Resolva as inequações:

1.
$$\log_3(5x-2) < \log_3 4$$

$a=3 \rightarrow a>1$ $\qquad f(x)<g(x)$

$5-2x<4$

$5x<6$

$x<6/5$

Pela definição de logaritmos

$5x-2>0 \rightarrow 5x>2 \rightarrow x>2/5$

$S=\{x \in \mathbb{R} / 2/5 < x < 6/5\}$

2.
$$\log_{\frac{1}{2}}(3x-1) \geq \log_{\frac{1}{2}}(2x+3)$$

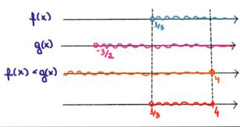
$a=1/2 \rightarrow 0<a<1$ $\qquad f(x)<g(x)$

$(3x-1) \leq (2x+3)$

$3x-2x \leq 3+1$

$x \leq 4$

$f(x)$	$g(x)$
$3x-1>0$	$2x+3>0$
$3x>1$	$2x>-3$
$x>1/3$	$x>-3/2$



$S=\{x \in \mathbb{R} / 1/3 < x \leq 4\}$

3.
$$\log_{\frac{1}{2}}(x^2-1) > \log_{\frac{1}{2}}(3x+9)$$

$a=1/2 \rightarrow 0<a<1$ $\qquad f(x)<g(x)$

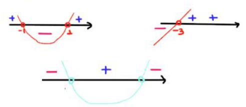
$3x+9>x^2-1$

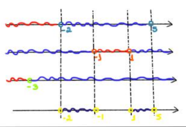
$-x^2+3x+10>0$

$-2 + 5 = -b/a = 3$

$-2 \cdot 5 = c/a = -10$

$f(x)$	$g(x)$
$x^2-1>0$	$3x+9>0$
$x>\sqrt{1}$	$3x>-9$
$x>\pm 1$	$x>-3$





$S=\{x \in \mathbb{R} / -2<x<-1 \text{ ou } 1<x<5\}$

4.
$$\log_{10}(x^2-x-2) < \log_{10}(x-4)$$

$a=10 \rightarrow a>1$

$f(x)<g(x)$

$x^2-x-2<x-4$

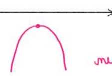
$-2+4<-x^2+x+x$

$-x^2+2x-2>0$

$\Delta = b^2-4.a.c$

$\Delta = 2^2-4.-1.-2$

$\Delta = -4$



nunca toca o eixo x

Não existem valores que satisfaçam a inequação

$S=\{\emptyset\}$

5.
$$\log_2(3x+5) > 3$$

$\log_a b > c \qquad b>a^c \rightarrow$ definição de log

$a=2 \ (a>1) \rightarrow a^c<b$

$2^3<3x+5$

$8-5<3x$

$3<3x \rightarrow x>1$

$S=\{x \in \mathbb{R} / x>1\}$

6.
$$\log_2(x^2+x-2) \leq 2 \qquad a>1, \text{ pois } a=2$$

$\log_2(x^2+x-2) \leq 2 \cdot \underbrace{\log_2 2}_{=1}$

$x^2+x-2 \leq 2^2$

$x^2+x-2 \leq 4 \rightarrow x^2+x-6 \leq 0$

$2 + (-3) = -b/a = -1$

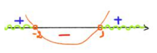
$2 \cdot (-3) = c/a = -6$

Da definição de log: logaritmando > 1

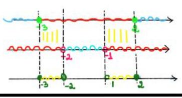
$x^2+x-2>0$

$-2 + 1 = -b/a = -1$

$-2 \cdot 1 = c/a = -2$



Intersecção das duas condições:



$S=\{x \in \mathbb{R} / -3 \leq x < -2 \ \cup \ 1<x \leq 2\}$

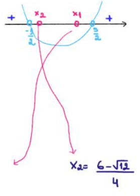
7.
$$\log_{\frac{1}{2}}(2x^2-6x+3) < 1$$

$a<1$

$2x^2-6x+3>(1/2)^1$

$2x^2-6x+5/2>0$

raízes: $(1/2, 5/2)$



Da definição de log: logaritmando > 1

$2x^2-6x+3>0$

$x_1 = \frac{6+\sqrt{12}}{4} \qquad x_2 = \frac{6-\sqrt{12}}{4}$

$S=\{x \in \mathbb{R} / 1/2>x \text{ ou } x>5/2$

8.
$$\log_{10}(x^2+3x+3) > 0$$

$a=10$

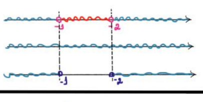
$x^2+3x+3>10^0$

$x^2+3x+3-1>0$

$x^2+3x+2 \qquad$ raízes $(-1, -2)$

$x^2+3x+3>0$

raízes → não existem raízes reais



$S=\{x \in \mathbb{R} / x<-2 \text{ ou } x>-1\}$

9.
$$3 \cdot (\log_3 x)^2 + 5 \cdot \log_3 x - 2 \leq 0$$

$\log_3 x = m$

$3m^2+5m-2 \leq 0$

raízes: $m_1 \leq 1/3$

$m_2 \geq -2$

$\log x \leq 1/3$ ou $\log_3 x \geq -2$

$x \leq 3^{1/3} \qquad x \geq 3^{-2}$

$S=\{x \in \mathbb{R} / 1/9 \leq x \leq \sqrt[3]{3}\}$

10.
$$\left(\log_{\frac{1}{2}} x\right)^2 - 3 \cdot \log_{\frac{1}{2}} x - 4 > 0$$

$\log_{1/2} x = m$

$m^2-3m-4>0$

raízes: $m_1=-1$

$m_2=4$

$\log_{1/2} x < -1 \qquad x > 1/2^{-1}$

$0<a<1$

$\log_{1/2} x > 4 \qquad x < 1/2^4$

$S=\{x \in \mathbb{R} / 0<x<1/16 \text{ ou } x>2\}$