

GELSON IEZZI

FUNDAMENTOS DE MATEMÁTICA ELEMENTAR

Trigonometria

3

LIVRO PARA ANÁLISE
DO PROFESSOR
• VENDA PROIBIDA •

ABRELIVROS

ASSOCIAÇÃO BRASILEIRA
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 **Atual**
Editora

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Trigonometria

3

COMPLEMENTO PARA O PROFESSOR

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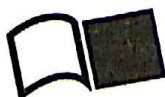
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Apresentação

Este livro é o *Complemento para o Professor* do volume 3, Trigonometria, da coleção *Fundamentos de Matemática Elementar*.

Cada volume desta coleção tem um complemento para o professor, com o objetivo de apresentar a solução dos exercícios mais complicados do livro e sugerir alguns encaminhamentos aos alunos.

É nossa intenção aperfeiçoar continuamente os *Complementos*. Estamos abertos às sugestões e críticas, que nos devem ser encaminhadas por intermédio da Editora.

Agradecemos à professora Erileide Maria de Sobral Souza a colaboração na redação das soluções que são apresentadas neste *Complemento*.

Os Autores.

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CAPÍTULO II — Razões trigonométricas no triângulo retângulo

6. $\operatorname{tg} \hat{B} = \frac{b}{c} = \frac{\sqrt{5}}{2} \Rightarrow b = \frac{c\sqrt{5}}{2}$
 $b^2 + c^2 = a^2 \Rightarrow \frac{5c^2}{4} + c^2 = 36 \Rightarrow c = 4$ e então $b = 2\sqrt{5}$

14. a) $\operatorname{sen} 20^\circ 15' = \operatorname{sen} 20^\circ + \frac{15}{60} (\operatorname{sen} 21^\circ - \operatorname{sen} 20^\circ) =$
 $= 0,34202 + \frac{15}{60} (0,35837 - 0,34202) = 0,34610$

b) $\cos 15^\circ 30' = \cos 15^\circ + \frac{30}{60} (\cos 16^\circ - \cos 15^\circ) =$
 $= 0,96593 + \frac{30}{60} (0,96126 - 0,96593) = 0,96358$

c) $\operatorname{tg} 12^\circ 40' = \operatorname{tg} 12^\circ + \frac{40}{60} (\operatorname{tg} 13^\circ - \operatorname{tg} 12^\circ) =$
 $= 0,21256 + \frac{40}{60} (0,23087 - 0,21256) = 0,22476$

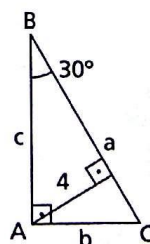
d) $\operatorname{sen} 50^\circ 12' = \operatorname{sen} 50^\circ + \frac{12}{60} (\operatorname{sen} 51^\circ - \operatorname{sen} 50^\circ) =$
 $= 0,76604 + \frac{12}{60} (0,77715 - 0,76604) = 0,76826$

e) $\cos 70^\circ 27' = \cos 70^\circ + \frac{27}{60} (\cos 71^\circ - \cos 70^\circ) =$
 $= 0,34202 + \frac{27}{60} (0,32557 - 0,34202) = 0,33462$

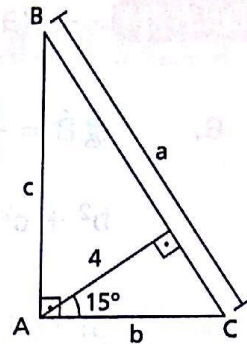
f) $\operatorname{tg} 80^\circ 35' = \operatorname{tg} 80^\circ + \frac{35}{60} (\operatorname{tg} 81^\circ - \operatorname{tg} 80^\circ) =$
 $= 5,67128 + \frac{35}{60} (6,31375 - 5,67128) = 6,04605$

15. $b = 4 \cdot \operatorname{tg} 35^\circ \Rightarrow b = 2,80084 \text{ cm}$
 $a = \frac{4}{\cos 35^\circ} \Rightarrow a = 4,88311 \text{ cm}$

16. $c \cdot \operatorname{sen} 30^\circ = 4 \Rightarrow c = 8$
 $b = c \cdot \operatorname{tg} 30^\circ \Rightarrow b = \frac{8\sqrt{3}}{3}$
 $a^2 = b^2 + c^2 \Rightarrow a = \frac{16\sqrt{3}}{3}$

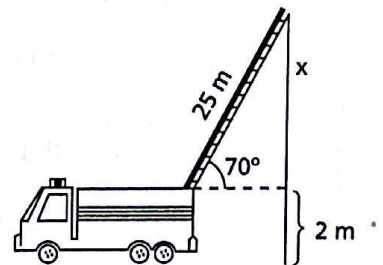


17. $b \cdot \cos 15^\circ = 4 \Rightarrow b = 4\sqrt{2}(\sqrt{3} - 1)$
 $c \cdot \cos 75^\circ = 4 \Rightarrow c = 4\sqrt{2}(\sqrt{3} + 1)$
 $a^2 = b^2 + c^2 \Rightarrow a = 16$

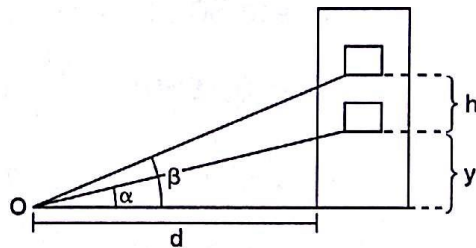


18. $h = y \cdot \operatorname{tg} 30^\circ$ e $h = x \cdot \operatorname{tg} 60^\circ \Rightarrow \frac{x}{y} = \frac{1}{3}$

19. $x = 25 \cdot \operatorname{sen} 70^\circ \Rightarrow x = 23,49$ m
 $x + 2 = 25,49$ m

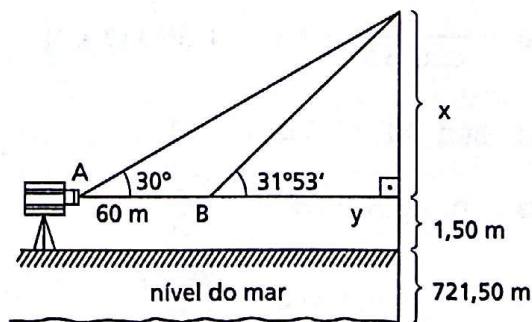


21. $d = \frac{h+y}{\operatorname{tg} \beta}$ e $d = \frac{y}{\operatorname{tg} \alpha} \Rightarrow$
 $\Rightarrow h = d(\operatorname{tg} \beta - \operatorname{tg} \alpha)$



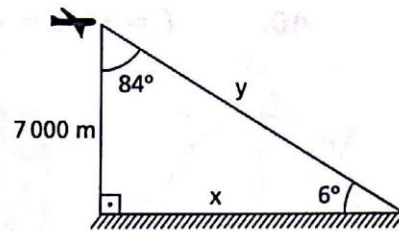
22. $AB = \frac{H-h}{\operatorname{tg} \beta}$ e $AB = \frac{h}{\operatorname{tg} \alpha} \Rightarrow H = h \left(\frac{\operatorname{tg} \beta}{\operatorname{tg} \alpha} + 1 \right)$

23. $x = y \cdot \operatorname{tg} 31^\circ 53'$ e $x = (60 + y) \cdot \operatorname{tg} 30^\circ \Rightarrow x = 503,57$ m
 $503,57 + 1,50 + 721,50 = 1226,57$ m

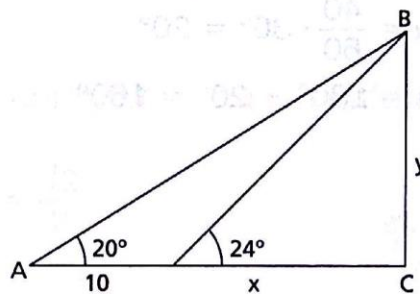


24. $x = 7000 \cdot \operatorname{tg} 84^\circ \Rightarrow x = 66,60 \text{ km}$

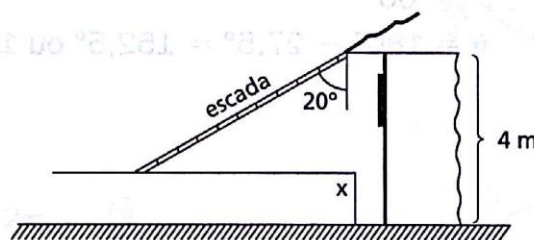
$y = \frac{7000}{\cos 84^\circ} \Rightarrow y = 66,97 \text{ km}$



25. $y = x \cdot \operatorname{tg} 24^\circ$ e $y = (10 + x) \cdot \operatorname{tg} 20^\circ \Rightarrow x = 44,72 \text{ m}$



26. $4 - x = 3 \cdot \cos 20^\circ \Rightarrow x = 1,18 \text{ m}$

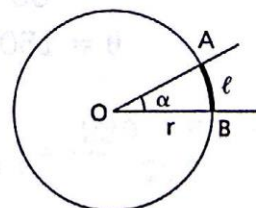


CAPÍTULO III — Arcos e ângulos

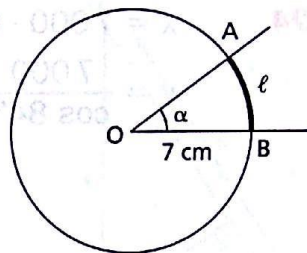
35.
$$\left. \begin{aligned} a - b &= \frac{\pi}{12} \\ a + b &= \frac{7\pi}{4} \end{aligned} \right\} \Rightarrow a = \frac{165\pi}{180} = \frac{11\pi}{12} \text{ rad}, b = \frac{150\pi}{180} = \frac{5\pi}{6} \text{ rad}$$

36.
$$\left. \begin{aligned} a + b + c &= 13^\circ \\ a + b + 2c &= 15^\circ \\ a + 2b + c &= 20^\circ \end{aligned} \right\} \Rightarrow a = 4^\circ, b = 7^\circ \text{ e } c = 2^\circ$$

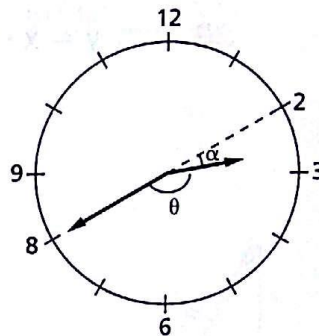
39. $\ell = \frac{2\pi r}{3}$
 $\alpha = \frac{\ell}{r} \text{ rad} \Rightarrow \alpha = \frac{2\pi}{3} \text{ rad ou } 120^\circ$



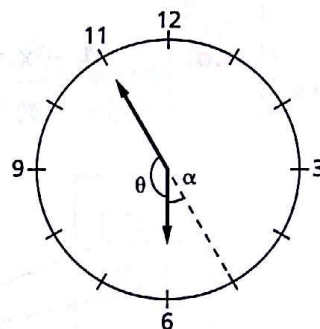
40. $l = \alpha \cdot r = 4,5 \cdot 7 = 31,5 \text{ cm}$



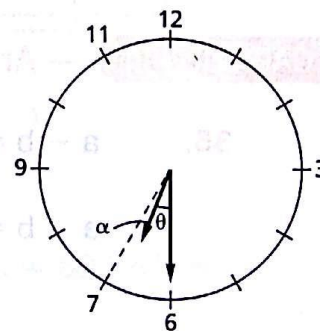
42. a) $\alpha = \frac{40}{60} \cdot 30^\circ = 20^\circ$
 $\theta = 180^\circ - 20^\circ = 160^\circ$



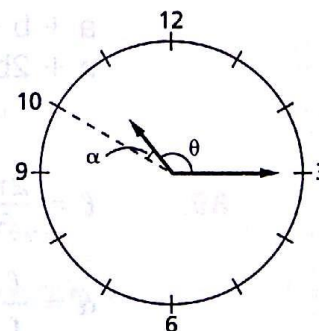
b) $\alpha = \frac{55}{60} \cdot 30^\circ = 27,5^\circ$
 $\theta = 180^\circ - 27,5^\circ = 152,5^\circ \text{ ou } 152^\circ 30'$



c) $\alpha = \frac{30 \cdot 30^\circ}{60} = 15^\circ$
 $\theta = 30^\circ - 15^\circ = 15^\circ$

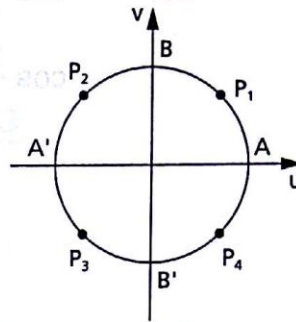


d) $\alpha = \frac{15}{60} \cdot 30^\circ = 7,5^\circ$
 $\theta = 150^\circ - 7,5^\circ = 142,5^\circ \text{ ou } 142^\circ 30'$



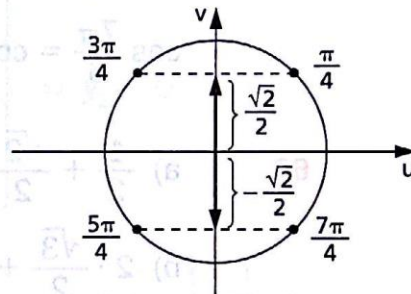
44. $\widehat{AP}_1 = \frac{1}{8} \cdot 2\pi = \frac{\pi}{4} \text{ rad}$

Imagens de x	A	P ₁	B	P ₂	A'	P ₃	B'	P ₄
x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$

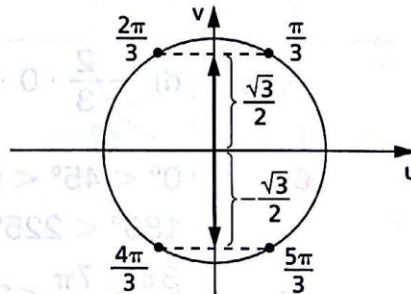


CAPÍTULO IV — Razões trigonométricas na circunferência

49. $\text{sen } \frac{\pi}{4} = \text{sen } \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$
 $\text{sen } \frac{5\pi}{4} = \text{sen } \frac{7\pi}{4} = -\frac{\sqrt{2}}{2}$



51. $\text{sen } \frac{\pi}{3} = \text{sen } \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$
 $\text{sen } \frac{4\pi}{3} = \text{sen } \frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$



52. a) $\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} - 0 = \frac{\sqrt{3} + \sqrt{2}}{2}$

b) $2 \cdot \frac{1}{2} + \frac{1}{2} \cdot \left(-\frac{\sqrt{2}}{2}\right) = \frac{4 - \sqrt{2}}{4}$

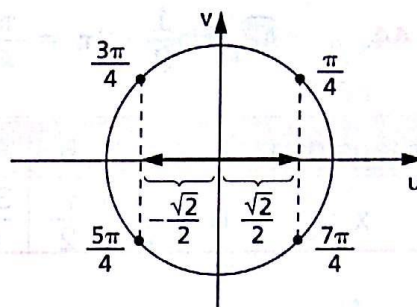
c) $3(1) - 2\left(-\frac{\sqrt{2}}{2}\right) + \frac{1}{2} \cdot 0 = 3 + \sqrt{2}$

d) $-\frac{2}{3}(-1) + \frac{3}{5} \cdot \left(-\frac{\sqrt{3}}{2}\right) - \frac{6}{7}\left(-\frac{1}{2}\right) = \frac{230 - 63\sqrt{3}}{210}$

57.

$$\cos \frac{\pi}{4} = \cos \frac{7\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos \frac{3\pi}{4} = \cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$$

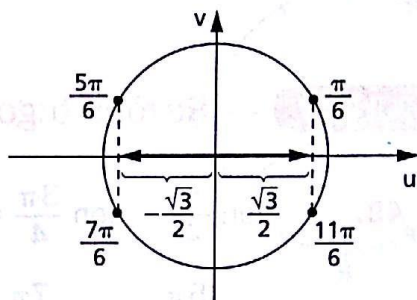


58.

$$\cos \frac{\pi}{6} = \cos \frac{11\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\cos \frac{5\pi}{6} = -\cos \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\cos \frac{7\pi}{6} = \cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$$



60.

a) $\frac{1}{2} + \frac{\sqrt{2}}{2} - 1 = \frac{-1 + \sqrt{2}}{2}$

b) $2 \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{4\sqrt{3} + \sqrt{2}}{4}$

c) $3 \cdot 0 - 2 \left(-\frac{\sqrt{2}}{2} \right) + \frac{1}{2} (-1) = \frac{2\sqrt{2} - 1}{2}$

d) $-\frac{2}{3} \cdot 0 + \frac{3}{5} \left(\frac{1}{2} \right) - \frac{6}{7} \left(-\frac{\sqrt{3}}{2} \right) = \frac{21 + 30\sqrt{3}}{70}$

63.

$0^\circ < 45^\circ < 90^\circ \Rightarrow (\text{sen } 45^\circ > 0 \text{ e } \text{cos } 45^\circ > 0) \Rightarrow y_1 > 0$

$180^\circ < 225^\circ < 270^\circ \Rightarrow (\text{sen } 225^\circ < 0 \text{ e } \text{cos } 225^\circ < 0) \Rightarrow y_2 < 0$

$\frac{3\pi}{2} < \frac{7\pi}{4} < 2\pi \Rightarrow \left(\text{sen } \frac{7\pi}{4} < 0 \text{ e } \text{cos } \frac{7\pi}{4} > 0 \text{ e } \left| \text{sen } \frac{7\pi}{4} \right| = \left| \text{cos } \frac{7\pi}{4} \right| \right) \Rightarrow$

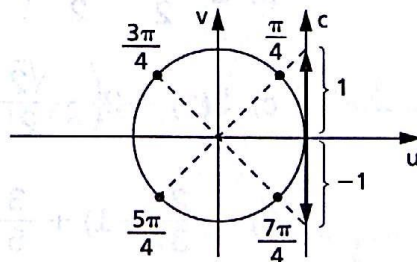
$\Rightarrow y_3 = 0$

$270^\circ < 300^\circ < 315^\circ < 360^\circ \Rightarrow (|\text{sen } 300^\circ| > |\text{cos } 300^\circ|;$
 $\text{sen } 300^\circ < 0 \text{ e } \text{cos } 300^\circ > 0) \Rightarrow y_4 < 0$

66.

