EXERCÍCIOS DE REVISÃO I

1. Calcule a integral $\int 2x \cdot \sec^2 x^2 dx$.

2. (**EFOMM)** O valor do $\lim_{x\to 0}\left(\frac{\sqrt{x+a}-\sqrt{a}}{x}\right)$ é:

a) $\frac{1}{\sqrt{a}}$

b) $\sqrt{a}$

c) $\frac{1}{2\sqrt{a}}$

d) $2\sqrt{a}$

e) 0

3. Calcule:

$$\lim_{x\to 5}\frac{\sqrt{x+11}-2\sqrt{x-1}}{x^2-25}$$

4. Se $\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$, então

a) $a=1$ e $b=4$

b) $a=1$ e $b=-4$

c) $a=2$ e $b=-3$

d) $a=2$ e $b=3$

e) $a=b=2$

5. (EFOMM 2002) Calcule $\lim_{x\to 0}\frac{e^{5x}-1}{x}$.

a) e^5

b) 0

c) e

d) 1

e) 5

6. (EFOMM 2001) O valor de $\lim\limits_{x \to 2} \dfrac{x-2}{\sqrt[3]{3x-5}-1}$:

a) 1

b) 2

c) 3

d) 4

e) 5

7. (EN 2008) O valor de $\int 4 \operatorname{sen} 2x \cos^2 x \, dx$ é

a) $-\dfrac{\cos 2x}{2} + \dfrac{\cos 4x}{4} + C$

b) $-\cos 2x - \dfrac{\operatorname{sen}^2 2x}{2} + C$

c) $-\dfrac{4\cos^3 x}{3} + C$

d) $-\dfrac{3}{2}\cos 2x + C$

e) $-\cos 2x - \dfrac{\cos 4x}{4} + C$

8. (EN 2004) Seja p uma constante real positiva. A integral $\int e^{\frac{1n(2px)}{2}} dx$ é igual a:

a) $\dfrac{2}{3}(2px)^{3/2} + c$

b) $p(2px)^{-1/2} + c$

c) $\dfrac{1}{3}(2px)^{3/2} + c$

d) $\dfrac{2}{3}x(2px)^{1/2} + c$

e) $\dfrac{1}{3}x(2px)^{-1/2} + c$

9. (EFOMM 1999) Resolvendo $\int 3\mathrm{sen}(x/2)dx$, encontramos:

a) $6\cos\frac{x}{2}+c$

b) $3\cos\frac{x}{2}+c$

c) $-3\cos\frac{x}{2}+c$

d) $-6\cos\frac{x}{2}+c$

e) $\cos\frac{x}{2}\cdot\mathrm{sen}\frac{x}{2}+c$

10. (EN 1989) $\int_0^1 \frac{x}{2-2x^2+x^4}dx$ é igual a:

a) $-\frac{\pi}{8}$

b) $-\frac{\pi}{4}$

c) $\frac{\pi}{8}$

d) $\frac{\pi}{4}$

e) 0

11. Calcule os limites

a) $\lim_{x\to 0}\frac{\ln(1+x)}{x}$

b) $\lim_{x\to 0}\frac{\log(1+10x)}{x}$

c) $\lim_{x\to\infty}(\ln(2x+1)-\ln(x+2))$

12. Determine os limites:

a) $\lim_{x\to 0}\frac{\mathrm{sen}\,3x}{x}$

b) $\lim_{x \to 0} \dfrac{tg 3x}{\text{sen } 4x}$

c) $\lim_{n \to \infty} \dfrac{1}{n^2} + \dfrac{2}{n^2} + \dfrac{3}{n^2} + \cdots + \dfrac{n-1}{n^2}$

13. Analise as afirmativas:

I. $\lim\limits_{x \to \infty} \dfrac{x-1}{x+1} = 1$

II. $\lim\limits_{x \to \infty} \left(\dfrac{x-1}{x+1}\right)^{x} = 1$

III. $\lim\limits_{x \to 0} \left(\dfrac{x^2 - 2x + 3}{x^2 - 3x + 2}\right)^{\frac{\text{sen } x}{x}} = \dfrac{3}{2}$

IV. $\lim\limits_{x \to 0} (1 + \text{sen } x)^{\frac{1}{x}} = 1$

Quantas são verdadeiras?

