \sqrt{x^2+4x+4} = 7$$

$$\sqrt{(x-1)^2} + \sqrt{(x+2)^2} = 7$$

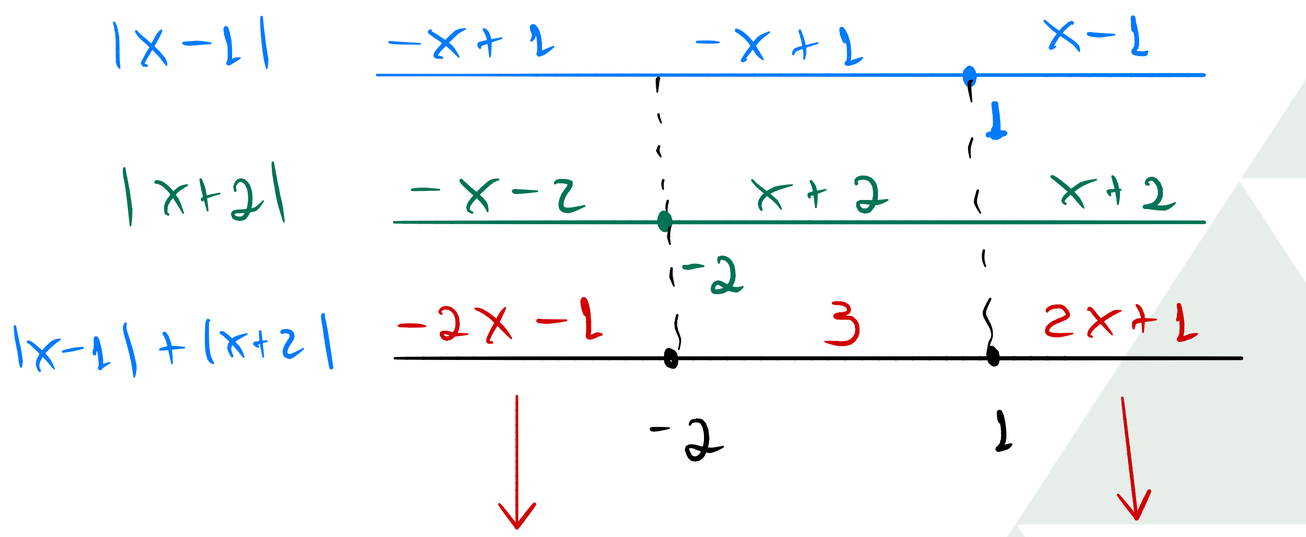
$$|x-1| + |x+2| = 7$$

$$|x-1| = \begin{cases} x-1 & \text{SE } x-1 \geq 0 \\ -(x-1) & \text{SE } x-1 < 0 \end{cases}$$

$$|x-1| = \begin{cases} x-1 & \text{SE } x \geq 1 \\ -x+1 & \text{SE } x < 1 \end{cases}$$

$$|x+2| = \begin{cases} x+2 & \text{SE } x+2 \geq 0 \\ -(x+2) & \text{SE } x+2 < 0 \end{cases}$$

$$|x+2| = \begin{cases} x+2 & \text{SE } x \geq -2 \\ -x-2 & \text{SE } x < -2 \end{cases}$$