$\text{tg } \frac{\pi}{4} = \text{tg } \frac{5\pi}{4} = 1$

$\text{tg } \frac{3\pi}{4} = \text{tg } \frac{7\pi}{4} = -1$



69.

a) $\sqrt{3} + 1 - 0 = 1 + \sqrt{3}$

b) $2 \cdot \frac{\sqrt{3}}{3} + \frac{1}{2}(-1) = \frac{4\sqrt{3} - 3}{6}$

c) $-2(1) + \frac{1}{2} \cdot (0) - \frac{1}{3} \left(-\frac{\sqrt{3}}{3} \right) = \frac{-18 + \sqrt{3}}{9}$

d) $\frac{3}{5}(-\sqrt{3}) - \frac{6}{7} \left(\frac{\sqrt{3}}{3} \right) - \frac{2}{3}(0) = \frac{-31\sqrt{3}}{35}$

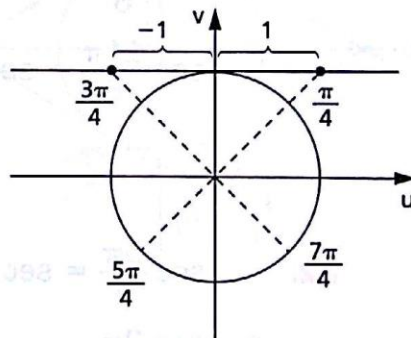
71.

a) $\left. \begin{array}{l} 180^\circ < 269^\circ < 270^\circ \Rightarrow \operatorname{tg} 269^\circ > 0 \\ 90^\circ < 178^\circ < 180^\circ \Rightarrow \operatorname{sen} 178^\circ > 0 \end{array} \right\} \Rightarrow y_1 > 0$

b) $\left. \begin{array}{l} 0 < \frac{5\pi}{11} < \frac{\pi}{2} \Rightarrow \operatorname{sen} \frac{5\pi}{11} > 0 \\ \frac{3\pi}{2} < \frac{23\pi}{12} < 2\pi \Rightarrow \operatorname{cos} \frac{23\pi}{12} > 0 \\ \frac{3\pi}{2} < \frac{12\pi}{7} < 2\pi \Rightarrow \operatorname{tg} \frac{12\pi}{7} < 0 \end{array} \right\} \Rightarrow y_2 < 0$

74. $\operatorname{cotg} \frac{\pi}{4} = \operatorname{cotg} \frac{5\pi}{4} = 1$

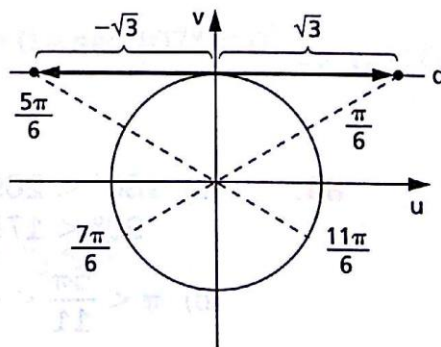
$\operatorname{cotg} \frac{3\pi}{4} = \operatorname{cotg} \frac{7\pi}{4} = -1$



75. $\operatorname{cotg} \frac{\pi}{6} = \operatorname{cotg} \frac{7\pi}{6} = \sqrt{3}$

$\operatorname{cotg} \frac{5\pi}{6} = -\operatorname{cotg} \frac{\pi}{6} = -\sqrt{3}$

$\operatorname{cotg} \frac{5\pi}{6} = \operatorname{cotg} \frac{11\pi}{6} = -\sqrt{3}$



77.

a) $\frac{\sqrt{3}}{3} + 1 + \sqrt{3} = \frac{3 + 4\sqrt{3}}{3}$

b) $2 \left(\frac{-\sqrt{3}}{3} \right) - \frac{1}{2} (-\sqrt{3}) = -\frac{\sqrt{3}}{6}$

$$c) \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} - (-\sqrt{3}) + \sqrt{3} = \frac{5\sqrt{3} + \sqrt{2}}{2}$$

$$d) \frac{3}{5} \left(-\frac{\sqrt{3}}{3} \right) - \frac{6}{7} \cdot \sqrt{3} - \frac{2}{3}(-1) + \frac{4}{5} \cdot \left(-\frac{\sqrt{2}}{2} \right) = \frac{-111\sqrt{3} - 42\sqrt{2} + 70}{105}$$

79. a) $180^\circ < 269^\circ < 270^\circ \Rightarrow \cotg 269^\circ > 0$
 $90^\circ < 178^\circ < 180^\circ \Rightarrow \text{sen } 178^\circ > 0$ } $\Rightarrow y_1 > 0$

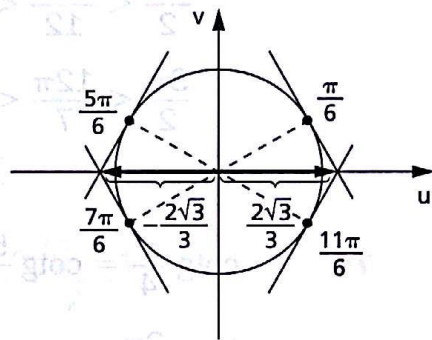
b) $\frac{\pi}{2} < \frac{5\pi}{11} < \pi \Rightarrow \text{sen } \frac{5\pi}{11} > 0$
 $\frac{3\pi}{2} < \frac{23\pi}{12} < 2\pi \Rightarrow \cos \frac{23\pi}{12} > 0$
 $\frac{3\pi}{2} < \frac{12\pi}{7} < 2\pi \Rightarrow \cotg \frac{12\pi}{7} < 0$ } $\Rightarrow y_2 < 0$

81.

$$\sec \frac{5\pi}{6} = -\sec \frac{\pi}{6} = -\frac{2\sqrt{3}}{3}$$

$$\sec \frac{7\pi}{6} = -\sec \frac{\pi}{6} = -\frac{2\sqrt{3}}{3}$$

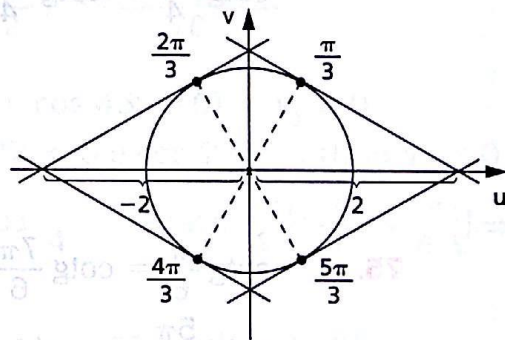
$$\sec \frac{11\pi}{6} = \sec \frac{\pi}{6} = \frac{2\sqrt{3}}{3}$$



82.

$$\sec \frac{5\pi}{3} = \sec \frac{\pi}{3} = 2$$

$$\sec \frac{2\pi}{3} = \sec \frac{4\pi}{3} = -2$$

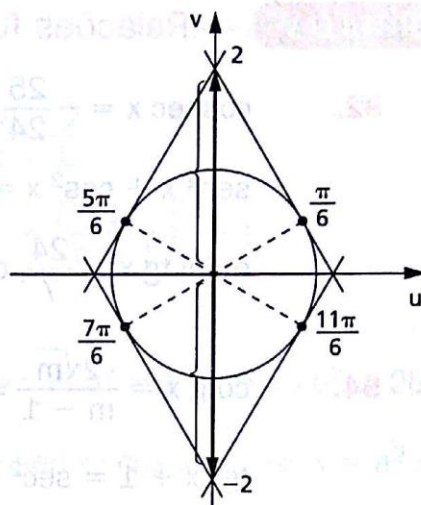


84.

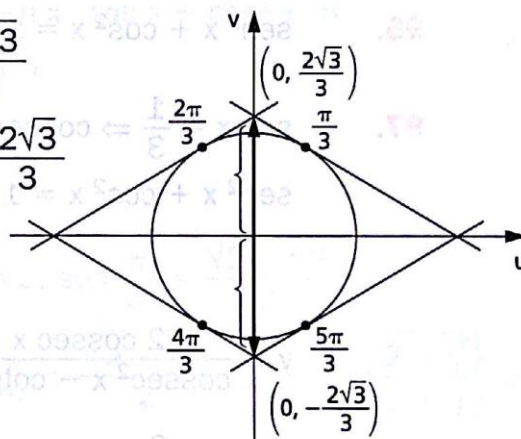
a) $180^\circ < 269^\circ < 270^\circ \Rightarrow \sec 269^\circ < -1$
 $90^\circ < 178^\circ < 180^\circ \Rightarrow 0 < \text{sen } 178^\circ < 1$ } $\Rightarrow y_1 < 0$

b) $\pi < \frac{5\pi}{11} < \frac{\pi}{2} \Rightarrow \text{sen } \frac{5\pi}{11} > 0$
 $\frac{3\pi}{2} < \frac{23\pi}{12} < 2\pi \Rightarrow \cos \frac{23\pi}{12} > 0$
 $\frac{3\pi}{2} < \frac{12\pi}{7} < 2\pi \Rightarrow \sec \frac{17\pi}{7} > 0$ } $\Rightarrow y_2 > 0$

86. $\operatorname{cosec} \frac{5\pi}{6} = \operatorname{cosec} \frac{\pi}{6} = 2$
 $\operatorname{cosec} \frac{7\pi}{6} = \operatorname{cosec} \frac{11\pi}{6} = -2$



87. $\operatorname{cosec} \frac{2\pi}{3} = \operatorname{cosec} \frac{\pi}{3} = \frac{2\sqrt{3}}{3}$
 $\operatorname{cosec} \frac{4\pi}{3} = \operatorname{cosec} \frac{5\pi}{3} = -\frac{2\sqrt{3}}{3}$



- 89.** a) $90^\circ < 91^\circ < 180^\circ \Rightarrow \cos 91^\circ < 0 \text{ e } \operatorname{cosec} 91^\circ > 0 \Rightarrow y_1 > 0$
 $|\cos 91^\circ| < |\operatorname{cosec} 91^\circ|$
- b) $90^\circ < 107^\circ < 180^\circ \Rightarrow \operatorname{sen} 107^\circ > 0 \text{ e } \sec 107^\circ < 0 \Rightarrow y_2 < 0$
 $|\cos 107^\circ| < |\sec 107^\circ|$
- c) $0 < \frac{\pi}{7} < \frac{\pi}{2} \Rightarrow \operatorname{cotg} \frac{\pi}{7} > 0$
 $\pi < \frac{7\pi}{6} < \frac{3\pi}{2} \Rightarrow \operatorname{tg} \frac{7\pi}{6} > 0$
 $\pi < \frac{9\pi}{8} < \frac{3\pi}{2} \Rightarrow \sec \frac{9\pi}{8} < 0$ $\Rightarrow y_3 < 0$

90. $\left(2 + \frac{1}{2}\right)\left(\frac{\sqrt{2}}{2} - 2\right) = 1,25(\sqrt{2} - 4)$

CAPÍTULO V — Relações fundamentais

92. $\operatorname{cosec} x = -\frac{25}{24} \Rightarrow \operatorname{sen} x = -\frac{24}{25}$

$\operatorname{sen}^2 x + \operatorname{cos}^2 x = 1 \Rightarrow \operatorname{cos} x = -\frac{7}{25}$

e daí $\operatorname{tg} x = \frac{24}{7}$, $\operatorname{cotg} x = \frac{7}{24}$ e $\operatorname{sec} x = -\frac{25}{7}$

94. $\operatorname{cotg} x = \frac{2\sqrt{m}}{m-1} \Rightarrow \operatorname{tg} x = \frac{(m-1)\sqrt{m}}{2m}$

$\operatorname{tg}^2 x + 1 = \operatorname{sec}^2 x \Rightarrow \operatorname{sec} x = \pm \frac{m+1}{2\sqrt{m}} \Rightarrow \operatorname{cos} x = \pm \frac{2\sqrt{m}}{m+1}$

95. $\operatorname{sen}^2 x + \operatorname{cos}^2 x = 1 \Rightarrow \operatorname{cos} x = \pm \frac{(a^2 - b^2)}{a^2 + b^2} \Rightarrow \operatorname{sec} x = \pm \frac{(a^2 + b^2)}{a^2 - b^2}$

97. $\operatorname{sen} x = \frac{1}{3} \Rightarrow \operatorname{cosec} x = 3$

$\left. \begin{array}{l} \operatorname{sen}^2 x + \operatorname{cos}^2 x = 1 \Rightarrow \operatorname{cos} x = \frac{2\sqrt{2}}{3} \\ \operatorname{sen} x = \frac{1}{3} \end{array} \right\} \Rightarrow \operatorname{cotg} x = 2\sqrt{2}$

$y = \frac{2 \operatorname{cosec} x}{\operatorname{cosec}^2 x - \operatorname{cotg}^2 x} = \frac{2 \cdot 3}{9 - 8} \Rightarrow y = 6$

99. $\operatorname{cos} x = \frac{2}{5} \Rightarrow \operatorname{sec} x = \frac{5}{2}$; $\operatorname{sec}^2 x = \operatorname{tg}^2 x + 1 \Rightarrow \operatorname{tg}^2 x = \frac{21}{4}$

$y = \left(1 + \frac{21}{4}\right)^2 + \left(1 - \frac{21}{4}\right)^2 \Rightarrow y = \frac{457}{8}$

101. $5 \operatorname{sec} x - 3(\operatorname{sec}^2 x - 1) = 1 \Rightarrow 3 \operatorname{sec}^2 x - 5 \operatorname{sec} x - 2 = 0$

e daí $\operatorname{sec} x = -\frac{1}{3} \Rightarrow \operatorname{cos} x = -3$ (não convém), $\operatorname{sec} x = 2 \Rightarrow$

$\Rightarrow \operatorname{cos} x = \frac{1}{2}$

$\operatorname{sen}^2 x + \operatorname{cos}^2 x = 1 \Rightarrow \operatorname{sen} x = \pm \frac{\sqrt{3}}{2}$

102. $\operatorname{sen}^2 x + \operatorname{cos}^2 x - 5 \operatorname{sen} x \operatorname{cos} x = 3 \Rightarrow -5 \operatorname{sen} x \operatorname{cos} x = 2 \Rightarrow$

$\Rightarrow \frac{-5 \operatorname{sen} x \operatorname{cos} x}{\operatorname{cos}^2 x} = \frac{2}{\operatorname{cos}^2 x} \Rightarrow -5 \operatorname{tg} x = 2(1 + \operatorname{tg}^2 x) \Rightarrow$

$\Rightarrow 2 \operatorname{tg}^2 x + 5 \operatorname{tg} x + 2 = 0 \Rightarrow \operatorname{tg} x = -2$ ou $\operatorname{tg} x = -\frac{1}{2}$

104. $\cotg x = \frac{1}{\operatorname{tg} x} \Rightarrow \frac{m}{3} = \frac{1}{m-2} \Rightarrow m^2 - 2m - 3 = 0 \Rightarrow m = 3 \text{ ou } m = -1$

105. $\operatorname{cossec} x = \frac{a+1}{\sqrt{a+2}} \Rightarrow \operatorname{sen} x = \frac{\sqrt{a+2}}{a+1}$

$\operatorname{sen}^2 x + \operatorname{cos}^2 x = 1 \Rightarrow \left(\frac{\sqrt{a+2}}{a+1}\right)^2 + \left(\frac{1}{a+1}\right)^2 = 1 \Rightarrow$
 $\Rightarrow a^2 + a - 2 = 0 \Rightarrow a = 1$

108. $(\operatorname{sen} x + \operatorname{cos} x)^2 = 1 + 2 \operatorname{sen} x \operatorname{cos} x \Rightarrow a^2 = 1 + 2b \Rightarrow a^2 - 2b = 1$

110. $\operatorname{sen} x + \operatorname{cos} x = a \Rightarrow \operatorname{sen}^2 x + 2 \cdot \operatorname{sen} x \cdot \operatorname{cos} x + \operatorname{cos}^2 x = a^2 \Rightarrow$
 $\Rightarrow \operatorname{sen} x \cdot \operatorname{cos} x = \frac{a^2 - 1}{2}$

$y = (\operatorname{sen} x + \operatorname{cos} x)(\operatorname{sen}^2 x - \operatorname{sen} x \cdot \operatorname{cos} x + \operatorname{cos}^2 x) =$
 $= a \cdot \left(1 - \frac{a^2 - 1}{2}\right) = \frac{a(3 - a^2)}{2}$

CAPÍTULO VI — Arcos notáveis

112. $\ell_8 = \sqrt{1(2 - \sqrt{4 - 2})} = \sqrt{2 - \sqrt{2}}; \operatorname{sen} \frac{\pi}{8} = \frac{\sqrt{2 - \sqrt{2}}}{2}$

$\operatorname{sen}^2 \frac{\pi}{8} + \operatorname{cos}^2 \frac{\pi}{8} = 1 \Rightarrow \operatorname{cos} \frac{\pi}{8} = \frac{\sqrt{2 + \sqrt{2}}}{2}; \operatorname{tg} \frac{\pi}{8} = \sqrt{\frac{2 - \sqrt{2}}{2 + \sqrt{2}}} \Rightarrow$
 $\Rightarrow \operatorname{tg} \frac{\pi}{8} = -1 + \sqrt{2}$

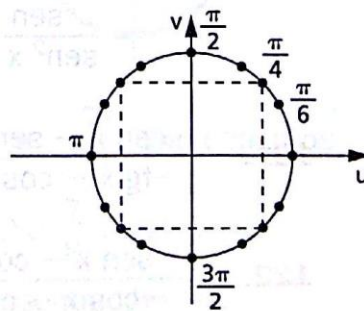
113. $\operatorname{sen} \frac{\pi}{5} = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$ e $\operatorname{sen}^2 \frac{\pi}{5} + \operatorname{cos}^2 \frac{\pi}{5} = 1 \Rightarrow \operatorname{cos} \frac{\pi}{5} = \frac{\sqrt{6 + 2\sqrt{5}}}{4}$

$\operatorname{tg} \frac{\pi}{5} = \sqrt{5 - 2\sqrt{5}}; \operatorname{sen} \frac{\pi}{10} = \frac{\sqrt{5} - 1}{4}$ e $\operatorname{sen}^2 \frac{\pi}{10} + \operatorname{cos}^2 \frac{\pi}{10} = 1 \Rightarrow$
 $\Rightarrow \operatorname{cos} \frac{\pi}{10} = \frac{\sqrt{10 + 2\sqrt{5}}}{4}; \operatorname{tg} \frac{\pi}{10} = \frac{\sqrt{5} - 1}{\sqrt{10 + 2\sqrt{5}}} = \frac{\sqrt{25 - 10\sqrt{5}}}{5}$

115. $A = \left\{0, \frac{1}{2}, \frac{\sqrt{3}}{2}, 1, -\frac{1}{2}, -\frac{\sqrt{3}}{2}, -1\right\}$

$B = \left\{1, \frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2}, -1\right\}$

$A \cap B = \{-1, 0, 1\}$



CAPÍTULO VII — Redução ao 1º quadrante

118. $\operatorname{sen}\left(x + \frac{\pi}{2}\right) = \operatorname{sen}\left[\pi - \left(\frac{\pi}{2} + x\right)\right] = \operatorname{sen}\left(\frac{\pi}{2} - x\right) = \cos x = \frac{3}{5}$

119. a) $\operatorname{sen}^2 x + \cos^2 x = 1 \Rightarrow \cos x = \frac{\sqrt{3}}{2}$

b) $\cos\left(x + \frac{\pi}{2}\right) = -\cos\left[\pi - \left(\frac{\pi}{2} + x\right)\right] = -\cos\left(\frac{\pi}{2} - x\right) =$
 $= -\operatorname{sen} x = -\frac{1}{2}$

c) $\operatorname{sen}\left(x + \frac{\pi}{2}\right) = \operatorname{sen}\left[\pi - \left(\frac{\pi}{2} + x\right)\right] = \operatorname{sen}\left(\frac{\pi}{2} - x\right) = \cos x = \frac{\sqrt{3}}{2}$

d) $\operatorname{tg}\left(x + \frac{\pi}{2}\right) = \frac{\operatorname{sen}\left(x + \frac{\pi}{2}\right)}{\cos\left(x + \frac{\pi}{2}\right)} = -\sqrt{3}$

e) $\operatorname{cotg}\left(x + \frac{\pi}{2}\right) = \frac{1}{\operatorname{tg}\left(x + \frac{\pi}{2}\right)} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

f) $\sec\left(x + \frac{\pi}{2}\right) = \frac{1}{\cos\left(x + \frac{\pi}{2}\right)} = -2$

g) $\operatorname{cossec}\left(x + \frac{\pi}{2}\right) = \frac{1}{\operatorname{sen}\left(x + \frac{\pi}{2}\right)} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

120. a) $[\operatorname{sen} x + \operatorname{sen} x][\operatorname{cotg} x + \operatorname{cotg} x] = 2 \operatorname{sen} x \cdot 2 \operatorname{cotg} x =$
 $= 4 \operatorname{sen} x \cdot \frac{\cos x}{\operatorname{sen} x} = 4 \cos x$

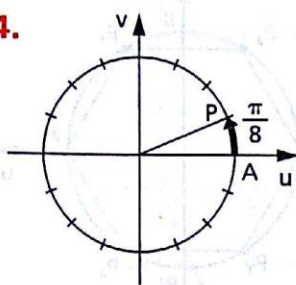
b) $\frac{\operatorname{tg} x - \sec x}{[\operatorname{tg} x + \operatorname{cossec} x] \cdot \operatorname{sen} x} = \frac{\frac{\operatorname{sen} x - 1}{\cos x}}{\left(\frac{\operatorname{sen}^2 x + \cos x}{\cos x \cdot \operatorname{sen} x}\right) \cdot \operatorname{sen} x} =$
 $= \frac{\operatorname{sen} x - 1}{\operatorname{sen}^2 x + \cos x}$

121. $\frac{\operatorname{sen} x - \operatorname{sen} x + \operatorname{tg} x}{-\operatorname{tg} x - \cos x + \cos x} = \frac{\operatorname{tg} x}{-\operatorname{tg} x} = -1$

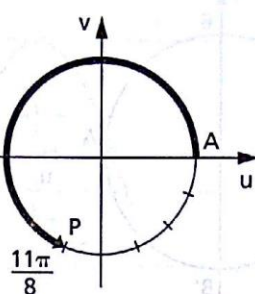
122. $\frac{-\operatorname{sen} x - \cos x + \cos x + 3 \operatorname{sen} x}{-\cos x - \cos x - \operatorname{sen} x + \operatorname{sen} x} = \frac{2 \operatorname{sen} x}{-2 \cos x} = -\operatorname{tg} x$

CAPÍTULO VIII — Funções circulares

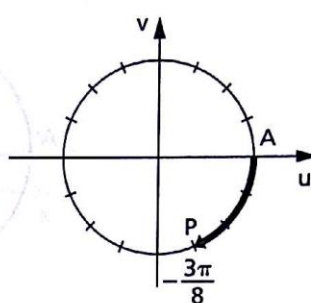
124.