a) 0

b) 1

c) 2

d) 3

e) 4

14. O valor do limite $\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2 + 14} + x}{\sqrt{x^2 - 2} + x} = -7$ é:

a) 1

b) –1

c) 0

d) –7

e) 14

15. Sendo $A = \lim\limits_{x \to 0} \left[\dfrac{2\sqrt{x \cdot \text{sen } 6x}}{\text{cossec } 6x(1 - \cos^2 6x)}\right]$ e $B = \lim\limits_{x \to \log_2 3} \left[2^{(2x+1)}\right]$, $\dfrac{A^2 B}{2}$ vale:

a) $2\sqrt{3}$

b) 6

c) 12

d) $6\sqrt{3}$

e) 18

16. Sabendo-se que y = $\lim\limits_{x \to 0} e^{\sqrt[x]{1+2x}}$, o logaritmo neperiano de y vale:

a) e^2

b) $\sqrt{e}$

c) e^e

d) 2e

e) -3e

17. Calcule os limites abaixo:

a) $\lim\limits_{x \to 0} (\cos x)^{\frac{1}{x}}$

b) $\lim\limits_{x \to 0} (\cos x)^{\frac{1}{x^2}}$

18. (EN 2003) De um ponto P do cais, João observa um barco AB ancorado. Para um sistema de eixos ortogonais os pontos A e B têm coordenadas respectivamente iguais a (0, 20) e (0, 40), enquanto P encontra-se no semieixo positivo das abscissa. Se o ângulo APB de observação é máximo, então a abscissa de P é igual a:

a) $20\sqrt{2}$

b) $20\sqrt{3}$

c) 20

d) 15

e) 10

GABARITO

1.

SE $u = x^2 \Rightarrow du = 2x\,dx$

teremos

$$\int 2x \cdot \sec^2 x^2 dx =$$
$$= \int \sec^2 u\, du$$
$$= tg\ u + c$$
$$= tg\, x^2 + c$$

2.

RESPOSTA: C

$$\lim_{x\to 0}\left(\frac{\sqrt{x+a}-\sqrt{a}}{x}\right) = \lim_{x\to 0}\left(\frac{\sqrt{x+a}-\sqrt{a}}{x}\cdot\frac{\sqrt{x+a}+\sqrt{a}}{\sqrt{x+a}+\sqrt{a}}\right) = \lim_{x\to 0}\left(\frac{\left(\sqrt{x+a}\right)^2-\left(\sqrt{a}\right)^2}{x\left(\sqrt{x+a}+\sqrt{a}\right)}\right) =$$

$$= \lim_{x\to 0}\left(\frac{x+\cancel{a}-\cancel{a}}{x\left(\sqrt{x+a}+\sqrt{a}\right)}\right) = \lim_{x\to 0}\left(\frac{1}{\sqrt{x+a}+\sqrt{a}}\right) = \frac{1}{\sqrt{0+a}+\sqrt{a}} = \frac{1}{2\sqrt{a}}$$

3.

$$\lim_{x\to 5}\frac{\sqrt{x+11}-2\sqrt{x-1}}{x^2-25} = \lim_{x\to 5}\frac{\sqrt{x+11}-2\sqrt{x-1}}{x^2-25}\cdot\frac{\sqrt{x+11}+2\sqrt{x-1}}{\sqrt{x+11}+2\sqrt{x-1}} =$$

$$= \lim_{x\to 5}\frac{-3\cancel{(x-5)}}{(x+5)\cancel{(x-5)}\left(\sqrt{x+11}+2\sqrt{x-1}\right)} = \frac{-3}{(5+5)\left(\sqrt{5+11}+2\sqrt{5-1}\right)} = -\frac{3}{80}$$

4.

RESPOSTA: B

$$\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right) = 4 \Leftrightarrow$$

$$\lim_{x\to\infty}\left(\frac{(1-a)x^2+(1-a-b)x+(1-b)}{x+1}\right) = 4 \Leftrightarrow$$

$$\begin{cases} 1-a=0 \Leftrightarrow a=1 \\ 1-a-b=4 \Leftrightarrow b=-4 \end{cases}$$

5.

RESPOSTA: D

Como o limite é da forma $\left(\frac{0}{0}\right)$, vamos aplicar o teorema de L'Hôpital.

$$\lim_{x\to 0}\frac{e^{5x}-1}{x}=\lim_{x\to 0}\frac{e^{5x}\cdot 5}{1}=\lim_{x\to 0}\left(5e^{5x}\right)=5\cdot e^{5\cdot 0}=5$$

6.

RESPOSTA: A

Como o limite é da forma $\left(\frac{0}{0}\right)$, vamos aplicar o teorema de L'Hôpital.

$$\lim_{x\to 2}\frac{x-2}{\sqrt[3]{3x-5}-1}=\lim_{x\to 2}\frac{1}{\dfrac{3}{3\sqrt[3]{(3x-5)^2}}}=\lim_{x\to 2}\sqrt[3]{(3x-5)^2}=\sqrt[3]{(3\cdot 2-5)^2}=1$$

7.