$$-2x-1 = 7$$

$$-2x = 8$$

$$\boxed{x = -4}$$

$$2x+1 = 7$$

$$2x = 6$$

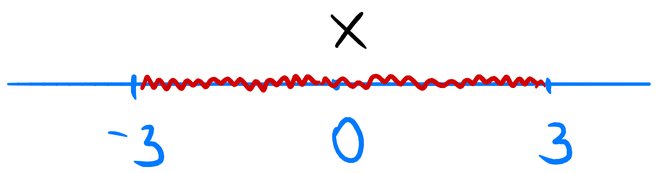
$$\boxed{x = 3}$$

$$S = \{-4, 3\}$$

MESTRES DA MATEMÁTICA

3) INEQUAÇÕES MODULARES

a) $|x| \leq 3$



$-3 \leq x \leq 3$

$x \geq -3$ e $x \leq 3$

b) $|x| \geq 3$

$-3 \geq x \geq 3$

$x \leq -3$ ou $x \geq 3$

c) $|2x-1| < 7$

$-7 < 2x-1 < 7$

$2x-1 > -7$

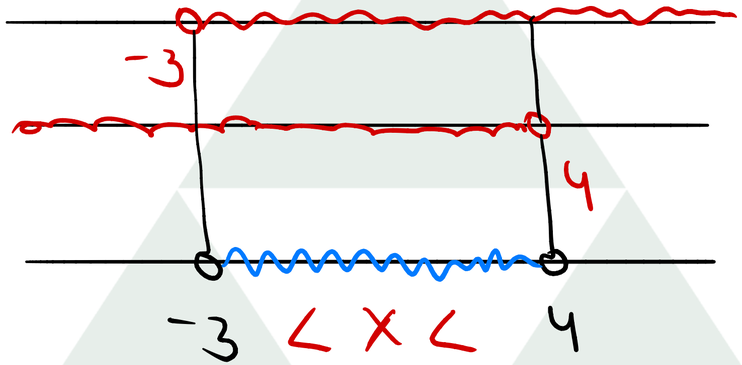