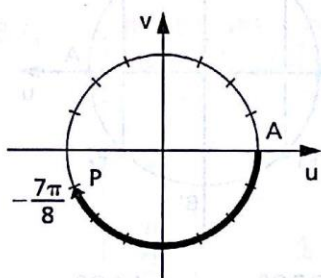
$\widehat{AP} = \frac{1}{16}$ do ciclo, no sentido anti-horário



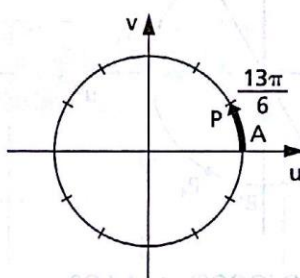
$\widehat{AP} = \frac{11}{16}$ do ciclo, no sentido anti-horário



$\widehat{AP} = \frac{3}{16}$ do ciclo, no sentido horário

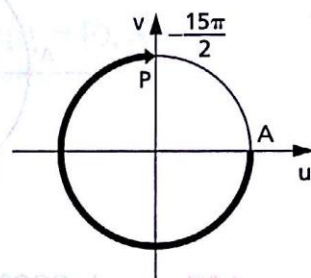


$\widehat{AP} = \frac{7}{16}$ do ciclo, no sentido horário



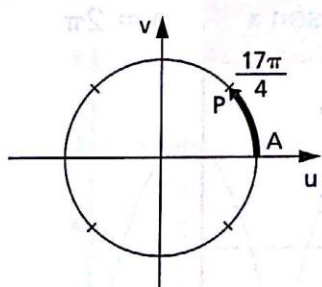
$\frac{13\pi}{6}$ e $\frac{\pi}{6}$ são côngruos

$\widehat{AP} = \frac{1}{12}$ do ciclo, no sentido anti-horário



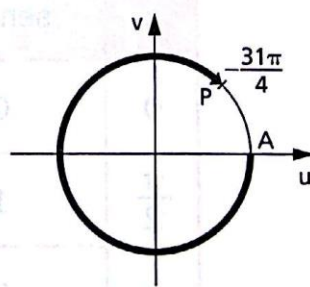
$-\frac{15\pi}{2}$ e $-\frac{3\pi}{2}$ são côngruos

$\widehat{AP} = \frac{3}{4}$ do ciclo, no sentido horário



$\frac{17\pi}{4}$ e $\frac{\pi}{4}$ são côngruos

\widehat{AP} é $\frac{1}{8}$ do ciclo, no sentido anti-horário



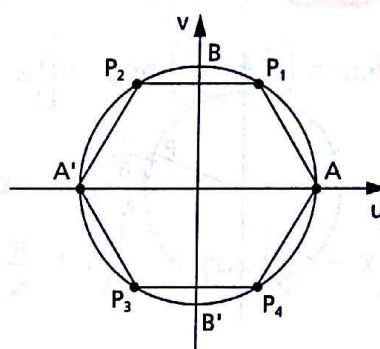
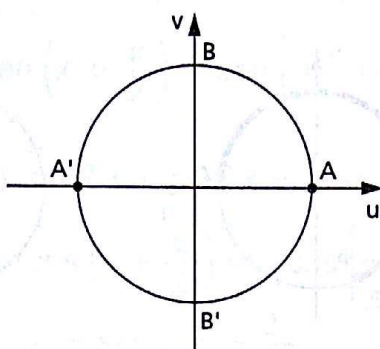
$-\frac{31\pi}{4}$ e $-\frac{7\pi}{4}$ são côngruos

\widehat{AP} é $\frac{7}{8}$ do ciclo, no sentido horário

126.

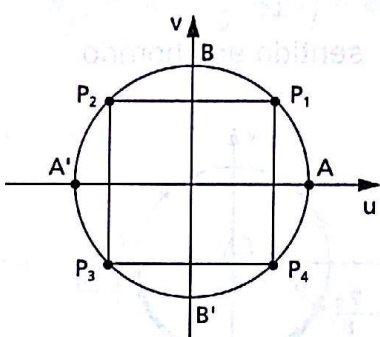
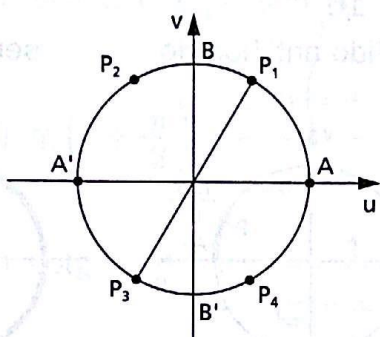
$$E = \{A', A\}$$

$$F = \{A, P_1, P_2, A', P_3, P_4\}$$



$$G = \{P_1, P_3\}$$

$$H = \{P_1, P_2, P_3, P_4\}$$



127.

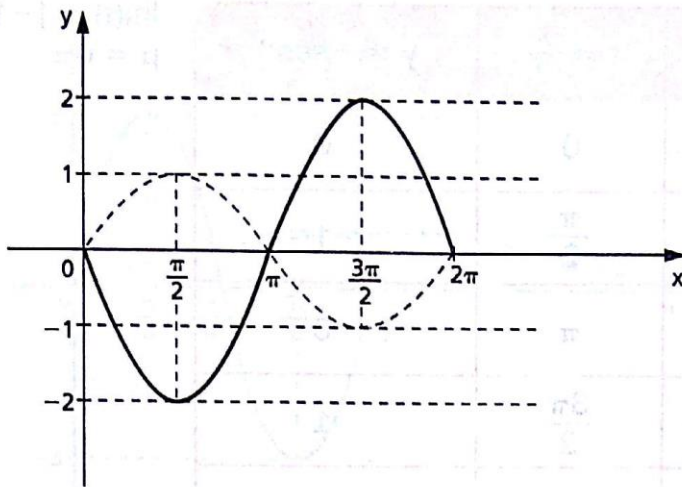
- a) $830^\circ = 2(360^\circ) + 110^\circ \Rightarrow \sin 830^\circ = \sin 110^\circ = \sin 70^\circ$
 $1195^\circ = 3(360^\circ) + 115^\circ \Rightarrow \sin 1195^\circ = \sin 115^\circ = \sin 65^\circ$
 $0^\circ < x < 90^\circ \Rightarrow \sin x \text{ é crescente} \Rightarrow \sin 830^\circ > \sin 1195^\circ$
- b) $-535^\circ = -360^\circ - 175^\circ \Rightarrow \cos(-535^\circ) = \cos(-175^\circ) = -\cos 5^\circ$
 $\cos 190^\circ = -\cos 10^\circ; 0^\circ < x < 90^\circ \Rightarrow \cos x \text{ é decrescente} \Rightarrow$
 $\Rightarrow \cos 5^\circ > \cos 10^\circ \Rightarrow \cos 190^\circ > \cos(-535^\circ)$

130.

x	sen x	y = -2 sen x
0	0	0
$\frac{\pi}{2}$	1	-2
π	0	0
$\frac{3\pi}{2}$	-1	2
2π	0	0

$$\text{Im}(f) = [-2, 2]$$

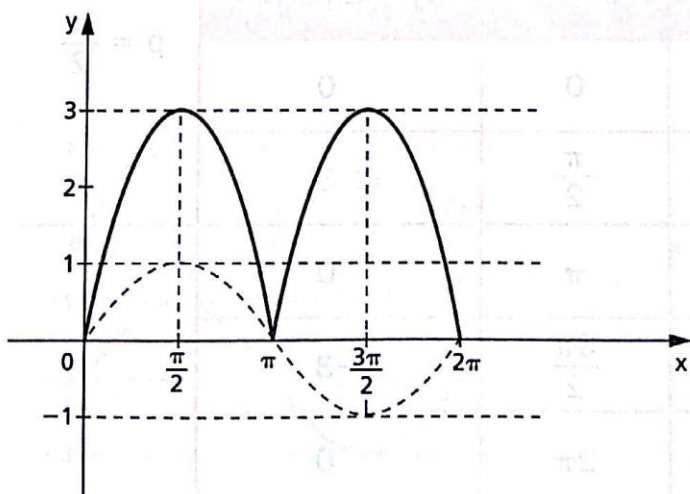
$$p = 2\pi$$



132.

x	sen x	$y = 3 \text{ sen } x $
0	0	0
$\frac{\pi}{2}$	1	3
π	0	0
$\frac{3\pi}{2}$	-1	3
2π	0	0

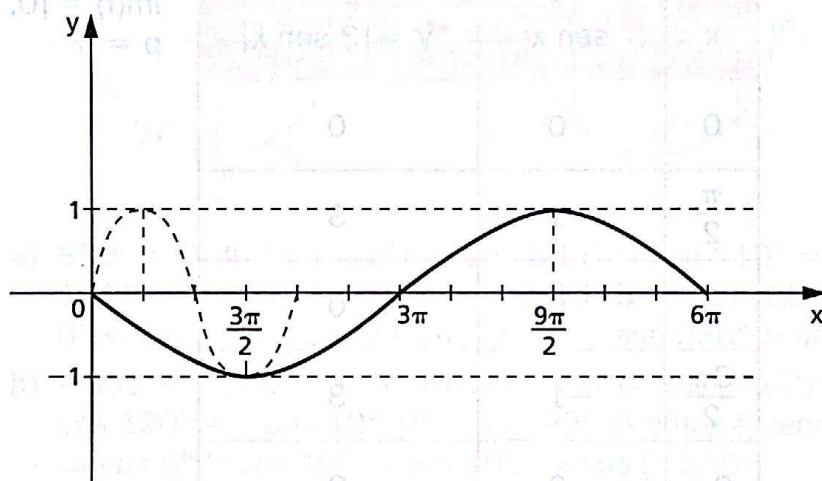
$\text{Im}(f) = [0, 3]$
 $p = \pi$



136.

x	$t = \frac{x}{3}$	$y = -\text{sen } t$
0	0	0
$\frac{3\pi}{2}$	$\frac{\pi}{2}$	-1
3π	π	0
$\frac{9\pi}{2}$	$\frac{3\pi}{2}$	1
6π	2π	0

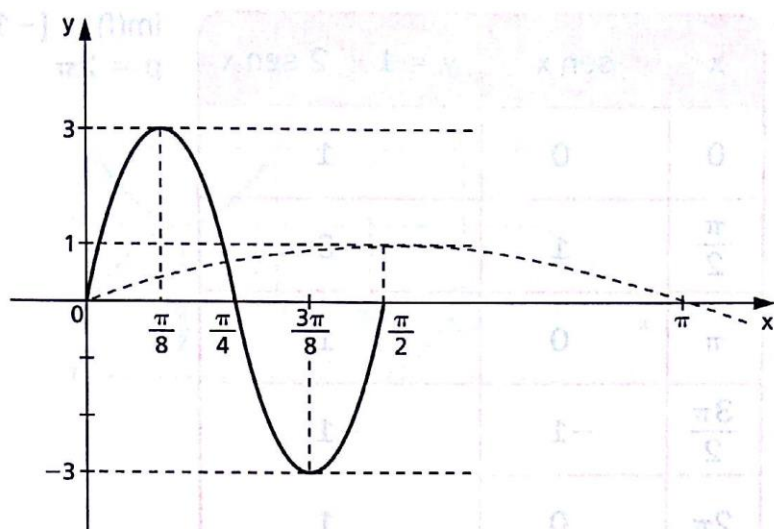
$\text{Im}(f) = [-1, 1]$
 $p = 6\pi$



137.

x	$t = 4x$	$y = 3 \text{ sen } t$
0	0	0
$\frac{\pi}{8}$	$\frac{\pi}{2}$	3
$\frac{\pi}{4}$	π	0
$\frac{3\pi}{8}$	$\frac{3\pi}{2}$	-3
$\frac{\pi}{2}$	2π	0

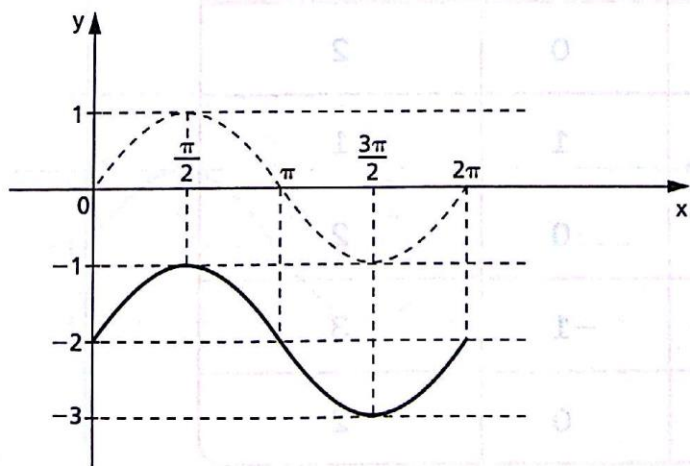
$\text{Im}(f) = [-3, 3]$
 $p = \frac{\pi}{2}$



139.

x	sen x	$y = -2 + \text{sen } x$
0	0	-2
$\frac{\pi}{2}$	1	-1
π	0	-2
$\frac{3\pi}{2}$	-1	-3
2π	0	-2

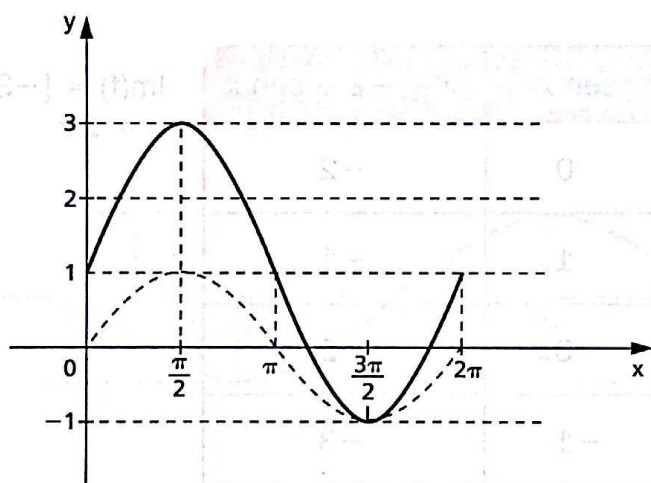
$\text{Im}(f) = [-3, -1]$
 $p = 2\pi$



140.

x	sen x	$y = 1 + 2 \text{ sen } x$
0	0	1
$\frac{\pi}{2}$	1	3
π	0	1
$\frac{3\pi}{2}$	-1	-1
2π	0	1

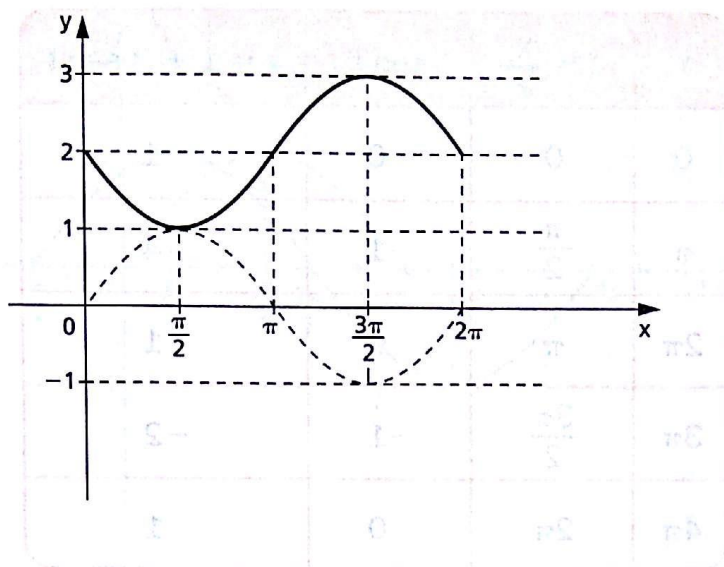
$\text{Im}(f) = [-1, 3]$
 $p = 2\pi$



141.

x	sen x	$y = 2 - \text{sen } x$
0	0	2
$\frac{\pi}{2}$	1	1
π	0	2
$\frac{3\pi}{2}$	-1	3
2π	0	2

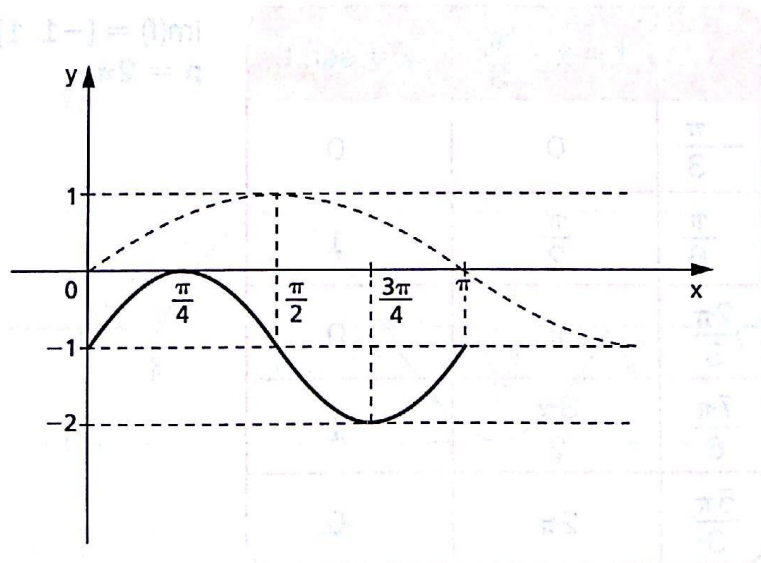
$\text{Im}(f) = [1, 3]$
 $p = 2\pi$



142.

x	$t = 2x$	$\text{sen } t$	$y = -1 + \text{sen } t$
0	0	0	-1
$\frac{\pi}{4}$	$\frac{\pi}{2}$	1	0
$\frac{\pi}{2}$	π	0	-1
$\frac{3\pi}{4}$	$\frac{3\pi}{2}$	-1	-2
π	2π	0	-1

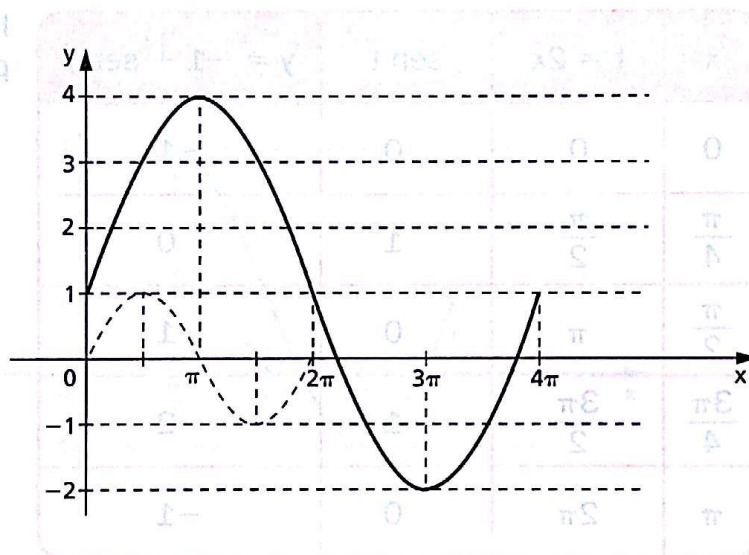
$\text{Im}(f) = [-2, 0]$
 $p = \pi$



143.

x	$t = \frac{x}{2}$	sen t	$y = 1 + 3 \text{ sen } t$
0	0	0	1
π	$\frac{\pi}{2}$	1	4
2π	π	0	1
3π	$\frac{3\pi}{2}$	-1	-2
4π	2π	0	1

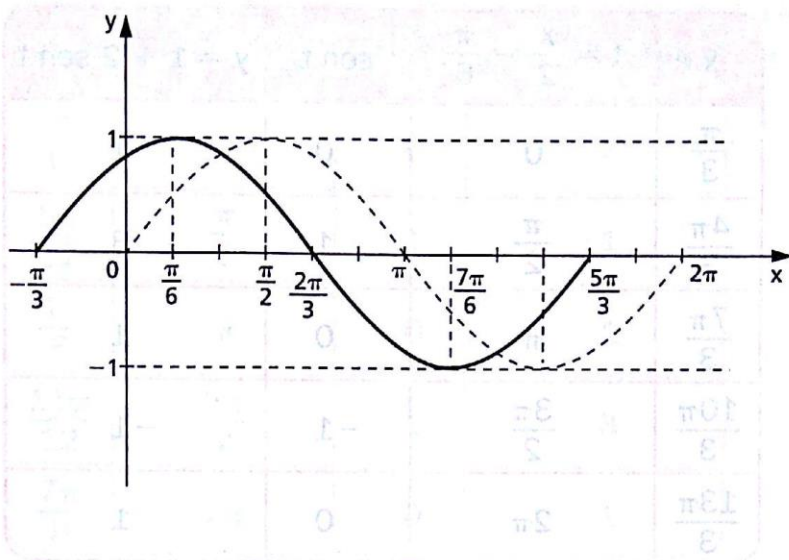
$\text{Im}(f) = [-2, 4]$
 $p = 4\pi$



145.

x	$t = x + \frac{\pi}{3}$	$y = \text{sen } t$
$-\frac{\pi}{3}$	0	0
$\frac{\pi}{6}$	$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	π	0
$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	-1
$\frac{5\pi}{3}$	2π	0

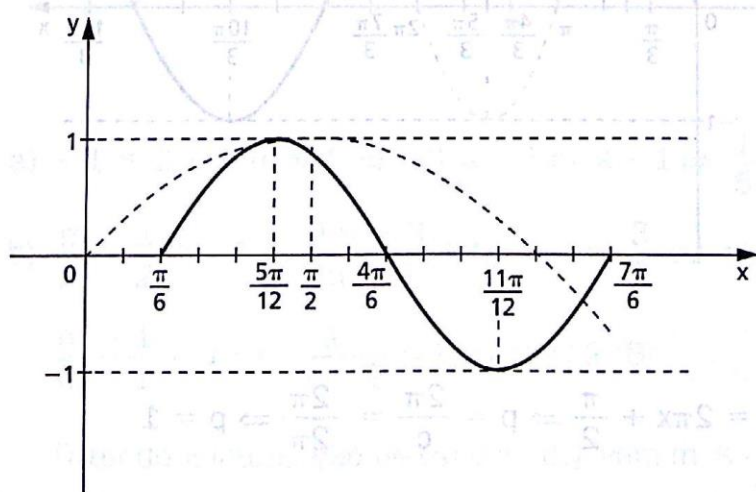
$\text{Im}(f) = [-1, 1]$
 $p = 2\pi$



146.