RESPOSTA: E

$$\int 4\,\text{sen}\,2x\cos^2 x\,dx=\int 4\,\text{sen}\,2x\cdot\frac{\cos 2x+1}{2}dx=\int 2\,\text{sen}\,2x\cos 2x\,dx+2\int \text{sen}\,2x\,dx$$
$$=\int \text{sen}\,4x\,dx+2\int \text{sen}\,2x\,dx=-\frac{\cos 4x}{4}+2\cdot\left(-\frac{\cos 2x}{2}\right)+C=-\frac{\cos 4x}{4}-\cos 2x+C$$

8.

RESPOSTA: D

$$\int e^{\frac{1n(2px)}{2}}dx=$$
$$\int e^{\ln\sqrt{2px}}dx=$$
$$\int \sqrt{2px}\,dx=$$
$$\sqrt{2p}\int x^{1/2}dx=$$
$$\sqrt{2p}\cdot\frac{x^{3/2}}{3/2}+c=\frac{2}{3}x\cdot(2px)^{1/2}+c$$

9.

RESPOSTA: D

10.

RESPOSTA: C

11.

a) 1

b) log

c) 1

12.

a) 3

b) 3/4

c) 1/2

13.

RESPOSTA: C (V-F-V-F)

I) VERDADEIRA

$$\lim_{x\to\infty}\frac{x-1}{x+1}=\lim_{x\to\infty}\frac{1-\frac{1}{x}}{1+\frac{1}{x}}=1$$

II) FALSA

$$\lim_{x\to\infty}\left(\frac{x-1}{x+1}\right)^{x}=\lim_{x\to\infty}\left(1-\frac{2}{x+1}\right)^{x}=\lim_{x\to\infty}\left[\left(1+\frac{1}{-\frac{x+1}{2}}\right)^{-\frac{x+1}{2}}\right]^{-2}\cdot\left(\frac{1-\frac{1}{x}}{1+\frac{1}{x}}\right)^{-1}=$$

$$=e^{-2}\cdot 1=\frac{1}{e^{2}}$$

III) VERDADEIRA

$$\lim_{x\to 0}\left(\frac{x^{2}-2x+3}{x^{2}-3x+2}\right)^{\frac{\operatorname{sen}x}{x}}=\lim_{x\to 0}\left(\frac{x^{2}-2x+3}{x^{2}-3x+2}\right)^{\frac{\operatorname{sen}x}{x}}=\left(\frac{3}{2}\right)^{1}=\frac{3}{2}$$

IV) FALSA

$$\lim_{x\to 0}(1+\operatorname{sen}x)^{\frac{1}{x}}=\lim_{x\to 0}\left[(1+\operatorname{sen}x)^{\frac{1}{\operatorname{sen}x}}\right]^{\frac{\operatorname{sen}x}{x}}=e^{1}=e$$

14.

RESPOSTA: D

$$\lim_{x\to-\infty}\frac{\sqrt{x^2+14}+x}{\sqrt{x^2-2}+x}\overset{t=-x}{=}\lim_{t\to+\infty}\frac{\sqrt{t^2+14}-t}{\sqrt{t^2-2}-t}=$$

$$\lim_{t\to+\infty}\frac{\left(\sqrt{t^2+14}-t\right)\left(\sqrt{t^2+14}+t\right)}{\sqrt{t^2+14}+t}\cdot\frac{\sqrt{t^2-2}+t}{\left(\sqrt{t^2-2}-t\right)\left(\sqrt{t^2-2}+t\right)}=$$

$$=\lim_{t\to+\infty}\frac{t^2+14-t^2}{\sqrt{t^2+14}+t}\cdot\frac{\sqrt{t^2-2}+t}{t^2-2-t^2}=-7\lim_{t\to+\infty}\frac{\sqrt{t^2-2}+t}{\sqrt{t^2+14}+t}=-7\lim_{t\to+\infty}\frac{\sqrt{1-\frac{2}{t^2}}+1}{\sqrt{1+\frac{14}{t^2}}+1}=-7\cdot\frac{1+1}{1+1}=-7$$

15.