$2x > -6$

$x > -3$

$2x-1 < 7$

$2x < 8$

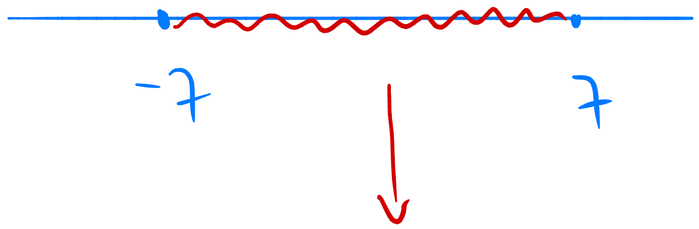
$x < 4$



$-3 < x < 4$

$-3 < x < 4$

2º modo) $|2x-1| < 7$



$-7 < 2x-1 < 7$

$-7+1 < 2x-1+1 < 7+1$

$-6 < 2x < 8 \quad (\div 2)$

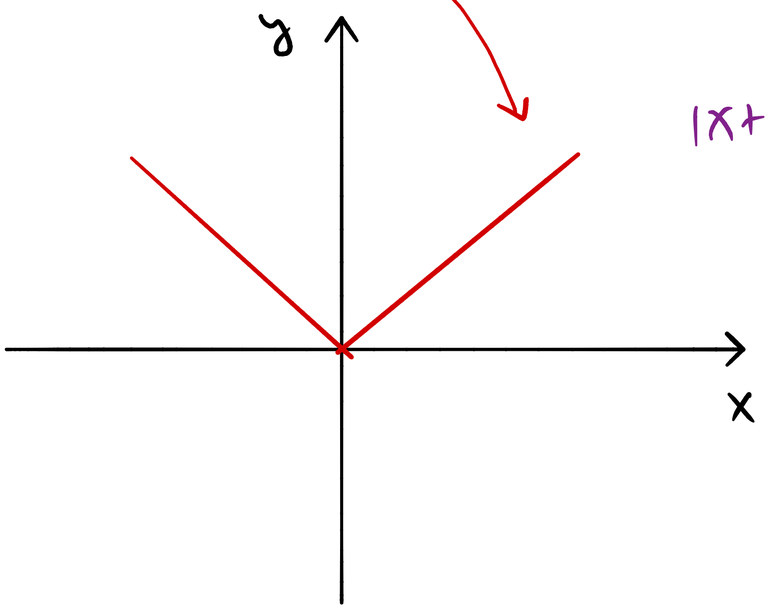
$$\boxed{-3 < x < 4}$$

$x > -3$ e $x < 4$

$-7 < 2x-1 < 7$

4) GRÁFICOS

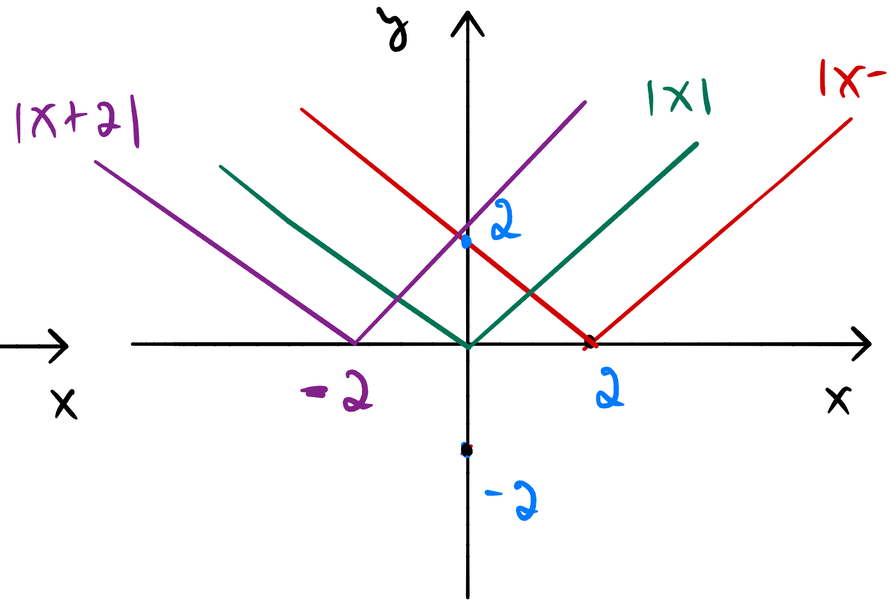