x	$t = 2x - \frac{\pi}{3}$	$y = \text{sen } t$
$\frac{\pi}{6}$	0	0
$\frac{5\pi}{12}$	$\frac{\pi}{2}$	1
$\frac{4\pi}{6}$	π	0
$\frac{11\pi}{12}$	$\frac{3\pi}{2}$	-1
$\frac{7\pi}{6}$	2π	0

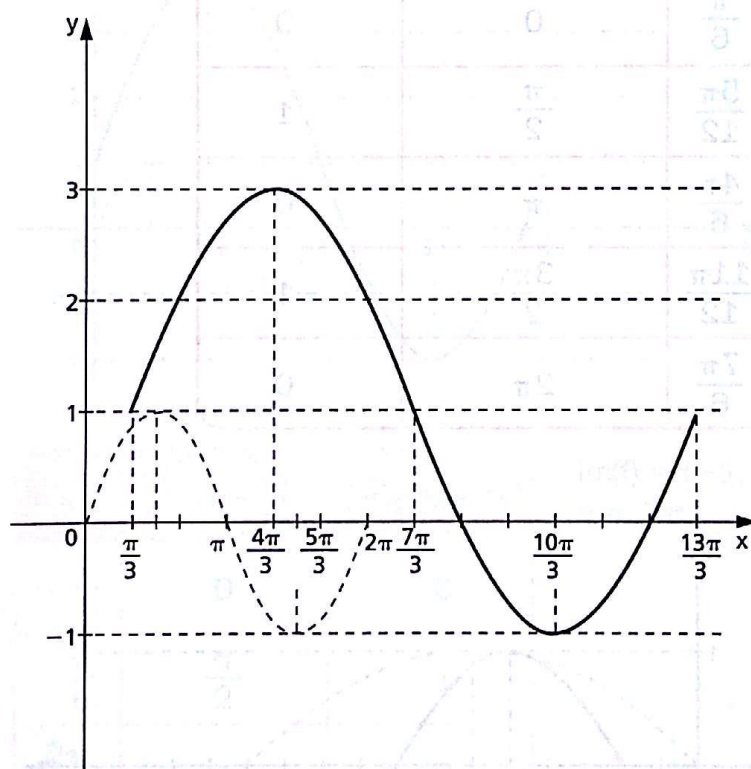
$\text{Im}(f) = [-1, 1]$
 $p = \pi$



147.

x	$t = \frac{x}{2} - \frac{\pi}{6}$	$\text{sen } t$	$y = 1 + 2 \text{sen } t$
$\frac{\pi}{3}$	0	0	1
$\frac{4\pi}{3}$	$\frac{\pi}{2}$	1	3
$\frac{7\pi}{3}$	π	0	1
$\frac{10\pi}{3}$	$\frac{3\pi}{2}$	-1	-1
$\frac{13\pi}{3}$	2π	0	1

$\text{Im}(f) = [-1, 3]$
 $p = 4\pi$

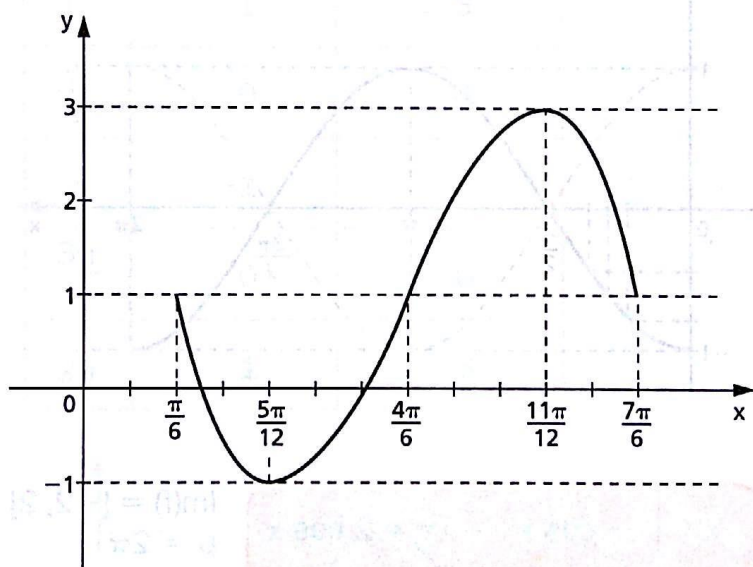


149. $t = 2\pi x + \frac{\pi}{2} \Rightarrow p = \frac{2\pi}{c} = \frac{2\pi}{2\pi} \Rightarrow p = 1$

150.

x	$t = 2x - \frac{\pi}{3}$	sen t	$y = 1 - 2 \text{ sen } t$
$\frac{\pi}{6}$	0	0	1
$\frac{5\pi}{12}$	$\frac{\pi}{2}$	1	-1
$\frac{4\pi}{6}$	π	0	1
$\frac{11\pi}{12}$	$\frac{3\pi}{2}$	-1	3
$\frac{7\pi}{6}$	2π	0	1

$\text{Im}(f) = [-1, 3]$
 $p = \pi$



152.

a) $-1 \leq 2 - 5m \leq 1 \Rightarrow -3 \leq -5m \leq -1 \Rightarrow \frac{1}{5} \leq m \leq \frac{3}{5}$

b) $\frac{m-1}{m-2} \geq -1 \Rightarrow \frac{2m-3}{m-2} \geq 0 \Rightarrow m \leq \frac{3}{2}$ ou $m > 2$ (A)

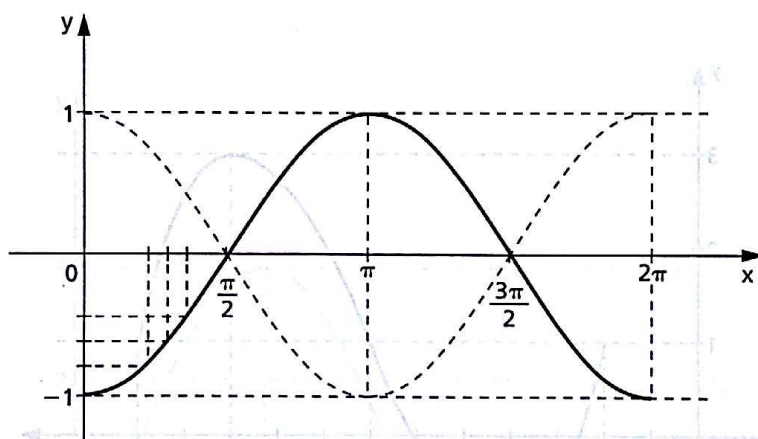
$\frac{m-1}{m-2} \leq 1 \Rightarrow \frac{1}{m-2} \leq 0 \Rightarrow m < 2$ (B)

Fazendo a interseção de (A) com (B), vem $m \leq \frac{3}{2}$.

153.

x	cos x	y = -cos x
0	1	-1
$\frac{\pi}{2}$	0	0
π	-1	1
$\frac{3\pi}{2}$	0	0
2π	1	-1

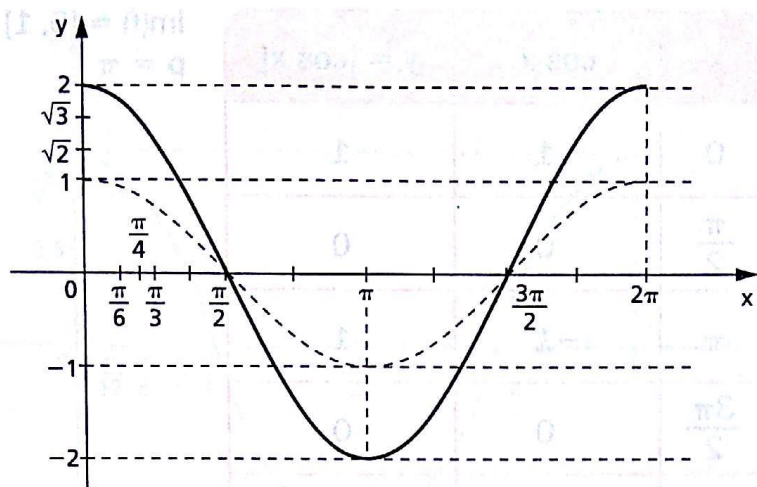
Im(f) = [-1, 1]
p = 2π



154.

x	cos x	y = 2 cos x
0	1	2
$\frac{\pi}{2}$	0	0
π	-1	-2
$\frac{3\pi}{2}$	0	0
2π	1	2

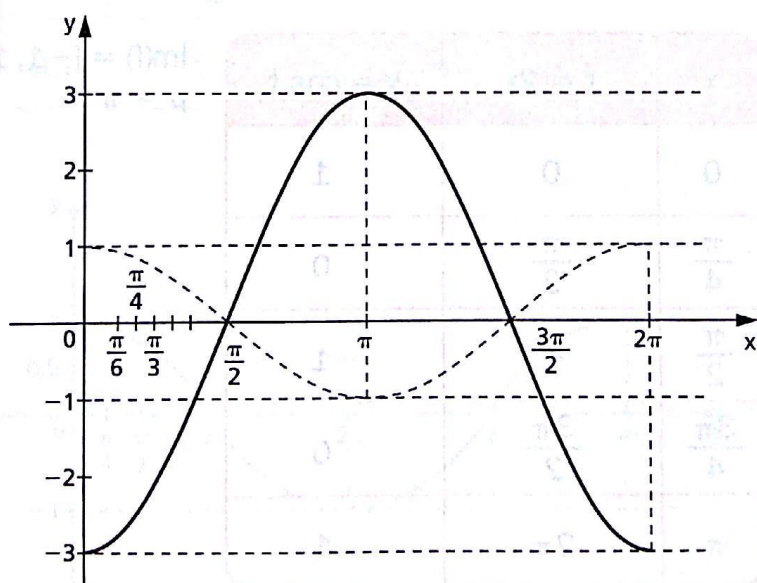
Im(f) = [-2, 2]
p = 2π



155.

x	cos x	y = -3 cos x
0	1	-3
$\frac{\pi}{2}$	0	0
π	-1	3
$\frac{3\pi}{2}$	0	0
2π	1	-3

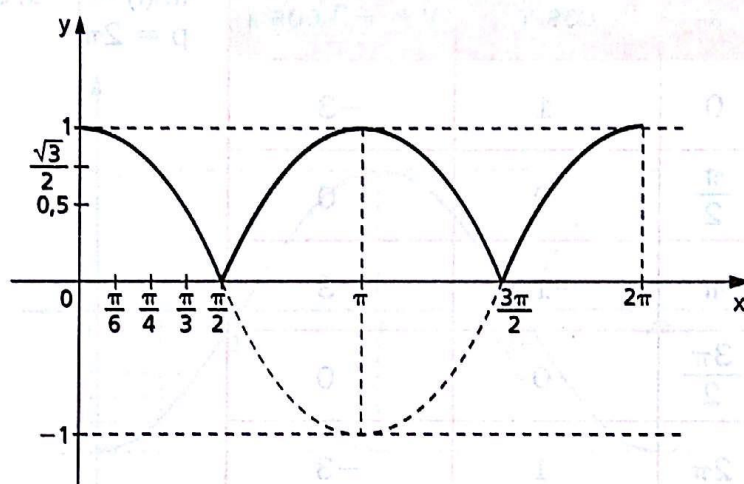
$\text{Im}(f) = [-3, 3]$
 $p = 2\pi$



156.

x	cos x	y = cos x
0	1	1
$\frac{\pi}{2}$	0	0
π	-1	1
$\frac{3\pi}{2}$	0	0
2π	1	1

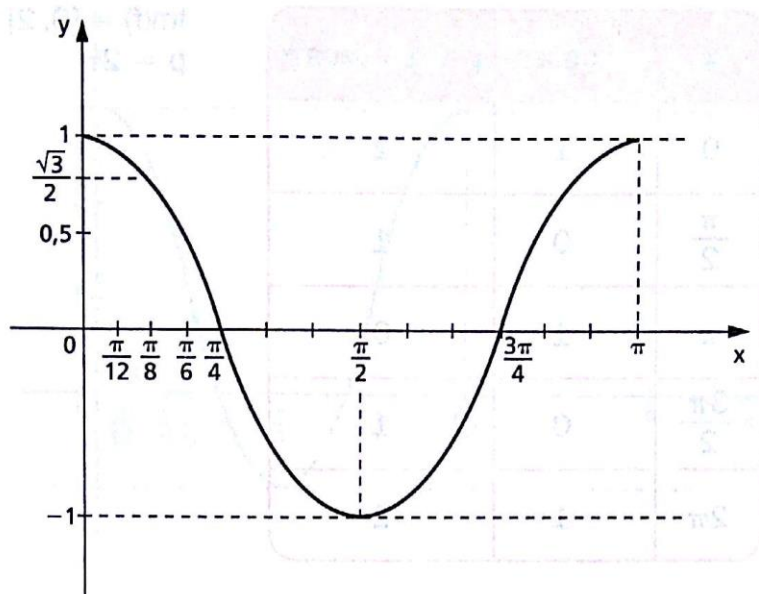
$\text{Im}(f) = [0, 1]$
 $p = \pi$



157.

x	t = 2x	y = cos t
0	0	1
$\frac{\pi}{4}$	$\frac{\pi}{2}$	0
$\frac{\pi}{2}$	π	-1
$\frac{3\pi}{4}$	$\frac{3\pi}{2}$	0
π	2π	1

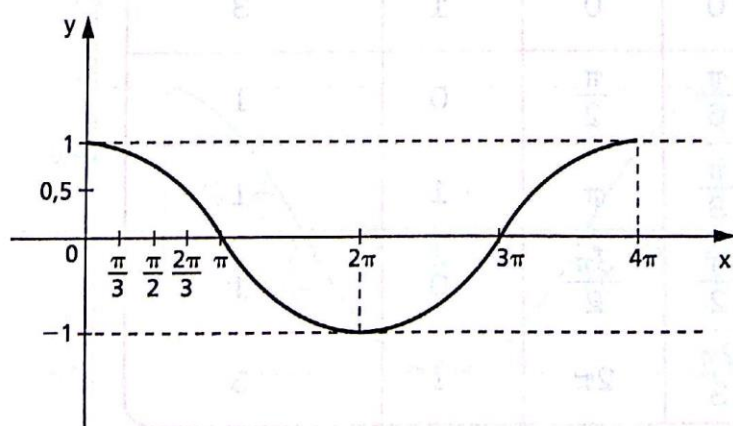
$\text{Im}(f) = [-1, 1]$
 $p = \pi$



158.

x	$t = \frac{x}{2}$	$y = \cos t$
0	0	1
π	$\frac{\pi}{2}$	0
2π	π	-1
3π	$\frac{3\pi}{2}$	0
4π	2π	1

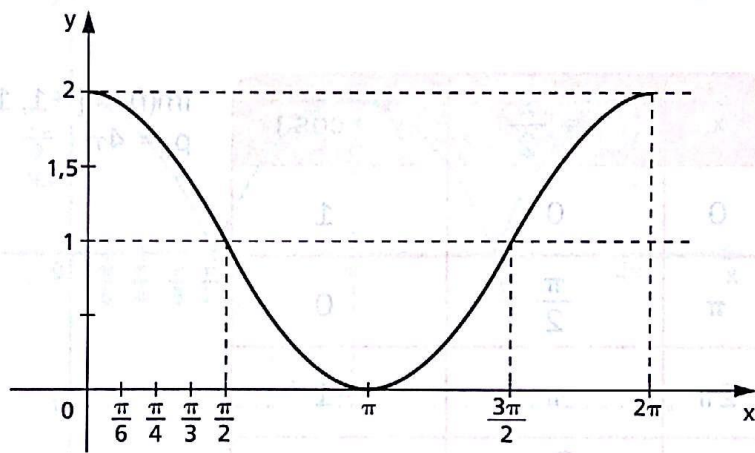
$\text{Im}(f) = [-1, 1]$
 $p = 4\pi$



159.

x	cos x	y = 1 + cos x
0	1	2
$\frac{\pi}{2}$	0	1
π	-1	0
$\frac{3\pi}{2}$	0	1
2π	1	2

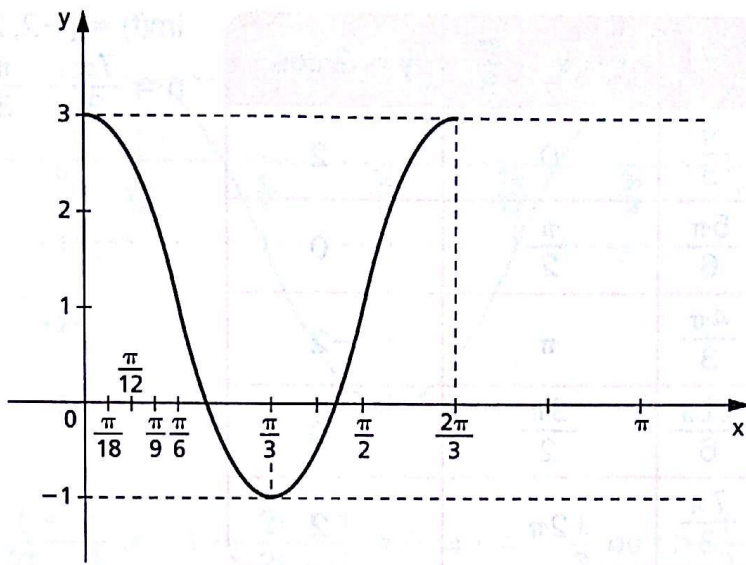
$\text{Im}(f) = [0, 2]$
 $p = 2\pi$



160.

x	t = 3x	cos t	y = 1 + 2 cos t
0	0	1	3
$\frac{\pi}{6}$	$\frac{\pi}{2}$	0	1
$\frac{\pi}{3}$	π	-1	-1
$\frac{\pi}{2}$	$\frac{3\pi}{2}$	0	1
$\frac{2\pi}{3}$	2π	1	3

$\text{Im}(f) = [-1, 3]$
 $p = \frac{2\pi}{3}$

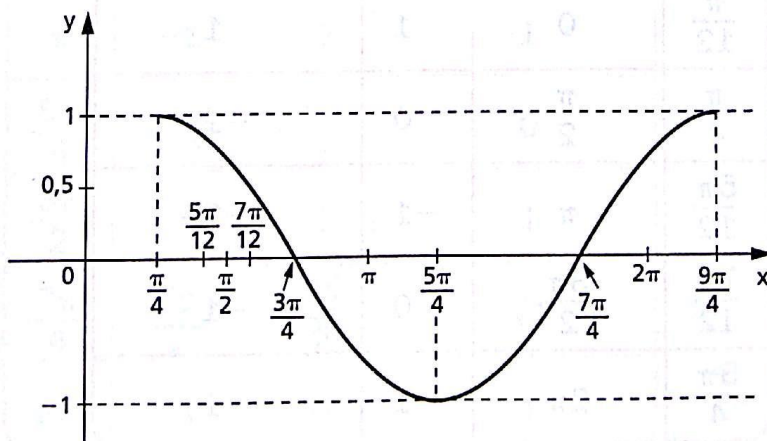


161.

x	$t = x - \frac{\pi}{4}$	$y = \cos t$
$\frac{\pi}{4}$	0	1
$\frac{3\pi}{4}$	$\frac{\pi}{2}$	0
$\frac{5\pi}{4}$	π	-1
$\frac{7\pi}{4}$	$\frac{3\pi}{2}$	0
$\frac{9\pi}{4}$	2π	1

$$\text{Im}(f) = [-1, 1]$$

$$p = \frac{9\pi}{4} - \frac{\pi}{4} = 2\pi$$

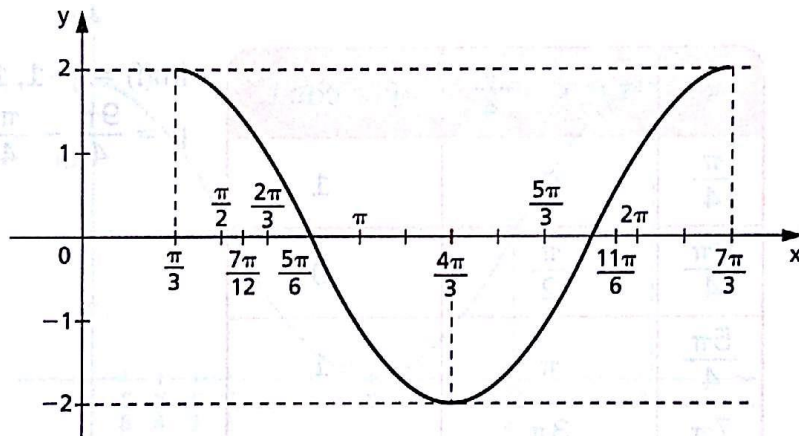


162.