RESPOSTA: B

$$A=2\lim_{x\to0}\left[\frac{\sqrt{x.\text{sen}\,6x}}{\frac{1}{\text{sen}\,6x}(\text{sen}^2\,6x)}\right]=2\lim_{x\to0}\left(\frac{\sqrt{x.\text{sen}\,6x}}{\text{sen}\,6x}\right)=2\lim_{x\to0}\left(\sqrt{\frac{x\,\text{sen}\,6x}{\text{sen}^2\,6x}}\right)=$$

$$=2\lim_{x\to0}\left(\sqrt{\frac{x}{\text{sen}\,6x}.\frac{6}{6}}\right)=2\lim_{x\to0}\left(\frac{1}{\sqrt{6}}.\sqrt{\frac{6x}{\text{sen}\,6x}}\right)=\frac{2}{\sqrt{6}}\left(\lim_{x\to0}\sqrt{\frac{6x}{\text{sen}\,6x}}\right)=$$

$$=\frac{\sqrt{6}}{3}.\frac{1}{\lim_{x\to0}\sqrt{\frac{\text{sen}\,6x}{6x}}}=\frac{\sqrt{6}}{3}.\underbrace{\sqrt{\lim_{x\to0}\frac{\text{sen}\,6x}{6x}}}_{1}=\frac{\sqrt{6}}{3}$$

$$B=2^{\lim_{x\to\log_2 3}(2x+1)}=2^{(2\log_2 3\,+\,1)}=2^{(\log_2 9+\log_2 2)}=2^{\log_2 18}=18$$

$$\Rightarrow\frac{A^2.B}{2}=\frac{\frac{2}{3}\cdot18}{2}=6$$

16.

RESPOSTA: A

y = $\lim_{x\to0}e^{(1+2x)^{\frac{1}{x}}}=e^{\lim_{x\to0}(1+2x)^{\frac{1}{x}}}$.

Fazendo u = 2x $\Rightarrow$ x = $\frac{u}{2}$. Se x $\to$0, então u $\to$ 0. Logo:

y = $e^{\lim\limits_{u\to 0}(1+u)^{\frac{2}{u}}} = e^{\left[\lim\limits_{u\to 0}(1+u)^{\frac{1}{u}}\right]^2} = e^{e^2} \Rightarrow$

$\Rightarrow$ ln y = ln $e^{e^2} = e^2$

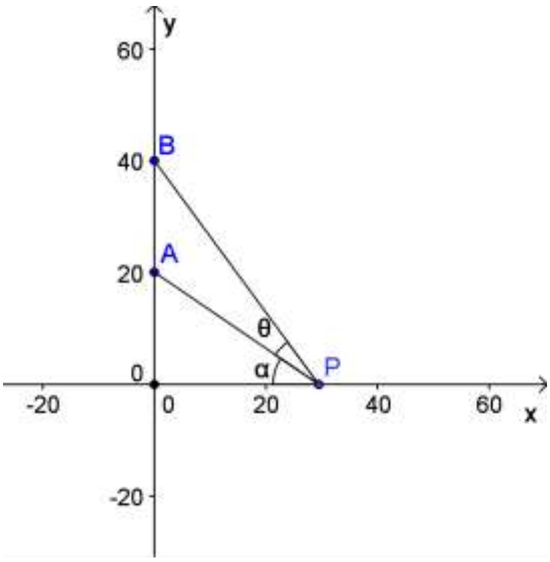
17.

a) 1

b) $\frac{1}{\sqrt{e}}$

18.

RESPOSTA: A



$$P(p,0) \Rightarrow \operatorname{tg}\alpha = \frac{20}{p},\ \operatorname{tg}(\alpha+\theta) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\theta}{1-\operatorname{tg}\alpha\cdot\operatorname{tg}\theta} = \frac{40}{p} \Rightarrow \frac{\frac{20}{p}+\operatorname{tg}\theta}{1-\frac{20}{p}\cdot\operatorname{tg}\theta} = \frac{40}{p}$$

$$\Leftrightarrow \frac{20+p\cdot\operatorname{tg}\theta}{p-20\cdot\operatorname{tg}\theta} = \frac{40}{p} \Leftrightarrow 20p + p^2\cdot\operatorname{tg}\theta = 40p - 800\cdot\operatorname{tg}\theta \Leftrightarrow \operatorname{tg}\theta = \frac{20p}{p^2+800}$$

O ângulo θ é máximo quando $\operatorname{tg}\theta$ é máximo, ou seja, quando $\frac{20p}{p^2+800}$ assume seu valor máximo.

$$\left(\frac{20p}{p^2+800}\right)' = \frac{20\cdot(p^2+800)-20p\cdot 2p}{(p^2+800)^2} = 0 \Leftrightarrow p^2 = 800 \Rightarrow p = 20\sqrt{2}$$

Observe que essa abscissa refere-se a um ponto de máximo, pois a derivada é positiva antes e negativa depois do ponto.