a) $f(x) = |x|$



$y = x$

b) $f(x) = |x-2|$

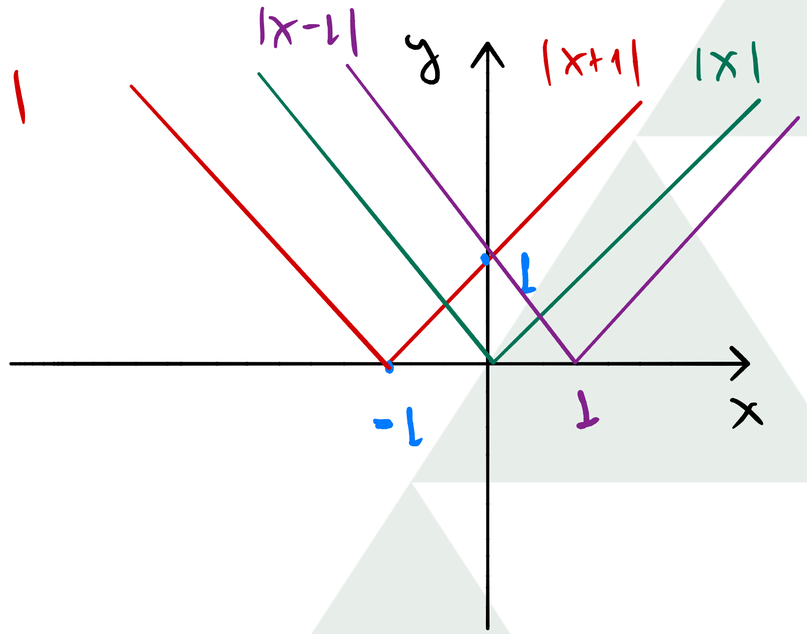
$|x+2|$



$y = x - 2$

x	y
0	-2
2	0

c) $f(x) = |x+1|$

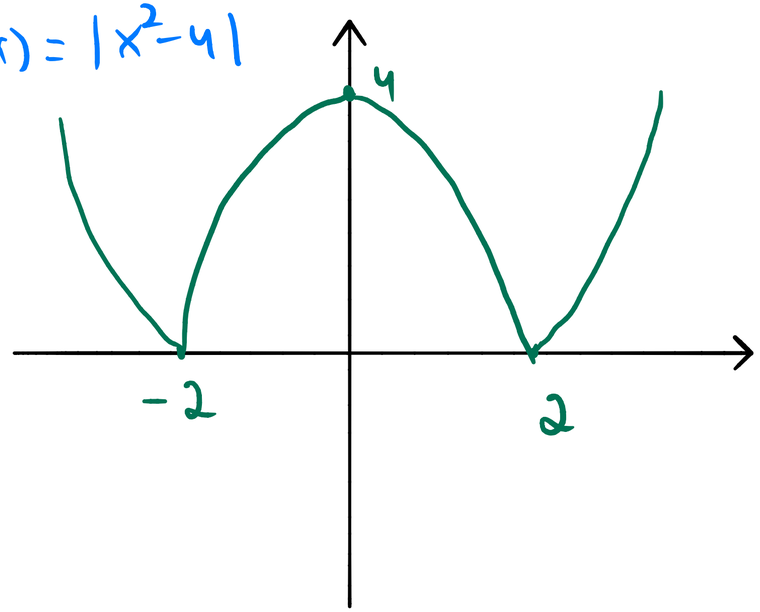


$y = x + 1$

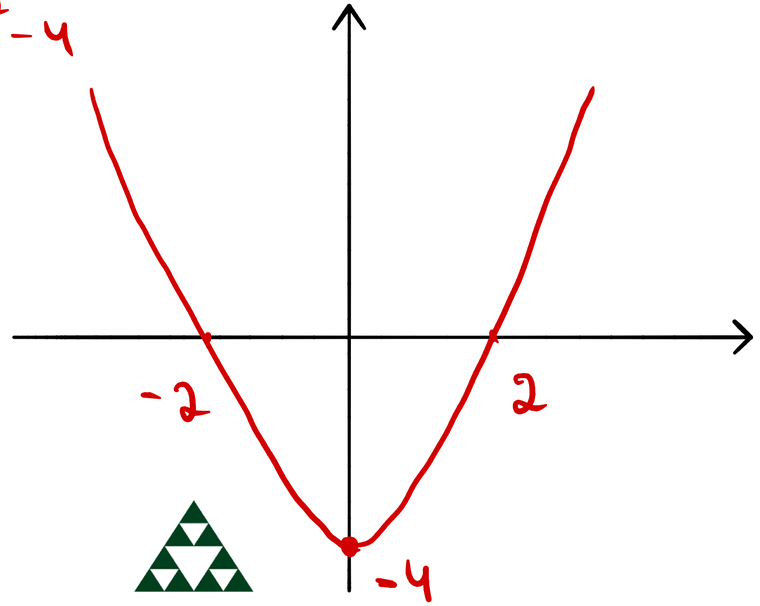
x	y
0	1
-1	0

MESTRES DA MATEMÁTICA

d) $f(x) = |x^2 - 4|$

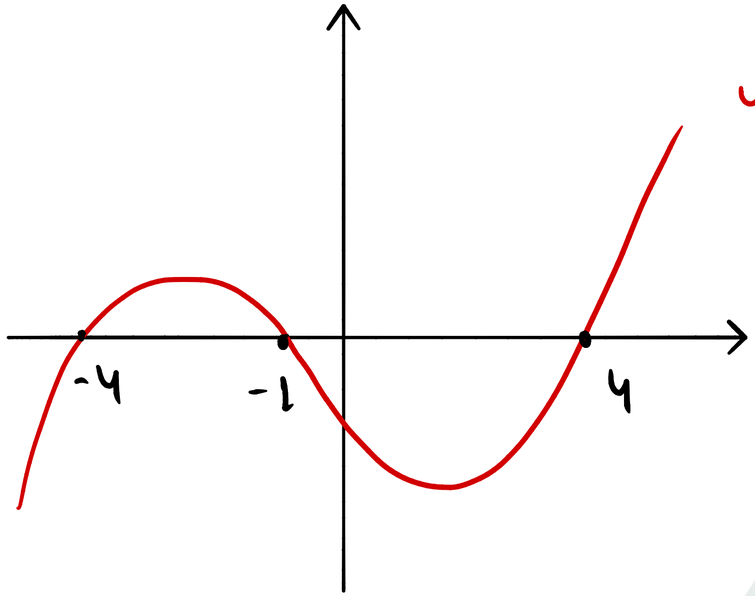


$y = x^2 - 4$

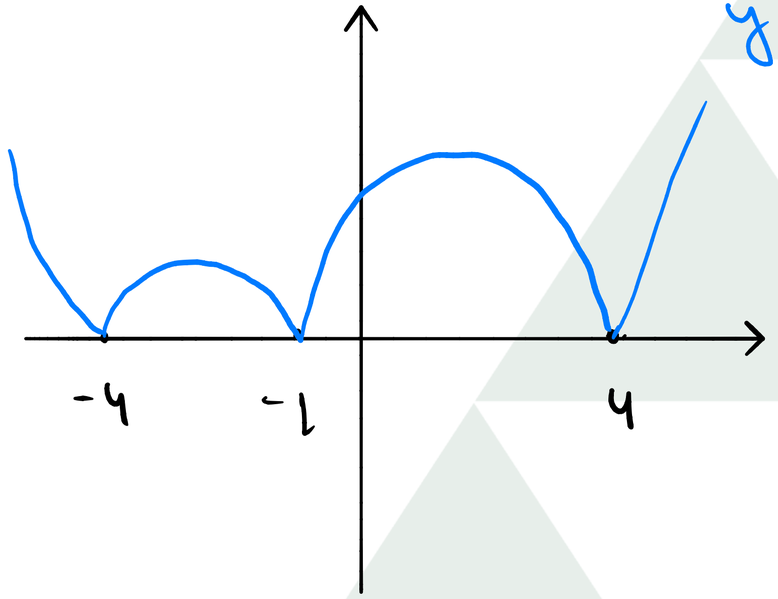


OBS:

$y = f(x)$



$y = |f(x)|$



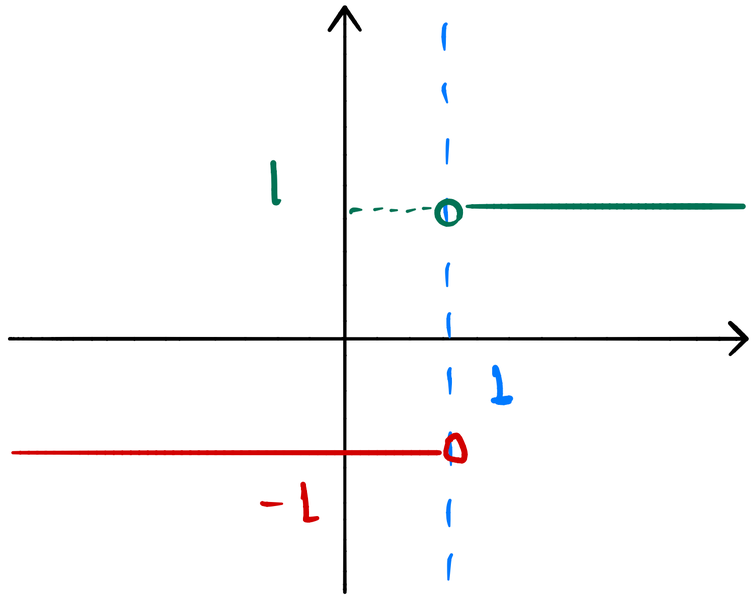
MESTRES DA MATEMÁTICA

e) $f(x) = \dfrac{|x-1|}{x-1}$

$x > 1$ ⟶ $f(x) = \dfrac{x-1}{x-1} = 1$

$D = \mathbb{R} - \{1\}$

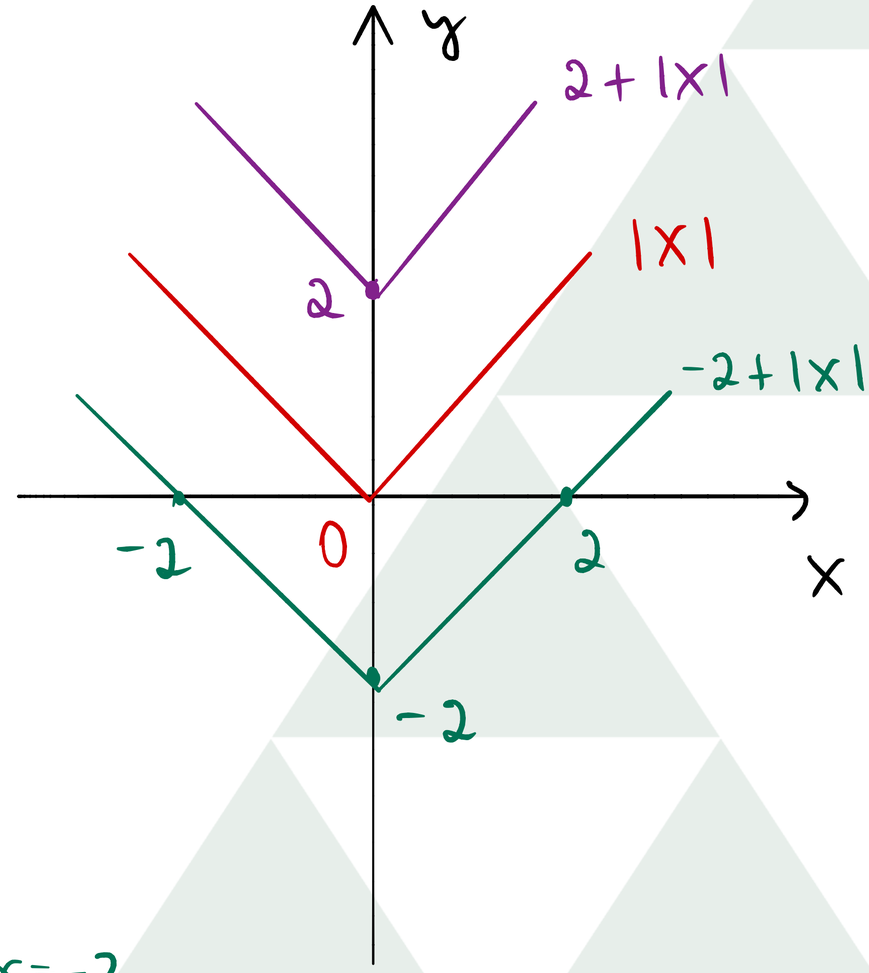
$x < 1$ ⟶ $f(x) = \dfrac{-(x-1)}{x-1} = -1$



$$|x-1| = \begin{cases} x-1 & \text{se } x > 1 \\ -(x-1) & \text{se } x < 1 \end{cases}$$