x	$t = x - \frac{\pi}{3}$	$y = 2 \cos t$
$\frac{\pi}{3}$	0	2
$\frac{5\pi}{6}$	$\frac{\pi}{2}$	0
$\frac{4\pi}{3}$	π	-2
$\frac{11\pi}{6}$	$\frac{3\pi}{2}$	0
$\frac{7\pi}{3}$	2π	2

$$\text{Im}(f) = [-2, 2]$$

$$p = \frac{7\pi}{3} - \frac{\pi}{3} = 2\pi$$

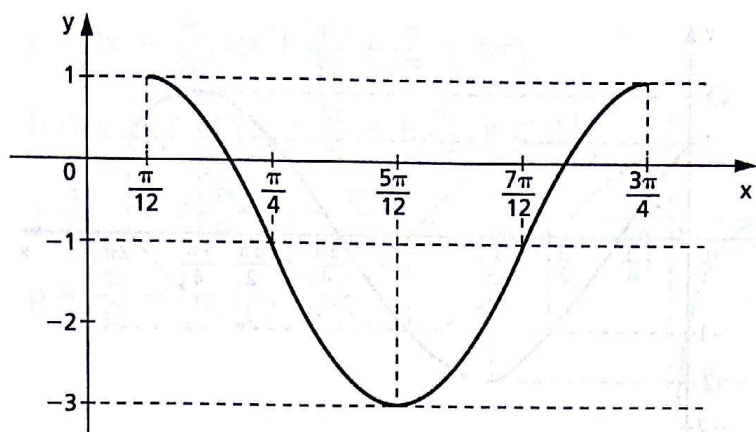


163.

x	$t = 3x - \frac{\pi}{4}$	$\cos t$	$y = -1 + 2 \cos t$
$\frac{\pi}{12}$	0	1	1
$\frac{\pi}{4}$	$\frac{\pi}{2}$	0	-1
$\frac{5\pi}{12}$	π	-1	-3
$\frac{7\pi}{12}$	$\frac{3\pi}{2}$	0	-1
$\frac{3\pi}{4}$	2π	1	1

$$\text{Im}(f) = [-3, 1]$$

$$p = \frac{3\pi}{4} - \frac{\pi}{12} = \frac{2\pi}{3}$$



164. $\frac{t+2}{2t-1} \geq -1 \Rightarrow \frac{3t+1}{2t-1} \geq 0 \Rightarrow t \leq -\frac{1}{3}$ ou $t > \frac{1}{2}$ (A)

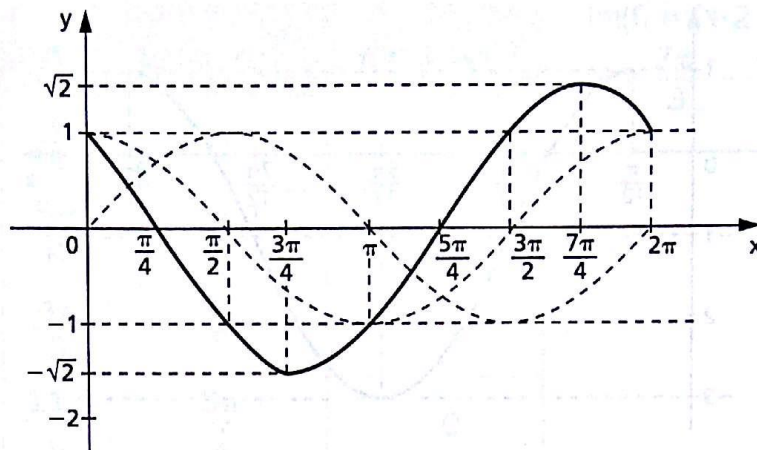
(B) $\frac{t+2}{2t-1} \leq 1 \Rightarrow \frac{-t+3}{2t-1} \leq 0 \Rightarrow t < \frac{1}{2}$ ou $t \geq 3$ (B)

Fazendo a interseção de (A) com (B), vem $t \leq -\frac{1}{3}$ ou $t \geq 3$.

166.

x	cos x	sen x	y = cos x - sen x
0	1	0	1
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	0
$\frac{\pi}{2}$	0	1	-1
$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$-\sqrt{2}$
π	-1	0	-1
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	0
$\frac{3\pi}{2}$	0	-1	1
$\frac{7\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	$\sqrt{2}$
2π	1	0	1

$p = 2\pi$



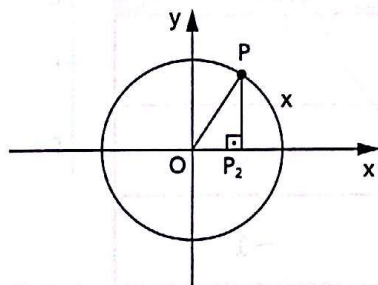
167. Solução 1

$$0 < x < \frac{\pi}{2} \Rightarrow \sin x > 0 \text{ e } \sin x < 1 \Rightarrow \sin^2 x < \sin x \quad (1)$$

$$0 < x < \frac{\pi}{2} \Rightarrow \cos x > 0 \text{ e } \cos x < 1 \Rightarrow \cos^2 x < \cos x \quad (2)$$

$$\text{De (1) + (2)} \Rightarrow \sin x + \cos x > 1.$$

Solução 2



No triângulo OP_2P , temos:

$$\overline{OP_2} + \overline{P_2P} > \overline{OP}$$

(um lado é sempre menor que a soma dos outros dois)

Então:

$$\cos x + \sin x > 1$$

168. $t = 4x; t = 0 \Rightarrow x = 0; t = 2\pi \Rightarrow x = \frac{\pi}{2}; p = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

169. A sequência é $\cos \alpha, -\cos \alpha, \cos \alpha, -\cos \alpha, \dots$; então: a soma dos 12 termos iniciais é zero.

171. a) $t = 3x; 3x \neq \frac{\pi}{2} + k\pi; D(f) = \left\{ x \in \mathbb{R} \mid x \neq \frac{\pi}{6} + k \frac{\pi}{3}, k \in \mathbb{Z} \right\}$

b) $t = 2x - \frac{\pi}{3}; 2x - \frac{\pi}{3} \neq \frac{\pi}{2} + k\pi; D(f) = \left\{ x \in \mathbb{R} \mid x \neq \frac{5\pi}{12} + k \frac{\pi}{2}, k \in \mathbb{Z} \right\}$

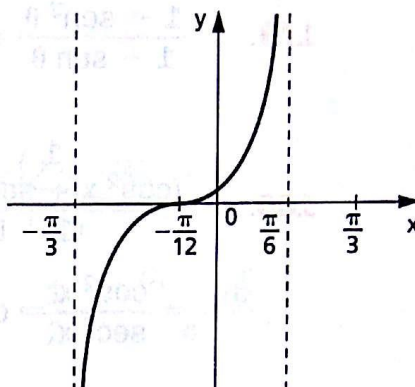
172. $\alpha^2 - 5\alpha + 4 \geq 0 \Rightarrow \alpha \leq 1 \text{ ou } \alpha \geq 4$

$$174. \quad t = 2x + \frac{\pi}{6}; 2x + \frac{\pi}{6} \neq \frac{\pi}{2} + k\pi;$$

$$D(f) = \left\{ x \in \mathbb{R} \mid x \neq \frac{\pi}{6} + k \frac{\pi}{2}, k \in \mathbb{Z} \right\}$$

$$-\frac{\pi}{2} < t < \frac{\pi}{2} \Rightarrow -\frac{\pi}{3} < x < \frac{\pi}{6};$$

$$p = \frac{\pi}{6} - \left(-\frac{\pi}{3}\right) = \frac{\pi}{2}$$



$$175. \quad t = x - \frac{\pi}{3}, x - \frac{\pi}{3} \neq k\pi, D(f) = \left\{ x \in \mathbb{R} \mid x \neq \frac{\pi}{3} + k\pi, k \in \mathbb{Z} \right\}$$

$$0 < x - \frac{\pi}{3} < \pi \Rightarrow \frac{\pi}{3} < x < \frac{4\pi}{3}; p = \frac{4\pi}{3} - \frac{\pi}{3} = \pi$$

$$t = 2x, 2x \neq \frac{\pi}{2} + k\pi, D(g) = \left\{ x \in \mathbb{R} \mid x \neq \frac{\pi}{4} + k \frac{\pi}{2}, k \in \mathbb{Z} \right\}$$

$$-\frac{\pi}{2} < 2x < \frac{3\pi}{2} \Rightarrow -\frac{\pi}{4} < x < \frac{3\pi}{4}; p = \frac{3\pi}{4} - \left(-\frac{\pi}{4}\right) = \pi$$

$$t = x + \frac{\pi}{4}, x + \frac{\pi}{4} \neq k\pi, D(h) = \left\{ x \in \mathbb{R} \mid x \neq -\frac{\pi}{4} + k\pi, k \in \mathbb{Z} \right\}$$

$$0 < x + \frac{\pi}{4} < 2\pi \Rightarrow -\frac{\pi}{4} < x < \frac{7\pi}{4}; p = \frac{7\pi}{4} - \left(-\frac{\pi}{4}\right) = 2\pi$$

$$176. \quad a) 2 - m \geq 0 \Rightarrow m \leq 2$$

$$b) 3m - 2 \leq -1 \Rightarrow m \leq \frac{1}{3} \text{ ou } 3m - 2 \geq 1 \Rightarrow m \geq 1$$

$$c) \frac{2m - 1}{1 - 3m} \leq -1 \Rightarrow \frac{-m}{1 - 3m} \leq 0 \Rightarrow 0 \leq m < \frac{1}{3}$$

ou

$$\frac{2m - 1}{1 - 3m} \geq 1 \Rightarrow \frac{5m - 2}{1 - 3m} \geq 0 \Rightarrow \frac{1}{3} < m \leq \frac{2}{5}$$

$$177. \quad \frac{1}{\left(1 + \frac{1}{\cos x}\right)} \cdot \frac{(1 + \cos x)^{\frac{1}{2}}}{(1 - \cos x)^{\frac{1}{2}}} = \frac{\cos x}{\sqrt{1 - \cos^2 x}} = \frac{\cos x}{\sqrt{\sin^2 x}} = \frac{\cos x}{|\sin x|}$$

$$178. \quad \frac{\frac{1}{\sin x} - \sin x}{\frac{1}{\cos x} - \cos x} = \frac{\cos^2 x}{\sin x} \cdot \frac{\cos x}{\sin^2 x} = \cotg^3 x$$

179.
$$\frac{1 - \operatorname{sen}^2 \theta}{1 - \operatorname{sen} \theta} = \frac{(1 + \operatorname{sen} \theta)(1 - \operatorname{sen} \theta)}{1 - \operatorname{sen} \theta} = 1 + \operatorname{sen} \theta$$

180.
$$\frac{1}{(\cos^2 x + \operatorname{sen}^2 x)(\cos^2 x - \operatorname{sen}^2 x)} = \frac{\cos^2 x - \cos^2 x \cdot \operatorname{tg}^2 x}{\sec^2 x \cdot (1 - \operatorname{tg}^2 x)}$$

$$= \frac{\cos^2 x}{\sec^2 x} = \cos^4 x$$

181.
$$\frac{\operatorname{sen}^2 x + (1 + \cos x)^2}{\operatorname{sen} x \cdot (1 + \cos x)} = \frac{2 + 2 \cos x}{\operatorname{sen} x \cdot (1 + \cos x)} = \frac{2}{\operatorname{sen} x} = 2 \operatorname{cosec} x$$

182.
$$\operatorname{sen} x - \operatorname{cosec} x = t \Rightarrow (\operatorname{sen} x - \operatorname{cosec} x)^2 = t^2 \Rightarrow$$

$$\Rightarrow \operatorname{sen}^2 x + \operatorname{cosec}^2 x = t^2 + 2$$

183.
$$\frac{\sec^2 x}{\operatorname{cosec}^2 x} = \frac{\operatorname{sen}^2 x}{1 - \operatorname{sen}^2 x} = \frac{\left(\frac{n-1}{n}\right)^2}{1 - \left(\frac{n-1}{n}\right)^2} = \frac{(n-1)^2}{2n-1}$$

184. a) $\operatorname{tg}(-x) = \frac{\operatorname{sen}(-x)}{\cos(-x)} = -\frac{\operatorname{sen} x}{\cos x} = -\operatorname{tg}(x)$; a função é ímpar

b) $\operatorname{cotg}(-x) = \frac{1}{\operatorname{tg}(-x)} = -\frac{1}{\operatorname{tg} x} = -\operatorname{cotg}(x)$; a função é ímpar

c) $\sec(-x) = \frac{1}{\cos(-x)} = \frac{1}{\cos x} = \sec x$; a função é par

d) $\operatorname{cosec}(-x) = \frac{1}{\operatorname{sen}(-x)} = -\frac{1}{\operatorname{sen} x} = -\operatorname{cosec} x$; a função é ímpar

185. a) $0 \in D(f)$, f é ímpar $\Rightarrow f(-0) = -f(0) \Rightarrow f(0) + f(0) = 0 \Rightarrow f(0) = 0$

b) f é ímpar $\Rightarrow f(-x) = -f(x)$
 f é par $\Rightarrow f(-x) = f(x)$ } $\Rightarrow -f(x) = f(x) \Rightarrow 2f(x) = 0 \Rightarrow f(x) = 0, \forall x$

186. $f(x) = f(-x) = 3 \Rightarrow f(x)$ é par; $g(x)$ é par, $\forall n$, pois

$$g(x) = \underbrace{f(x) \cdot f(x) \cdot \dots \cdot f(x)}_{n \text{ fatores}} = \underbrace{f(-x) \cdot f(-x) \cdot f(-x) \cdot \dots \cdot f(-x)}_{n \text{ fatores}} = g(-x), \forall x$$

CAPÍTULO IX — Transformações

$$188. \quad \cotg(120^\circ + 45^\circ) = \frac{-\frac{\sqrt{3}}{3} \cdot 1 - 1}{-\frac{\sqrt{3}}{3} + 1} = -(2 + \sqrt{3});$$

$$\cos(225^\circ + 30^\circ) = -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{2} - \sqrt{6}}{4};$$

$$\sec 225^\circ = \frac{1}{\cos 255^\circ} = -\sqrt{2} - \sqrt{6}$$

$$\sin(45^\circ - 30^\circ) = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4};$$

$$\operatorname{cosec} 15^\circ = \frac{1}{\sin 15^\circ} = \sqrt{6} + \sqrt{2}$$

$$189. \quad \operatorname{tg}(A - B) = \frac{2 - 1}{1 + 2 \cdot 1} = \frac{1}{3}$$

$$190. \quad \left. \begin{aligned} \sin(60^\circ + 45^\circ) &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} \quad (A) \\ \cos(30^\circ + 45^\circ) &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \quad (B) \end{aligned} \right\} A - B = \frac{\sqrt{2}}{2}$$

$$193. \quad \cos x = +\sqrt{1 - \sin^2 x} = \frac{8}{17}; \quad \cos y = -\sqrt{1 - \sin^2 y} = -\frac{4}{5};$$

$$\operatorname{tg} x = \frac{15}{8}; \quad \operatorname{tg} y = \frac{3}{4}$$

$$\sin(x + y) = \frac{15}{17} \cdot \left(-\frac{4}{5}\right) + \left(-\frac{3}{5}\right) \left(\frac{8}{17}\right) = -\frac{84}{85};$$

$$\cos(x + y) = \frac{13}{85}; \quad \operatorname{tg}(x + y) = -\frac{84}{13}$$

$$195. \quad \text{a) } f(x) = \cos 2x \cdot \cos 2x - \sin 2x \cdot \sin 2x = \cos 4x$$

$$D(f) = \mathbb{R}; \quad \operatorname{Im}(f) = [-1, 1]; \quad p = \frac{\pi}{2}$$

$$\text{b) } g(x) = 2 \cdot \sin \frac{\pi}{3} \cos x - 2 \cos \frac{\pi}{3} \sin x = 2 \cdot \sin \left(\frac{\pi}{3} - x\right)$$

$$D(g) = \mathbb{R}; \quad \operatorname{Im}(g) = [-2, 2]; \quad p = 2\pi$$

$$\text{c) } h(x) = \frac{\frac{\sin x}{\cos x} + \frac{\cos x}{\cos x}}{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}} = \frac{\operatorname{tg} x + 1}{1 - \operatorname{tg} x} = \operatorname{tg} \left(x + \frac{\pi}{4}\right)$$

$$x + \frac{\pi}{4} \neq \frac{\pi}{2} + k\pi \Rightarrow D(h) = \left\{ x \in \mathbb{R} \mid x \neq \frac{\pi}{4} + k\pi \right\}$$

$$\left. \begin{array}{l} x + \frac{\pi}{4} = -\frac{\pi}{2} \Rightarrow x = -\frac{3\pi}{4} \\ x + \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{4} \end{array} \right\} \Rightarrow p = \frac{\pi}{4} - \left(-\frac{3\pi}{4}\right) = \pi$$

196. $f(x) = \sin x (\cos 2x \cos 3x - \sin 2x \sin 3x) +$
 $+ \cos x (\sin 2x \cos 3x + \sin 3x \cos 2x) \Rightarrow$
 $\Rightarrow f(x) = \sin x \cos 5x + \cos x \sin 5x = \sin 6x; p = \frac{2\pi}{6} = \frac{\pi}{3}$

197. $\operatorname{tg}(75^\circ - 60^\circ) = \frac{(2 + \sqrt{3}) - \sqrt{3}}{1 + (2 + \sqrt{3})\sqrt{3}} = 2 - \sqrt{3}$

199. $\operatorname{tg} x + \operatorname{cotg} x = 3 \Rightarrow \frac{\sin^2 x + \cos^2 x}{\cos x \cdot \sin x} = 3 \Rightarrow \sin x \cdot \cos x = \frac{1}{3} \Rightarrow$
 $\Rightarrow 2 \cdot \sin x \cdot \cos x = \frac{2}{3} \Rightarrow \sin 2x = \frac{2}{3}$

201. a) $\sin\left(\frac{\pi}{2} + 2\alpha\right) = \cos 2\alpha = 1 - 2 \sin^2 \alpha \Rightarrow \sin\left(\frac{\pi}{2} + 2\alpha\right) = \frac{1}{9}$

b) $\left. \begin{array}{l} \cos\left(\frac{\pi}{4} + \alpha\right) = \frac{\sqrt{2}}{2} \cos \alpha - \frac{\sqrt{2}}{2} \sin \alpha \\ \cos \alpha = \sqrt{1 - \sin^2 \alpha} \Rightarrow \cos \alpha = \frac{\sqrt{5}}{3} \end{array} \right\} \Rightarrow \cos\left(\frac{\pi}{4} + \alpha\right) = \frac{\sqrt{10} - 2\sqrt{2}}{6}$

203. $\sin x = -\sqrt{1 - \left(\frac{3}{5}\right)^2} \Rightarrow \sin x = -\frac{4}{5};$

$$\sin 3x = 3 \cdot \left(-\frac{4}{5}\right) - 4 \cdot \left(-\frac{4}{5}\right)^3 = -\frac{44}{125}$$

204. $\cos x = -\sqrt{1 - \left(\frac{12}{13}\right)^2} \Rightarrow \cos x = -\frac{5}{13};$

$$\cos 3x = 4 \cdot \left(-\frac{5}{13}\right)^3 - 3 \cdot \left(-\frac{5}{13}\right) = \frac{2035}{2197}$$

205. $\operatorname{tg} x = \sqrt{\sec^2 x - 1} = \frac{\sqrt{7}}{3} \Rightarrow \operatorname{tg} 3x = \frac{3 \cdot \frac{\sqrt{7}}{3} - \left(\frac{\sqrt{7}}{3}\right)^3}{1 - 3\left(\frac{\sqrt{7}}{3}\right)^2} = -\frac{5\sqrt{7}}{9}$

206. $\left(\sin^2 \frac{\pi}{12} - \cos^2 \frac{\pi}{12}\right) + \operatorname{tg} \frac{\pi}{3} + \operatorname{tg} \frac{14\pi}{3} = -\cos\left(2 \cdot \frac{\pi}{12}\right) + \operatorname{tg} \frac{\pi}{3} + \operatorname{tg} \frac{2\pi}{3} =$
 $= -\cos \frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

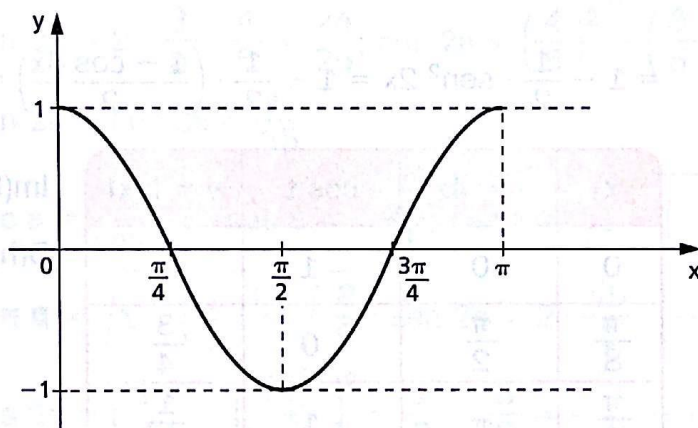
208. a) $f(x) = (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) = \cos 2x$

x	t = 2x	cos t
0	0	1
$\frac{\pi}{4}$	$\frac{\pi}{2}$	0
$\frac{\pi}{2}$	π	-1
$\frac{3\pi}{4}$	$\frac{3\pi}{2}$	0
π	2π	1

$\text{Im}(f) = [-1, 1]$

$D(f) = \mathbb{R}$

$p = \pi$



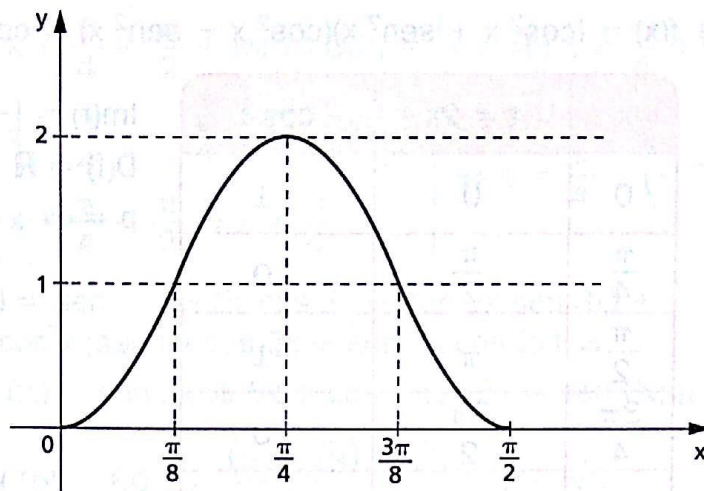
b) $g(x) = 2(2 \sin x \cos x)^2 = 2 \sin^2 2x = 1 - \cos 4x$

x	t = 4x	cos t	y = 1 - cos 4x
0	0	1	0
$\frac{\pi}{8}$	$\frac{\pi}{2}$	0	1
$\frac{\pi}{4}$	π	-1	2
$\frac{3\pi}{8}$	$\frac{3\pi}{2}$	0	1
$\frac{\pi}{2}$	2π	1	0

$\text{Im}(g) = [0, 2]$

$D(g) = \mathbb{R}$

$p = \frac{\pi}{2}$



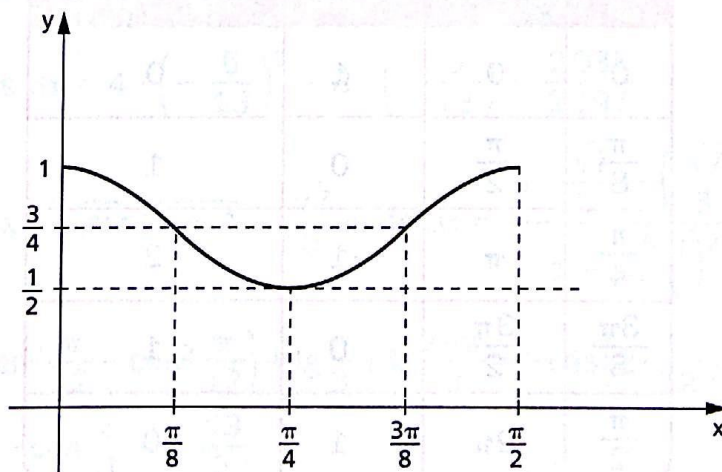
$$\begin{aligned}
 \text{c) } h(x) &= (\cos^2 x + \sin^2 x)^2 - 2 \cdot \cos^2 x \cdot \sin^2 x = 1 - 2 \cdot \left(\frac{\sin 2x}{2}\right)^2 = \\
 &= 1 - \frac{1}{2} \cdot \sin^2 2x = 1 - \frac{1}{2} \cdot \left(\frac{1 - \cos 4x}{2}\right) = \frac{3}{4} + \frac{1}{4} \cdot \cos 4x
 \end{aligned}$$

x	t = 4x	cos t	y = h(x)
0	0	1	1
$\frac{\pi}{8}$	$\frac{\pi}{2}$	0	$\frac{3}{4}$
$\frac{\pi}{4}$	π	-1	$\frac{1}{2}$
$\frac{3\pi}{8}$	$\frac{3\pi}{2}$	0	$\frac{3}{4}$
$\frac{\pi}{2}$	2π	1	1

$$\text{Im}(h) = \left[\frac{1}{2}, 1\right]$$

$$D(h) = \mathbb{R}$$

$$p = \frac{\pi}{2}$$



- 209.** a) $f(x) = \frac{1}{2} \operatorname{sen} 2x$; $p = \frac{2\pi}{2} \Rightarrow p = \pi$
 b) $g(x) = \frac{1 - \operatorname{tg}^2 2x}{\sec^2 2x} = (1 - \operatorname{tg}^2 2x) \cdot \cos^2 2x = \cos^2 2x - \operatorname{sen}^2 2x = \cos 4x$
 $p = \frac{2\pi}{4} \Rightarrow p = \frac{\pi}{2}$
 c) $h(x) = (\cos^2 x + \operatorname{sen}^2 x)(\cos^4 x - \cos^2 x \cdot \operatorname{sen}^2 x + \operatorname{sen}^4 x) =$
 $= (\cos^4 x + \operatorname{sen}^4 x) - \cos^2 x \cdot \operatorname{sen}^2 x =$
 $= (\cos^2 x + \operatorname{sen}^2 x)^2 - 3 \cdot \cos^2 x \cdot \operatorname{sen}^2 x =$
 $= 1 - \frac{3}{4} (\operatorname{sen} 2x)^2 = 1 - \frac{3}{4} \left(\frac{1 - \cos 4x}{2} \right) = \frac{5}{8} + \frac{3}{8} \cdot \cos 4x$
 $p = \frac{2\pi}{4} = \frac{\pi}{2}$
- 210.** $\operatorname{sen} 2a = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} = \frac{24}{25}$; $\cos 2a = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{7}{25}$;
 $\operatorname{sen} 2a + \cos 2a = \frac{31}{25}$
- 211.** $\sec a = \frac{1}{\cos a} \Rightarrow \cos a = -\frac{2}{3}$; $\operatorname{sen} a = \sqrt{1 - \left(-\frac{2}{3}\right)^2} = \frac{\sqrt{5}}{3}$;
 $\operatorname{sen} b = \sqrt{1 - \left(\frac{1}{3}\right)^2} = \sqrt{\frac{8}{9}}$; $\operatorname{sen} 2a = 2 \cdot \frac{\sqrt{5}}{3} \cdot \left(-\frac{2}{3}\right) = -\frac{4\sqrt{5}}{9}$;
 $\cos 2b = \left(\frac{1}{3}\right)^2 - \left(\sqrt{\frac{8}{9}}\right)^2 = \frac{1}{9} - \frac{8}{9} = -\frac{7}{9} \Rightarrow \cos 2b = -\frac{7}{9}$
- 212.** $\operatorname{tg} x = a \operatorname{cotg} x + b \cdot \frac{1}{\operatorname{tg} 2x} \Rightarrow \operatorname{tg} x = \frac{a}{\operatorname{tg} x} + \frac{b(1 - \operatorname{tg}^2 x)}{2 \operatorname{tg} x} \Rightarrow$
 $\Rightarrow 2 \operatorname{tg}^2 x - 2a = -b \operatorname{tg}^2 x + b \Rightarrow (2 = -b \text{ e } -2a = b) \Rightarrow b = -2 \text{ e } a = 1$
- 215.** $\cos \theta = -\sqrt{1 - \operatorname{sen}^2 \theta} \Rightarrow \cos \theta = -\frac{4}{5}$; $\operatorname{sen} \frac{\theta}{2} = +\sqrt{\frac{1 - \cos \theta}{2}} \Rightarrow$
 $\Rightarrow \operatorname{sen} \frac{\theta}{2} = \frac{3}{\sqrt{10}}$; $A = 25 \cdot \frac{3}{5} + \sqrt{10} \cdot \frac{3}{\sqrt{10}} \Rightarrow A = 18$
- 216.** $\cos \left(\frac{a_n}{2}\right) = \sqrt{\frac{1 + \cos a_n}{2}} \Rightarrow \cos \left(\frac{a_n}{2}\right) = \sqrt{\frac{1 + \frac{n}{n+1}}{2}} \Rightarrow$
 $\Rightarrow \cos \left(\frac{a_n}{2}\right) = \sqrt{\frac{2n+1}{2n+2}} \Rightarrow$
 $\Rightarrow \cos \left(\frac{a_n}{2}\right) = \frac{\sqrt{4n^2 + 6n + 2}}{2n+2}$

$$218. \quad \sin \frac{x}{2} = +\sqrt{\frac{1 - \cos x}{2}} = \frac{\sqrt{3}}{3}; \quad \operatorname{tg} \frac{x}{2} = +\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sqrt{2}}{2}$$

$$219. \quad \cos \frac{x}{2} = +\sqrt{\frac{1 + \cos x}{2}} = \frac{7}{5\sqrt{2}}; \quad \sin \frac{x}{4} = +\sqrt{\frac{1 - \cos \frac{x}{2}}{2}} =$$

$$= +\sqrt{\frac{10 - 7\sqrt{2}}{20}}$$

$$\cos \frac{x}{4} = +\sqrt{\frac{1 + \cos \frac{x}{2}}{2}} = \sqrt{\frac{10 + 7\sqrt{2}}{20}}; \quad \operatorname{tg} \frac{x}{4} = \sqrt{\frac{10 - 7\sqrt{2}}{10 + 7\sqrt{2}}} =$$

$$= 5\sqrt{2} - 7$$

$$220. \quad \sec x = 4 \Rightarrow \cos x = \frac{1}{4}$$

$$\frac{3\pi}{2} < x < 2\pi \Rightarrow \frac{3\pi}{4} < \frac{x}{2} < \pi$$

$$\operatorname{tg} \left(\frac{\pi}{2} + \frac{x}{2} \right) = -\operatorname{cotg} \frac{x}{2} = -\left(-\sqrt{\frac{1 + \cos x}{1 - \cos x}} \right) = \frac{\sqrt{15}}{3}$$

$$222. \quad f(x) = \frac{\sqrt{1 - \cos 2x}}{\sqrt{1 + \cos 2x}} = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} = |\operatorname{tg} x|$$

$$\operatorname{Im}(f) = \mathbb{R}_+; \quad p = \pi; \quad D(f) = \left\{ x \in \mathbb{R} \mid x \neq \frac{\pi}{2} + k\pi \right\}$$

$$223. \quad f(x) = \sqrt{1 + \cos 4x} = \sqrt{2} \cdot \sqrt{\frac{1 + \cos 4x}{2}} = \sqrt{2} \cdot |\cos 2x|$$

$$p = \frac{\pi}{2}$$

$$224. \quad \operatorname{tg} a = \frac{2 \cdot \operatorname{tg} \frac{a}{2}}{1 - \operatorname{tg}^2 \frac{a}{2}} = \frac{2 \cdot \frac{1}{2}}{1 - \left(\frac{1}{2}\right)^2} = \frac{4}{3}$$

$$225. \quad \operatorname{cotg} \frac{a}{2} = \frac{1}{\operatorname{tg} \frac{a}{2}} \Rightarrow \operatorname{tg} \frac{a}{2} = \frac{\sqrt{3}}{3}; \quad \operatorname{sen} a = \frac{2 \cdot \operatorname{tg} \frac{a}{2}}{1 + \operatorname{tg}^2 \frac{a}{2}} = \frac{\sqrt{3}}{2}$$

$$229. \quad \text{a) } y = 2 \operatorname{sen} \left(\frac{a + b + c - a + b - c}{2} \right) \cdot \cos \left(\frac{a + b + c + a - b + c}{2} \right) =$$

$$= 2 \operatorname{sen} b \cos (a + c)$$

$$\text{b) } y = 2 \cdot \cos \left(\frac{a + 2b + a}{2} \right) \cdot \cos \left(\frac{a + 2b - a}{2} \right) = 2 \cos (a + b) \cos b$$

$$\begin{aligned} \text{c) } y &= [\text{sen}(a + 3r) + \text{sen } a] + [\text{sen}(a + 2r) + \text{sen}(a + r)] = \\ &= 2 \cdot \text{sen} \frac{2a + 3r}{2} \cdot \cos \frac{3r}{2} + 2 \cdot \text{sen} \frac{2a + 3r}{2} \cdot \cos \frac{r}{2} = \\ &= 2 \cdot \text{sen} \frac{2a + 3r}{2} \cdot \left(\cos \frac{3r}{2} + \cos \frac{r}{2} \right) = 4 \cdot \text{sen} \frac{2a + 3r}{2} \cdot \cos r \cdot \cos \frac{r}{2} \end{aligned}$$

$$\begin{aligned} \text{d) } y &= [\cos(a + 3b) + \cos a] + [\cos(a + 2b) + \cos(a + b)] = \\ &= 2 \cdot \cos \frac{2a + 3b}{2} \cdot \cos \frac{3b}{2} + 2 \cdot \cos \frac{2a + 3b}{2} \cdot \cos \frac{b}{2} = \\ &= 2 \cdot \cos \frac{2a + 3b}{2} \cdot \left(\cos \frac{3b}{2} + \cos \frac{b}{2} \right) = 4 \cdot \cos \frac{2a + 3b}{2} \cdot \cos b \cdot \cos \frac{b}{2} \end{aligned}$$

$$\begin{aligned} \text{e) } y &= (\cos p + \cos q)(\cos p - \cos q) \\ y &= 2 \cdot \cos \left(\frac{p+q}{2} \right) \cdot \cos \left(\frac{p-q}{2} \right) \left[-2 \cdot \text{sen} \left(\frac{p+q}{2} \right) \cdot \text{sen} \left(\frac{p-q}{2} \right) \right] = \\ &= -\text{sen}(p+q) \cdot \text{sen}(p-q) \end{aligned}$$

$$\begin{aligned} \text{f) } y &= (\text{sen } p + \text{sen } q)(\text{sen } p - \text{sen } q) \\ y &= 2 \cdot \text{sen} \left(\frac{p+q}{2} \right) \cdot \cos \left(\frac{p-q}{2} \right) \cdot 2 \text{sen} \left(\frac{p-q}{2} \right) \cdot \cos \left(\frac{p+q}{2} \right) = \\ &= \text{sen}(p+q) \cdot \text{sen}(p-q) \end{aligned}$$

$$\begin{aligned} \text{g) } y &= \frac{1 + \cos 2p}{2} - \frac{1 - \cos 2q}{2} = \frac{1}{2} (\cos 2p + \cos 2q) = \\ &= \cos(p+q) \cdot \cos(p-q) \end{aligned}$$

$$\text{h) } y = \frac{2 \text{sen}(a+b) \cos(a-b)}{-2 \text{sen}(a+b) \text{sen}(a-b)} = -\text{cotg}(a-b)$$

$$\begin{aligned} \text{i) } y &= \frac{\text{sen} \frac{\pi}{2} + \text{sen } a}{\text{sen} \frac{\pi}{2} - \text{sen } a} = \frac{2 \text{sen} \left(\frac{\pi}{4} + \frac{a}{2} \right) \cdot \cos \left(\frac{\pi}{4} - \frac{a}{2} \right)}{2 \text{sen} \left(\frac{\pi}{4} - \frac{a}{2} \right) \cdot \cos \left(\frac{\pi}{4} + \frac{a}{2} \right)} = \\ &= \text{tg} \left(\frac{\pi}{4} + \frac{a}{2} \right) \cdot \text{cotg} \left(\frac{\pi}{4} - \frac{a}{2} \right) \end{aligned}$$

231. a) $\frac{p+q}{2} = \frac{7\pi}{8}; \frac{p-q}{2} = \frac{\pi}{8}; p = \pi$ e $q = \frac{3\pi}{4};$
 $y = \frac{1}{2} \cdot \left(2 \cos \frac{7\pi}{8} \cdot \cos \frac{\pi}{8} \right) = \frac{1}{2} \left(\cos \pi + \cos \frac{3\pi}{4} \right) = \frac{-2 - \sqrt{2}}{4}$

b) $\frac{p+q}{2} = \frac{13\pi}{12}; \frac{p-q}{2} = \frac{7\pi}{12}; p = \frac{5\pi}{3}$ e $q = \frac{\pi}{2};$
 $y = -\frac{1}{2} \left(-2 \text{sen} \frac{13\pi}{12} \cdot \text{sen} \frac{7\pi}{12} \right) = -\frac{1}{2} \left(\cos \frac{5\pi}{3} - \cos \frac{\pi}{2} \right) = -\frac{1}{4}$

c) $\frac{p+q}{2} = \frac{5\pi}{24}; \frac{p-q}{2} = \frac{\pi}{24}; p = \frac{\pi}{4}$ e $q = \frac{\pi}{6};$
 $y = \frac{1}{2} \left(2 \text{sen} \frac{5\pi}{24} \cdot \cos \frac{\pi}{24} \right) = \frac{1}{2} \left(\text{sen} \frac{\pi}{4} + \text{sen} \frac{\pi}{6} \right) = \frac{1 + \sqrt{2}}{4}$

$$\begin{aligned}
 \mathbf{233.} \quad & (\operatorname{tg} 81^\circ + \operatorname{tg} 9^\circ) - (\operatorname{tg} 63^\circ + \operatorname{tg} 27^\circ) = \\
 & = \frac{\operatorname{sen} 90^\circ}{\cos 81^\circ \cdot \cos 9^\circ} - \frac{\operatorname{sen} 90^\circ}{\cos 63^\circ \cdot \cos 27^\circ} = \\
 & = \frac{1}{\frac{1}{2}(\cos 90^\circ + \cos 72^\circ)} - \frac{1}{\frac{1}{2}(\cos 90^\circ + \cos 36^\circ)} = \\
 & = \frac{2}{\operatorname{sen} 18^\circ} - \frac{2}{\operatorname{sen} 54^\circ} = \frac{2(\operatorname{sen} 54^\circ - \operatorname{sen} 18^\circ)}{\operatorname{sen} 18^\circ \cdot \operatorname{sen} 54^\circ} = \\
 & = \frac{2 \cdot 2 \cdot \operatorname{sen} 18^\circ \cdot \cos 36^\circ}{\operatorname{sen} 18^\circ \cdot \operatorname{sen} 54^\circ} = 4
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{235.} \quad & f(x) = \operatorname{sen} 2x + \operatorname{sen} \left(\frac{\pi}{2} + 2x \right) = 2 \operatorname{sen} \left(2x + \frac{\pi}{4} \right) \cdot \cos \left(-\frac{\pi}{4} \right) \\
 & f(x) = \sqrt{2} \cdot \operatorname{sen} \left(2x + \frac{\pi}{4} \right); D(f) = \mathbb{R}; \operatorname{Im}(f) = [-\sqrt{2}, \sqrt{2}], p = \pi
 \end{aligned}$$

$$\mathbf{236.} \quad f(x) = \frac{\operatorname{tg} \frac{\pi}{4} + \operatorname{tg} x}{\operatorname{tg} \frac{\pi}{4} - \operatorname{tg} x} = \frac{\frac{\operatorname{sen} \left(\frac{\pi}{4} + x \right)}{\cos \frac{\pi}{4} \cdot \cos x}}{\frac{\operatorname{sen} \left(\frac{\pi}{4} - x \right)}{\cos \frac{\pi}{4} \cdot \cos x}} = \frac{\operatorname{sen} \left(\frac{\pi}{4} + x \right)}{\operatorname{sen} \left(\frac{\pi}{4} - x \right)} = \operatorname{tg} \left(\frac{\pi}{4} + x \right); p = \pi$$

$$\begin{aligned}
 \mathbf{237.} \quad & |\operatorname{sen} x - \operatorname{sen} y| = \left| 2 \operatorname{sen} \frac{x-y}{2} \cdot \cos \frac{x+y}{2} \right| = \\
 & = |2| \cdot \left| \operatorname{sen} \frac{x-y}{2} \right| \cdot \left| \cos \frac{x+y}{2} \right| \leq |2| \cdot \left| \frac{x-y}{2} \right| \cdot 1 = \\
 & = \left| 2 \cdot \left(\frac{x-y}{2} \right) \right| = |x-y| \Rightarrow |\operatorname{sen} x - \operatorname{sen} y| \leq |x-y|
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{238.} \quad & \text{a) } \operatorname{tg} b(\operatorname{tg} a + \operatorname{tg} c) + \operatorname{tg} c \cdot \operatorname{tg} a = \operatorname{cotg}(a+c) \cdot (\operatorname{tg} a + \operatorname{tg} c) + \operatorname{tg} c \cdot \operatorname{tg} a = \\
 & = \frac{\cos(a+c)}{\operatorname{sen}(a+c)} \cdot \frac{\operatorname{sen}(a+c)}{\cos a \cos c} + \frac{\operatorname{sen} c \operatorname{sen} a}{\cos c \cos a} = \\
 & = \frac{\cos a \cos c - \operatorname{sen} a \operatorname{sen} c + \operatorname{sen} c \operatorname{sen} a}{\cos a \cos c} = 1
 \end{aligned}$$