f) $f(x) = 2 + |x|$

$f(x) = -2 + |x|$

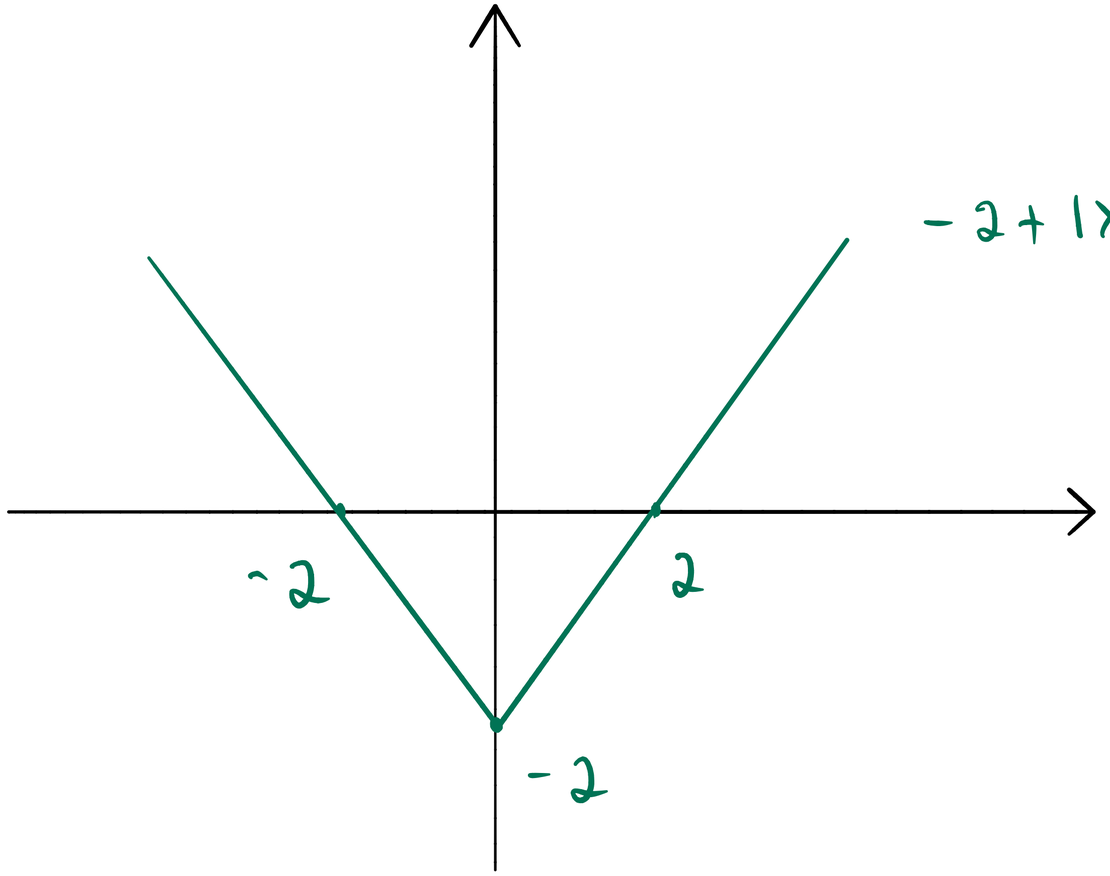


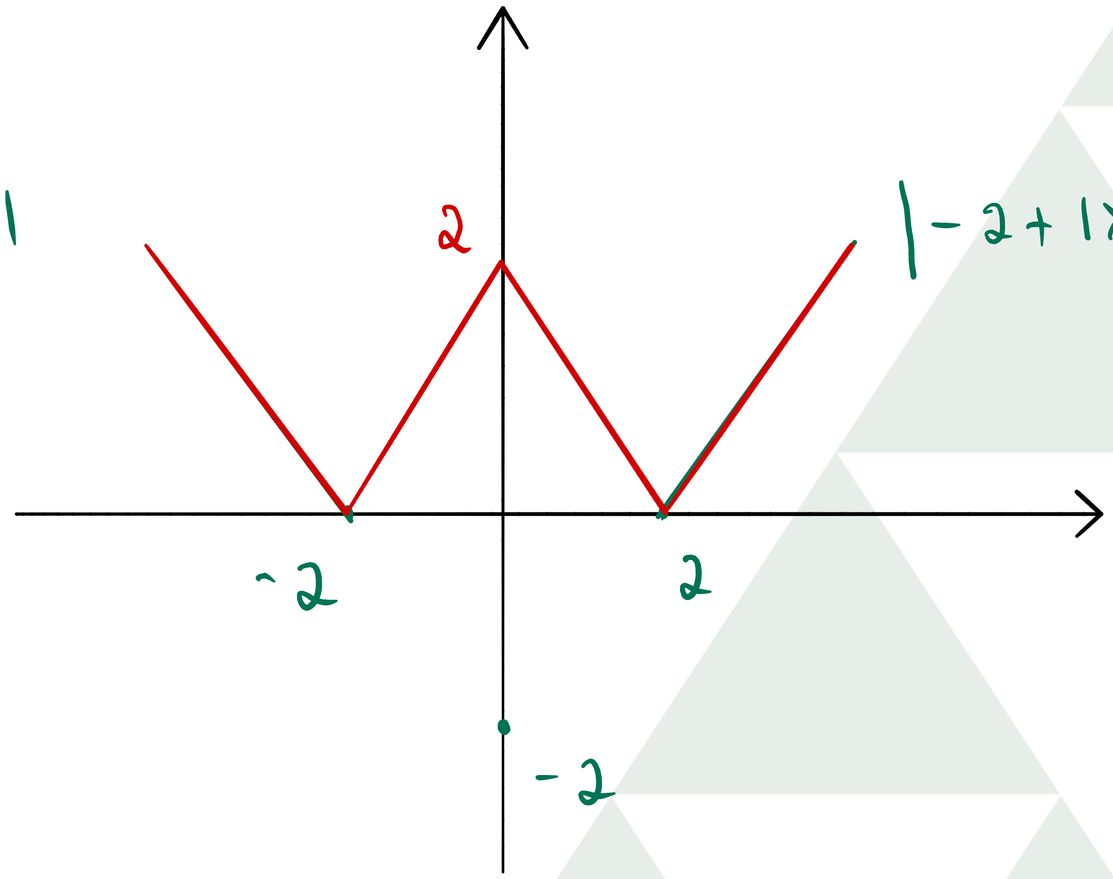
$-2 + |x| = 0$

$|x| = 2$

$x = 2$ ou $x = -2$

g) $f(x) = |-2+|x||$





MESTRES DA MATEMÁTICA