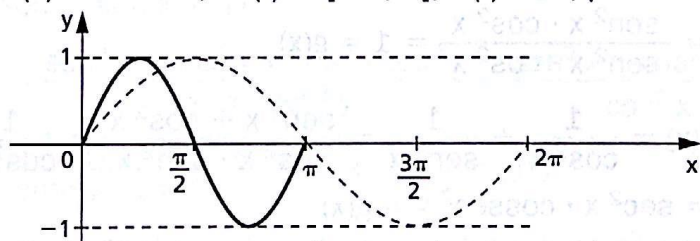
$$\begin{aligned}
 & \text{b) } \cos^2 a + \cos^2 b + \cos^2 c - 2 \cdot \operatorname{sen} a \cdot \operatorname{sen} b \cdot \operatorname{sen} c = \\
 & = \frac{1 + \cos 2a}{2} + \frac{1 + \cos 2b}{2} + (1 - \operatorname{sen}^2 c) + (-2 \cdot \operatorname{sen} a \cdot \operatorname{sen} b) \cdot \operatorname{sen} c = \\
 & = 1 + \cos(a+b) \cos(a-b) + 1 - \operatorname{sen}^2 c + [\cos(a+b) - \\
 & - \cos(a-b)] \operatorname{sen} c = \\
 & = 2 + \operatorname{sen} c \cdot \cos(a-b) - \operatorname{sen}^2 c + [\operatorname{sen} c - \cos(a-b)] \operatorname{sen} c = 2
 \end{aligned}$$

241. $\text{sen } 4A + \text{sen } 4B = -\text{sen } 4C \Rightarrow$
 $\Rightarrow 2 \text{sen } (2A + 2B) \cdot \cos (2A - 2B) = -2 \text{sen } 2C \cdot \cos 2C \Rightarrow$
 $\Rightarrow \text{sen } (360^\circ - 2C) \cdot \cos (2A - 2B) = -\text{sen } 2C \cdot \cos 2C \Rightarrow$
 $\Rightarrow \cos (2A - 2B) = \cos 2C \Rightarrow 2A - 2B = 2C \Rightarrow$
 $\Rightarrow \left(A = B + C = \frac{\pi}{2} \text{ ou } 2A - 2B = -2C \right) \Rightarrow B = A + C = \frac{\pi}{2}$

242. $A + B + C = \pi \Rightarrow C = \pi - (A + B) \Rightarrow 3C = 3\pi - (3A + 3B) \Rightarrow$
 $\Rightarrow \text{sen } 3C = -\text{sen } (3A + 3B)$, então:
 $(\text{sen } 3A + \text{sen } 3B) + \text{sen } 3C = 0 \Rightarrow$
 $\Rightarrow 2 \cdot \text{sen } \frac{3(A+B)}{2} \cdot \cos \frac{3(A-B)}{2} - 2 \cdot \text{sen } \frac{3(A+B)}{2} \cdot \cos \frac{3(A+B)}{2} = 0 \Rightarrow$
 $\Rightarrow 2 \cdot \text{sen } \frac{3(A+B)}{2} \cdot \left[\cos \frac{3(A-B)}{2} - \cos \frac{3(A+B)}{2} \right] = 0 \Rightarrow$
 $\Rightarrow 4 \cdot \text{sen } \frac{3(A+B)}{2} \cdot \text{sen } \frac{3A}{2} \cdot \text{sen } \frac{3B}{2} = 0 \Rightarrow$
 $\Rightarrow \text{sen } \frac{3(A+B)}{2} = 0 \text{ ou } \text{sen } \frac{3A}{2} = 0 \text{ ou } \text{sen } \frac{3B}{2} = 0 \Rightarrow$
 $\Rightarrow C = \frac{\pi}{3} \text{ ou } A = \frac{\pi}{3} \text{ ou } B = \frac{\pi}{3} \text{ (respectivamente)}$

244. $4 \text{sen } (x + 60^\circ) \cdot \cos (x + 30^\circ) =$
 $= 4 (\text{sen } x \cos 60^\circ + \text{sen } 60^\circ \cos x) \cdot (\cos x \cos 30^\circ - \text{sen } x \text{sen } 30^\circ) =$
 $= (\text{sen } x + \sqrt{3} \cos x) \cdot (\sqrt{3} \cos x - \text{sen } x) = (\sqrt{3} \cos x)^2 - (\text{sen } x)^2 =$
 $= 3 \cos^2 x - \text{sen}^2 x$

245. a) $f(x) = \text{sen } 2x$, $\text{Im}(f) = [-1, 1]$, $D(f) = \mathbb{R}$, $p = \pi$



b) $f(x) = \text{sen } 2x + \text{sen} \left(2x + \frac{\pi}{2} \right) = 2 \text{sen} \left(2x + \frac{\pi}{4} \right) \cdot \cos \left(-\frac{\pi}{4} \right) =$
 $= \sqrt{2} \text{sen} \left(2x + \frac{\pi}{4} \right)$

246. $\text{sen } u + \cos u = \text{sen } u + \text{sen} \left(\frac{\pi}{2} - u \right) = \sqrt{2} \cdot \cos \left(u - \frac{\pi}{4} \right) = \sqrt{2} \cos v \quad (1)$
 $\sqrt{2} \cdot \text{sen } u \cdot \cos u = \frac{2 \cdot \text{sen } u \cdot \cos u}{\sqrt{2}} = \frac{\text{sen } 2u}{\sqrt{2}} = \frac{\text{sen} \left(\frac{\pi}{2} - 2v \right)}{\sqrt{2}} = \frac{\cos 2v}{\sqrt{2}}$
 $S = \frac{\sqrt{2} \cdot \cos v}{\cos 2v} = \frac{2 \cdot \cos v}{\cos 2v} = \frac{2x}{2x^2 - 1}$
 $\sqrt{2}$

247. $n < 20 \cdot \cos^2 15 = 20 \cdot \frac{1 + \cos 30^\circ}{2} = 20 \cdot \left(\frac{1}{2} + \frac{\sqrt{3}}{4}\right) \cong 18,66;$
então $n = 18$

248. $f(x) = \cos 2x + \sin 2x = \sin\left(\frac{\pi}{2} + 2x\right) + \sin 2x = \sqrt{2} \sin\left(\frac{\pi}{4} + 2x\right)$
 $\text{Im}(f) = [-\sqrt{2}, \sqrt{2}]$

CAPÍTULO X — Identidades

253. a) $f(x) = (\cos^2 x + \sin^2 x)^2 = 1 = g(x)$

b) $f(x) = \frac{\sin x}{\operatorname{cosec} x} + \frac{\cos x}{\sec x} = \frac{\sin x}{\frac{1}{\sin x}} + \frac{\cos x}{\frac{1}{\cos x}} =$
 $= \sin^2 x + \cos^2 x = 1 = g(x)$

254. $f(x) = \operatorname{tg} x + \operatorname{cotg} x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} =$
 $= \frac{1}{\cos x} \cdot \frac{1}{\sin x} = \sec x \cdot \operatorname{cosec} x = g(x)$

255. $f(x) = \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)\left(\frac{1}{\cos x} - \cos x\right)\left(\frac{1}{\sin x} - \sin x\right) =$
 $= \left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right)\left(\frac{1 - \cos^2 x}{\cos x}\right)\left(\frac{1 - \sin^2 x}{\sin x}\right) =$
 $= \frac{\sin^2 x \cdot \cos^2 x}{\sin^2 x \cdot \cos^2 x} = 1 = g(x)$

256. $f(x) = \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \frac{\sin^2 x + \cos^2 x}{\cos^2 x \cdot \sin^2 x} = \frac{1}{\cos^2 x} \cdot \frac{1}{\sin^2 x} =$
 $= \sec^2 x \cdot \operatorname{cosec}^2 x = g(x)$

257. $f(x) = \frac{\operatorname{cotg}^2 x}{\operatorname{cosec}^2 x} = \frac{\cos^2 x}{\sin^2 x} : \frac{1}{\sin^2 x} = \cos^2 x = g(x)$

258. $f(x) = \frac{(\sin x - \cos x)(\sin^2 x + \cos^2 x + \sin x \cos x)}{\sin x - \cos x} = 1 + \sin x \cos x = g(x)$

259. $f(x) = 1 + \operatorname{cotg}^2 x + \operatorname{tg}^2 x = \sec^2 x + \operatorname{cotg}^2 x = g(x)$

260. $f(x) = 2\left(\sin x + \frac{\sin x}{\cos x}\right)\left(\cos x + \frac{\cos x}{\sin x}\right) =$
 $= 2\left[\frac{\sin x (\cos x + 1)}{\cos x}\right]\left[\frac{\cos x (\sin x + 1)}{\sin x}\right] =$
 $= 2(\cos x + 1)(\sin x + 1) = h(x)$

$$\begin{aligned}
 g(x) &= (1 + \operatorname{sen} x + \operatorname{cos} x)^2 = \\
 &= 1 + \operatorname{sen}^2 x + \operatorname{cos}^2 x + 2 \operatorname{sen} x \operatorname{cos} x + 2 \operatorname{sen} x + 2 \operatorname{cos} x = \\
 &= 2(1 + \operatorname{sen} x + \operatorname{cos} x + \operatorname{sen} x \operatorname{cos} x) = 2(1 + \operatorname{sen} x)(1 + \operatorname{cos} x) = h(x)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{261.} \quad f(x) &= 1 + 2 \operatorname{cotg} x + \operatorname{cotg}^2 x + 1 - 2 \operatorname{cotg} x + \operatorname{cotg}^2 x = 2 + 2 \operatorname{cotg}^2 x = \\
 &= 2(1 + \operatorname{cotg}^2 x) = 2 \operatorname{cossec}^2 x = g(x)
 \end{aligned}$$

$$\mathbf{262.} \quad f(x) = \frac{(1 - \operatorname{cos}^2 x)^2}{(1 - \operatorname{sen}^2 x)^2} = \left(\frac{\operatorname{sen}^2 x}{\operatorname{cos}^2 x}\right)^2 = \operatorname{tg}^4 x = g(x)$$

$$\begin{aligned}
 \mathbf{263.} \quad f(x) - g(x) &= \operatorname{cotg}^2 x - 2 \operatorname{cotg} x \cdot \operatorname{cos} x + \operatorname{cos}^2 x + 1 - 2 \cdot \operatorname{sen} x + \operatorname{sen}^2 x - \\
 &- 1 + 2 \cdot \operatorname{cossec} x - \operatorname{cossec}^2 x = \\
 &= -2 \cdot \frac{\operatorname{cos} x}{\operatorname{sen} x} \cdot \operatorname{cos} x - 2 \cdot \operatorname{sen} x + 2 \cdot \frac{1}{\operatorname{sen} x} = \\
 &= \frac{-2 \cdot \operatorname{cos}^2 x - 2 \cdot \operatorname{sen}^2 x + 2}{\operatorname{sen} x} = 0
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{264.} \quad f(x) - g(x) &= \frac{\operatorname{cos} x + \operatorname{cos} y}{\operatorname{sen} x - \operatorname{sen} y} - \frac{\operatorname{sen} x + \operatorname{sen} y}{\operatorname{cos} y - \operatorname{cos} x} = \\
 &= \frac{\operatorname{cos}^2 y - \operatorname{cos}^2 x - \operatorname{sen}^2 x + \operatorname{sen}^2 y}{(\operatorname{sen} x - \operatorname{sen} y)(\operatorname{cos} y - \operatorname{cos} x)} = 0
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{265.} \quad f(x) &= \frac{\operatorname{cos} x + \frac{\operatorname{cos} x}{\operatorname{sen} x}}{\frac{\operatorname{sen} x}{\operatorname{cos} x} + \frac{1}{\operatorname{cos} x}} = \frac{(\operatorname{sen} x \cdot \operatorname{cos} x + \operatorname{cos} x) \cdot \operatorname{cos} x}{\operatorname{sen} x (\operatorname{sen} x + 1)} = \\
 &= \frac{\operatorname{cos}^2 x (\operatorname{sen} x + 1)}{\operatorname{sen} x (\operatorname{sen} x + 1)} = \operatorname{cos} x \cdot \operatorname{cotg} x = g(x)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{266.} \quad f(x) &= \frac{\operatorname{sen}^2 x - \operatorname{cos}^2 y + \operatorname{cos}^2 x \operatorname{cos}^2 y}{\operatorname{cos}^2 x \cdot \operatorname{cos}^2 y} = \frac{\operatorname{sen}^2 x - \operatorname{cos}^2 y (1 - \operatorname{cos}^2 x)}{\operatorname{cos}^2 x \cdot \operatorname{cos}^2 y} = \\
 &= \frac{\operatorname{sen}^2 x (1 - \operatorname{cos}^2 y)}{\operatorname{cos}^2 x \cdot \operatorname{cos}^2 y} = \operatorname{tg}^2 x \cdot \operatorname{tg}^2 y = g(x)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{267.} \quad g(x) &= \operatorname{cossec}^2 x - 2 \cdot \operatorname{cossec} x \cdot \operatorname{cotg} x + \operatorname{cotg}^2 x = \\
 &= \frac{1}{\operatorname{sen}^2 x} - \frac{2 \operatorname{cos} x}{\operatorname{sen}^2 x} + \frac{\operatorname{cos}^2 x}{\operatorname{sen}^2 x} = \\
 &= \frac{(1 - \operatorname{cos} x)^2}{1 - \operatorname{cos}^2 x} = \frac{(1 - \operatorname{cos} x)^2}{(1 + \operatorname{cos} x)(1 - \operatorname{cos} x)} = f(x)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{268.} \quad f(x) &= \frac{\frac{\operatorname{cos} x}{\operatorname{sen} x} + \frac{\operatorname{cos} y}{\operatorname{sen} y}}{\frac{\operatorname{sen} x}{\operatorname{cos} x} + \frac{\operatorname{sen} y}{\operatorname{cos} y}} = \\
 &= \frac{(\operatorname{cos} x \cdot \operatorname{sen} y + \operatorname{cos} y \cdot \operatorname{sen} x)}{\operatorname{sen} x \cdot \operatorname{sen} y} \cdot \frac{(\operatorname{cos} x \cdot \operatorname{cos} y)}{(\operatorname{sen} x \cdot \operatorname{cos} y + \operatorname{sen} y \cdot \operatorname{cos} x)} = \\
 &= \operatorname{cotg} x \cdot \operatorname{cotg} y = g(x)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{269.} \quad f(x) &= \sec^2 x \cdot \sec^2 y + 2 \cdot \sec x \cdot \sec y \cdot \operatorname{tg} x \cdot \operatorname{tg} y + \operatorname{tg}^2 x \cdot \operatorname{tg}^2 y = \\
 &= (1 + \operatorname{tg}^2 x)(1 + \operatorname{tg}^2 y) + 2 \cdot \sec x \cdot \sec y \cdot \operatorname{tg} x \cdot \operatorname{tg} y + \operatorname{tg}^2 x \cdot \operatorname{tg}^2 y = \\
 &= 1 + \operatorname{tg}^2 x + \operatorname{tg}^2 y + \operatorname{tg}^2 x \cdot \operatorname{tg}^2 y + 2 \cdot \sec x \cdot \sec y \cdot \operatorname{tg} x \cdot \operatorname{tg} y + \operatorname{tg}^2 x \cdot \operatorname{tg}^2 y = \\
 &= 1 + \operatorname{tg}^2 y (1 + \operatorname{tg}^2 x) + 2 \cdot \sec x \cdot \sec y \cdot \operatorname{tg} x \cdot \operatorname{tg} y + \operatorname{tg}^2 x (1 + \operatorname{tg}^2 y) = \\
 &= 1 + \operatorname{tg}^2 y \cdot \sec^2 x + 2 \cdot \sec x \cdot \sec y \cdot \operatorname{tg} x \cdot \operatorname{tg} y + \operatorname{tg}^2 x \cdot \sec^2 y = g(x)
 \end{aligned}$$

$$\mathbf{270.} \quad f(x) - g(x) = \frac{(\sec x - \operatorname{tg} x)(\sec x + \operatorname{tg} x) - 1}{\sec x + \operatorname{tg} x} = \frac{\sec^2 x - \operatorname{tg}^2 x - 1}{\sec x + \operatorname{tg} x} = 0$$

$$\begin{aligned}
 \mathbf{271.} \quad f(x) &= (\operatorname{cosec}^2 x - \operatorname{cotg}^2 x)(\operatorname{cosec}^4 x + \operatorname{cosec}^2 x \cdot \operatorname{cotg}^2 x + \operatorname{cotg}^4 x) = \\
 &= (1 + \operatorname{cotg}^2 x - \operatorname{cotg}^2 x)[(1 + \operatorname{cotg}^2 x)^2 + \operatorname{cosec}^2 x \cdot \operatorname{cotg}^2 x + \operatorname{cotg}^4 x] = \\
 &= 1 + 2 \operatorname{cotg}^2 x + 2 \operatorname{cotg}^4 x + \operatorname{cosec}^2 x \cdot \operatorname{cotg}^2 x = \\
 &= 1 + 2 \operatorname{cotg}^2 x (1 + \operatorname{cotg}^2 x) + \operatorname{cosec}^2 x \cdot \operatorname{cotg}^2 x = \\
 &= 1 + 3 \operatorname{cotg}^2 x \cdot \operatorname{cosec}^2 x = g(x)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{273.} \quad f(x) &= \operatorname{tg}^2 (45^\circ + x) = \left[\frac{\operatorname{tg} 45^\circ + \operatorname{tg} x}{1 - \operatorname{tg} 45^\circ \operatorname{tg} x} \right]^2 = \left[\frac{\cos x + \operatorname{sen} x}{\cos x - \operatorname{sen} x} \right]^2 = \\
 &= \frac{1 + 2 \operatorname{sen} x \cos x}{1 - 2 \operatorname{sen} x \cos x} = g(x)
 \end{aligned}$$

$$\mathbf{275.} \quad \frac{\pi}{4} < a < \frac{\pi}{2} \Rightarrow 0,7 < \operatorname{sen} a < 1; 0 < \cos a < 0,71$$

$$\frac{\pi}{4} < b < \frac{\pi}{2} \Rightarrow 0,7 < \operatorname{sen} b < 1; 0 < \cos b < 0,71$$

$$\operatorname{sen} (a + b) - \operatorname{sen} a - \frac{4}{5} \operatorname{sen} b =$$

$$= \operatorname{sen} a (\cos b - 1) + \operatorname{sen} b (\cos a - 0,8) < 0 \Rightarrow$$

$$\Rightarrow \operatorname{sen} (a + b) < \operatorname{sen} a + \frac{4}{5} \operatorname{sen} b$$

$$\begin{aligned}
 \mathbf{276.} \quad f(x) &= \left[\operatorname{sen} A + \operatorname{sen} \left(\frac{\pi}{2} - A \right) \right]^4 = \left[\sqrt{2} \cdot \cos \left(A - \frac{\pi}{4} \right) \right]^4 = \\
 &= 4 \cos^4 \left(A - \frac{\pi}{4} \right) = g(x)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{278.} \quad \text{a)} \quad \operatorname{sen} B + \operatorname{sen} C - \operatorname{sen} A &= 2 \operatorname{sen} \frac{B+C}{2} \cdot \cos \frac{B-C}{2} - 2 \operatorname{sen} \frac{A}{2} \cdot \cos \frac{A}{2} = \\
 &= 2 \cos \frac{A}{2} \cdot \cos \frac{B-C}{2} - 2 \operatorname{sen} \frac{A}{2} \cdot \cos \frac{A}{2} = \\
 &= 2 \cos \frac{A}{2} \left[\cos \frac{B-C}{2} - \cos \frac{B+C}{2} \right] = \\
 &= 2 \cos \frac{A}{2} \left[-2 \operatorname{sen} \frac{B}{2} \operatorname{sen} \left(-\frac{C}{2} \right) \right] = 4 \cos \frac{A}{2} \operatorname{sen} \frac{B}{2} \operatorname{sen} \frac{C}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \cos B + \cos C - \cos A &= 2 \cos \frac{B+C}{2} \cos \frac{B-C}{2} - \left(1 - 2 \operatorname{sen}^2 \frac{A}{2}\right) = \\
 &= -1 + 2 \operatorname{sen} \frac{A}{2} \left(\cos \frac{B-C}{2} + \operatorname{sen} \frac{A}{2}\right) = \\
 &= -1 + 2 \operatorname{sen} \frac{A}{2} \left(\cos \frac{B-C}{2} + \cos \frac{B+C}{2}\right) = \\
 &= -1 + 2 \operatorname{sen} \frac{A}{2} \left[2 \cos \frac{B}{2} \cdot \cos \left(\frac{-C}{2}\right)\right] = \\
 &= -1 + 4 \cos \frac{B}{2} \cdot \cos \frac{C}{2} \cdot \operatorname{sen} \frac{A}{2} \\
 \text{c) } \cos 2A + \cos 2B + \cos 2C &= 2 \cos^2 A - 1 + 2 \cos(B+C) \cdot \cos(B-C) = \\
 &= 2 \cos^2 A - 1 + 2(-\cos A) \cos(B-C) = \\
 &= -1 + 2 \cos A [\cos A - \cos(B-C)] = \\
 &= -1 - 2 \cos A [\cos(B+C) + \cos(B-C)] = \\
 &= -1 - 2 \cos A (2 \cos B \cos C) = -1 - 4 \cos A \cos B \cos C \\
 \text{d) } \operatorname{sen}^2 A + \operatorname{sen}^2 B + \operatorname{sen}^2 C &= \\
 &= \frac{1 - \cos 2A}{2} + \frac{1 - \cos 2B}{2} + \frac{1 - \cos 2C}{2} = \\
 &= \frac{3 + 1 + 4 \cos A \cos B \cos C}{2} = 2(1 + \cos A \cos B \cos C) \\
 \text{e) } A + B + C = \pi &\Rightarrow A + B = \pi - C; \\
 \cotg(A+B) &= \cotg(\pi - C) = -\cotg C \Rightarrow \frac{\cotg A \cdot \cotg B - 1}{\cotg A + \cotg B} = \\
 &= -\cotg C \Rightarrow \cotg A \cdot \cotg B - 1 = -\cotg A \cdot \cotg C - \cotg B \cdot \cotg C \Rightarrow \\
 &\Rightarrow \frac{1}{\operatorname{tg} A \cdot \operatorname{tg} B} + \frac{1}{\operatorname{tg} B \cdot \operatorname{tg} C} + \frac{1}{\operatorname{tg} C \cdot \operatorname{tg} A} = 1
 \end{aligned}$$

279. a) $f(a) = \operatorname{sen} 4a = 2 \operatorname{sen} 2a \cdot \cos 2a = 4 \operatorname{sen} a \cdot \cos a (\cos^2 a - \operatorname{sen}^2 a) =$
 $= 4 \operatorname{sen} a \cdot \cos^3 a - 4 \operatorname{sen}^3 a \cdot \cos a = g(a)$

b) $f(a) = \cos 4a = 2 \cos^2 2a - 1 = 2(2 \cos^2 a - 1)^2 - 1 =$
 $= 8 \cos^4 a - 8 \cos^2 a + 1 = g(a)$

c) $f(a) = \frac{2 \cdot \operatorname{tg} 2a}{1 - \operatorname{tg}^2 2a} = (2 \cdot \operatorname{tg} 2a) \cdot \frac{1}{1 - \operatorname{tg}^2 2a} =$
 $= \frac{4 \cdot \operatorname{tg} a}{1 - \operatorname{tg}^2 a} \cdot \frac{1}{1 - \left(\frac{2 \cdot \operatorname{tg} a}{1 - \operatorname{tg}^2 a}\right)^2} =$
 $= \frac{4 \cdot \operatorname{tg} a}{1 - \operatorname{tg}^2 a} \cdot \frac{(1 - \operatorname{tg}^2 a)^2}{(1 - \operatorname{tg}^2 a)^2 - (2 \cdot \operatorname{tg} a)^2} = \frac{4 \cdot \operatorname{tg} a - 4 \cdot \operatorname{tg}^3 a}{1 - 6 \cdot \operatorname{tg}^2 a + \operatorname{tg}^4 a} = g(a)$

280. Prova pelo princípio da indução finita.

$$\text{Para } n = 1, \frac{\text{sen } 2a}{2 \text{ sen } a} = \frac{2 \text{ sen } a \cos a}{2 \text{ sen } a} = \cos a.$$

Admitindo para $n = k$:

$$\cos a \cdot \cos 2a \cdot \dots \cdot \cos (2^{k-1} \cdot a) = \frac{\text{sen } (2^k \cdot a)}{2^k \cdot \text{sen } a}$$

Provemos que vale para $n = k + 1$:

$$1 \cdot \cos a \cdot \cos 2a \cdot \dots \cdot \cos (2^{k-1} \cdot a) \cdot \cos (2^k \cdot a) =$$

$$\begin{aligned} & \text{H.I.} \\ & = \frac{\text{sen } (2^k \cdot a)}{2^k \cdot \text{sen } a} \cdot \cos (2^k \cdot a) = \frac{1 \cdot \text{sen } [2 \cdot (2^k \cdot a)]}{2 \cdot 2^k \cdot \text{sen } a} = \frac{\text{sen } (2^{k+1} \cdot a)}{2^{k+1} \cdot \text{sen } a} \end{aligned}$$

$$\mathbf{281.} \quad \text{a) } \cotg \frac{\alpha}{2} - \cotg \alpha = \frac{1}{\text{tg } \frac{\alpha}{2}} - \frac{1}{\text{tg } \alpha} =$$

$$= \frac{1}{\text{tg } \frac{\alpha}{2}} - \frac{1 - \text{tg}^2 \frac{\alpha}{2}}{2 \cdot \text{tg } \frac{\alpha}{2}} = \frac{1 + \text{tg}^2 \frac{\alpha}{2}}{2 \cdot \text{tg } \frac{\alpha}{2}} =$$

$$= \frac{\text{sec}^2 \frac{\alpha}{2}}{2 \cdot \text{tg } \frac{\alpha}{2}} = \frac{1}{2 \cdot \text{sen } \frac{\alpha}{2} \cdot \cos \frac{\alpha}{2}} = \frac{1}{\text{sen } \alpha}$$

$$\text{b) } \frac{1}{\text{sen } a} + \frac{1}{\text{sen } 2a} + \frac{1}{\text{sen } 4a} + \dots + \frac{1}{\text{sen } (2^n \cdot a)} =$$

$$= \left(\cotg \frac{a}{2} - \cotg a \right) + (\cotg a - \cotg 2a) + \dots +$$

$$+ (\cotg 2^{n-2}a - \cotg 2^{n-1}a) + (\cotg 2^{n-1}a - \cotg 2^n a) =$$

$$= \cotg \frac{a}{2} - \cotg 2^n a$$

$$\mathbf{282.} \quad \cos^2 x + \text{sen}^2 x + 2 \text{ sen } x \cos x + k \text{ sen } x \cos x - 1 = 0 \Rightarrow$$

$$\Rightarrow \text{sen } x \cos x (2 + k) = 0 \Rightarrow 2 + k = 0 \Rightarrow k = -2$$

$$\mathbf{285.} \quad \text{a) } \frac{(-\text{sen } x)(-\text{sen } x)}{-\frac{\text{sen } x}{\cos x}(-\cos x)} = \frac{\text{sen}^2 x}{\text{sen } x} = \text{sen } x$$

$$\text{b) } \frac{\text{sen } x (-\cotg x)}{-\text{tg } x (-\text{sen } x)} = -\frac{\cotg x}{\text{tg } x} = -\cotg^2 x$$

$$\text{c) } \frac{-\text{sec } x (-\cotg x)}{\text{cossec } x (-\cotg x)} = -\frac{\text{sen } x}{\cos x} = -\text{tg } x$$

$$\text{d) } -\cos x + \cos x + \cotg x = \cotg x$$

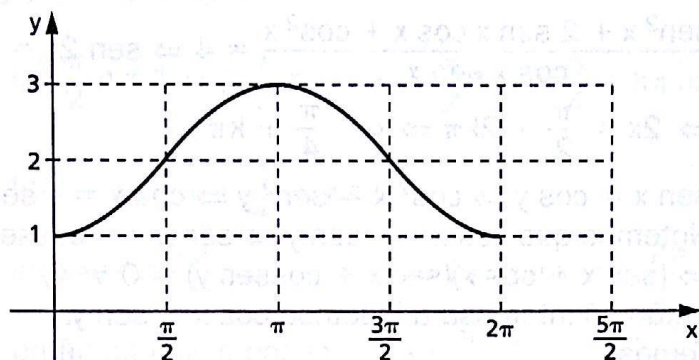
$$\mathbf{286.} \quad 1 - \text{sen } x \cdot \text{sen } x = 1 - \text{sen}^2 x = \cos^2 x$$

287.
$$-1 + \frac{(-\operatorname{sen} x) \cdot (-\operatorname{tg} x)}{-\cos x} = -1 - \frac{\operatorname{sen}^2 x}{\cos^2 x} = -1 - \operatorname{tg}^2 x = -\sec^2 x$$

288.
$$\frac{-a^2 - (a - b)^2 \cdot (-1) + 2ab}{b^2} = \frac{b^2}{b^2} = 1$$

289. $y = -\cos x + 2$

$\operatorname{Im}(f) = [1, 3]; p = 2\pi, D(f) = \mathbb{R}$



CAPÍTULO XI — Equações

293. $\operatorname{sen}^2 x = t \Rightarrow 4t^2 - 11t + 6 = 0 \Rightarrow t = 2$ ou $t = \frac{3}{4}$; $\operatorname{sen}^2 x = 2$ não serve;
 $\operatorname{sen}^2 x = \frac{3}{4} \Rightarrow \operatorname{sen} x = \pm \frac{\sqrt{3}}{2} \Rightarrow x = \pm \frac{\pi}{3} + k\pi$

295. a) $5x = 3x + 2k\pi \Rightarrow x = k\pi$
 ou
 $5x = \pi - 3x + 2k\pi \Rightarrow x = \frac{\pi}{8} + \frac{k\pi}{4}$
 b) $3x = 2x + 2k\pi \Rightarrow x = 2k\pi$
 ou
 $3x = \pi - 2x + 2k\pi \Rightarrow x = \frac{\pi}{5} + 2k\frac{\pi}{5}$

296. $\operatorname{sen} x < 0 \Rightarrow 2 \cdot \operatorname{sen} x \cdot (-\operatorname{sen} x) + 3 \cdot \operatorname{sen} x = 2 \Rightarrow$
 $\Rightarrow \exists x \in \mathbb{R} | 2 \operatorname{sen} x |\operatorname{sen} x| + 3 \operatorname{sen} x - 2 = 0$
 $\operatorname{sen} x > 0 \Rightarrow \operatorname{sen} x = t \Rightarrow 2t^2 + 3t - 2 = 0 \Rightarrow t = \frac{1}{2}$ ou $t = -2$;
 $\operatorname{sen} x = -2$ não serve; $\operatorname{sen} x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} + 2k\pi$ ou $x = \frac{5\pi}{6} + 2k\pi$

297. $\operatorname{sen}(x + y) = \operatorname{sen} 0 \Rightarrow \left. \begin{matrix} x + y = k\pi \\ x - y = \pi \end{matrix} \right\} \Rightarrow x = \frac{\pi}{2} + \frac{k\pi}{2}$ e $y = -\frac{\pi}{2} + \frac{k\pi}{2}$

- 302.** a) $3x = x + 2k\pi \Rightarrow x = k\pi$ ou $3x = -x + 2k\pi \Rightarrow x = \frac{k\pi}{2}$
 b) $5x = x + \frac{\pi}{3} + 2k\pi \Rightarrow x = \frac{\pi}{2} + \frac{k\pi}{12}$ ou $5x = -x - \frac{\pi}{3} + 2k\pi \Rightarrow$
 $\Rightarrow x = -\frac{\pi}{18} + \frac{k\pi}{3}$
- 303.** a) $(\sin x + \cos x)(\sec x + \operatorname{cosec} x) = 4 \Rightarrow (\sin x + \cos x) \left(\frac{\sin x + \cos x}{\cos x \sin x} \right) = 4$
 $\frac{\sin^2 x + 2 \sin x \cos x + \cos^2 x}{\cos x \sin x} = 4 \Rightarrow \sin 2x = 1 \Rightarrow$
 $\Rightarrow 2x = \frac{\pi}{2} + 2k\pi \Rightarrow x = \frac{\pi}{4} + k\pi$
 b) $\sin x = \cos y \Rightarrow \cos^2 x = \sin^2 y \Rightarrow \cos x = \pm \sin y$
 Notemos que $\cos x = -\sin y \Rightarrow \sec x = -\operatorname{cosec} y \Rightarrow$
 $\Rightarrow (\sin x + \cos x)(\sec x + \operatorname{cosec} y) = 0 \neq 4;$
 então só interessa a hipótese $\cos x = \sin y$.
 Temos:
 $(\sin x + \cos y)(\sec x + \operatorname{cosec} y) = 4 \Rightarrow$
 $\Rightarrow (\sin x + \sin x)(\sec x + \sec x) = 4 \Rightarrow \operatorname{tg} x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi$
 e daí $y = \frac{\pi}{4} + k\pi$.
- 304.** $\sin^2 x = y \Rightarrow y^3 + y^2 + y = 3 \Rightarrow (y - 1)(y^2 + 2y + 3) = 0 \Rightarrow y = 1$
 $\sin x = \pm 1 \Rightarrow x = \frac{\pi}{2} + k\pi \Rightarrow x = (2k + 1) \frac{\pi}{2}$
- 305.** $2 \sin \frac{\pi}{4} \cdot \cos x = \sqrt{2} \Rightarrow \cos x = 1 \Rightarrow x = 2k\pi$
- 306.** $x + y = \pi \Rightarrow x = \pi - y \Rightarrow \sin x = \sin y$
 $\sin x + \sin y = \log_{10} t^2 \Rightarrow 2 \sin x = 2 \log_{10} t \Rightarrow \sin x = \log_{10} t$
 $-1 \leq \log_{10} t \leq 1 \Rightarrow 0,1 \leq t \leq 10$
- 310.** a) $1 + \operatorname{tg}^2 x - 2 \operatorname{tg} x = 0, \operatorname{tg} x = t \Rightarrow t^2 - 2t + 1 = 0 \Rightarrow t = 1$
 $\operatorname{tg} x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi$
 b) $\operatorname{cosec}^2 x = 1 - \operatorname{cotg} x \Rightarrow 1 + \operatorname{cotg}^2 x = 1 - \operatorname{cotg} x \Rightarrow$
 $\Rightarrow \operatorname{cotg}^2 x = \operatorname{cotg} x = 0 \Rightarrow \operatorname{cotg} x = 0$ ou $\operatorname{cotg} x = -1 \Rightarrow$
 $\Rightarrow x = \frac{\pi}{2} + k\pi$ ou $x = \frac{3\pi}{4} + k\pi$
 c) $\sin 2x \cdot \cos \left(x + \frac{\pi}{4} \right) - \cos 2x \cdot \sin \left(x + \frac{\pi}{4} \right) = 0 \Rightarrow$
 $\Rightarrow \sin \left[2x - \left(x + \frac{\pi}{4} \right) \right] = 0 \Rightarrow \sin \left(x - \frac{\pi}{4} \right) = 0 \Rightarrow x = \frac{\pi}{4} + k\pi$

$$\begin{aligned}
 \text{d) } 1 + \operatorname{sen} 2x - \operatorname{tg} x - \operatorname{tg} x \operatorname{sen} 2x &= 1 + \operatorname{tg} x \Rightarrow \operatorname{sen} 2x = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg} x} \Rightarrow \\
 &\Rightarrow \frac{2 \operatorname{tg} x}{1 + \operatorname{tg}^2 x} = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg} x} \Rightarrow \operatorname{tg}^2 x + \operatorname{tg} x = 0 \Rightarrow \operatorname{tg} x (\operatorname{tg} x + 1) = 0 \Rightarrow \\
 &\Rightarrow \operatorname{tg} x = 0 \Rightarrow x = k\pi \text{ ou } \operatorname{tg} x = -1 \Rightarrow x = \frac{3\pi}{4} + k\pi
 \end{aligned}$$

$$\text{311. } \frac{1}{\operatorname{tg} x} = \frac{2 \operatorname{tg} x}{1 + \operatorname{tg}^2 x} \Rightarrow \operatorname{tg}^2 x = 1 \Rightarrow \operatorname{tg} x = \pm 1 \Rightarrow x = \pm \frac{\pi}{4} + k\pi$$

$$\text{312. } \operatorname{tg}^2 \frac{\pi}{2} p = 1 \Rightarrow \operatorname{tg} \frac{\pi}{2} p = \pm 1 \Rightarrow \frac{\pi}{2} p = \pm \frac{\pi}{4} + k\pi \Rightarrow p = \pm \frac{1}{2} + 2k, k \in \mathbb{Z}$$

$$\text{313. } \operatorname{tg} x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi \text{ ou}$$

$$\operatorname{sen} x = \pm \frac{\sqrt{3}}{2} \Rightarrow x = \pm \frac{\pi}{3} + 2k\pi \text{ ou } x = \pm \frac{2\pi}{3} + 2k\pi;$$

então as raízes positivas são $\frac{\pi}{4}, \frac{5\pi}{4}, \dots, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \dots$ e daí

$a = \frac{\pi}{4}$ (a menor delas). Temos:

$$\operatorname{sen}^4 a - \cos^2 a = \operatorname{sen}^4 \frac{\pi}{4} - \cos^2 \frac{\pi}{4} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$$

$$\text{314. } \Delta = 4 \operatorname{tg}^2 a + 4 = 4 \sec^2 a, x = \frac{2 \operatorname{tg} a \pm 2 \sec a}{2} \Rightarrow x = \frac{\operatorname{sen} a \pm 1}{\cos a}$$

$$\text{316. a) } \operatorname{sen} x + \operatorname{sen} \left(\frac{\pi}{2} - x \right) = -1 \Rightarrow \sqrt{2} \cos \left(x - \frac{\pi}{4} \right) = -1 \Rightarrow$$

$$\Rightarrow \cos \left(x - \frac{\pi}{4} \right) = -\frac{\sqrt{2}}{2} \Rightarrow \left[\cos \left(x - \frac{\pi}{4} \right) = \cos \frac{3\pi}{4} \Rightarrow$$

$$\Rightarrow x = \pi + 2k\pi \text{ ou } \cos \left(x - \frac{\pi}{4} \right) = \cos \frac{5\pi}{4} \Rightarrow x = \frac{3\pi}{2} + 2k\pi \right]$$

$$\text{b) } \operatorname{sen} x - \frac{1}{\sqrt{3}} \cos x = -1 \Rightarrow \operatorname{sen} x - \frac{\operatorname{sen} \frac{\pi}{6}}{\cos \frac{\pi}{6}} \cdot \cos x = -1 \Rightarrow$$

$$\Rightarrow \operatorname{sen} \left(x - \frac{\pi}{6} \right) = -\frac{\sqrt{3}}{2} \Rightarrow$$

$$\Rightarrow \left[\operatorname{sen} \left(x - \frac{\pi}{6} \right) = \operatorname{sen} \frac{4\pi}{3} \Rightarrow x = \frac{3\pi}{2} + 2k\pi \text{ ou}$$

$$\operatorname{sen} \left(x - \frac{\pi}{6} \right) = \operatorname{sen} \frac{5\pi}{3} \Rightarrow x = \frac{11\pi}{6} + 2k\pi \right]$$

318. a) $\text{sen } 4x = u$ e $\text{cos } x = v$, $\begin{cases} u + v = 1 & (1) \\ u^2 + v^2 = 1 & (2) \end{cases}$, (1) em (2) \Rightarrow

$$\Rightarrow u^2 + (1 - u)^2 = 1 \Rightarrow 2u^2 - 2u = 0.$$

Então: $u = 0$ e $v = 1 \Rightarrow \text{sen } 4x = 0$ e $\text{cos } 4x = 1 \Rightarrow x = \frac{k\pi}{2}$

ou $u = 1$ e $v = 0 \Rightarrow \text{sen } 4x = 1$ e $\text{cos } 4x = 0 \Rightarrow x = \frac{\pi}{8} + \frac{k\pi}{2}$

b) $\text{sen } x = 0$ e $\text{cos } x = \pm 1 \Rightarrow x = k\pi$

$\text{sen } x = \pm 1$ e $\text{cos } x = 0 \Rightarrow x = \frac{\pi}{2} + k\pi$

319. $\text{sen } 2x = u$ e $\text{cos } 2x = v$, $\begin{cases} u + v = 1 & (1) \\ u^2 + v^2 = 1 & (2) \end{cases}$, (1) em (2) \Rightarrow

$$\Rightarrow u^2 + (1 - u)^2 = 1 \Rightarrow 2u^2 - 2u = 0.$$

Então: $u = 0$ e $v = 1 \Rightarrow \text{sen } 2x = 0$ e $\text{cos } 2x = 1 \Rightarrow x = k\pi$

ou $u = 1$ e $v = 0 \Rightarrow \text{sen } 2x = 1$ e $\text{cos } 2x = 0 \Rightarrow x = \frac{\pi}{4} + k\pi$

321. a) $\text{sen } x = \frac{2t}{1+t^2}$ e $\text{cos } x = \frac{1-t^2}{1+t^2} \Rightarrow m \frac{(1-t^2)}{1+t^2} - (m+1) \frac{2t}{1+t^2} = m \Rightarrow$

$$\Rightarrow t(-2mt - 2m - 2) = 0 \Rightarrow t = 0 \text{ ou } -2mt - 2m - 2 = 0 \Rightarrow$$

$$\Rightarrow t = \frac{-(m+1)}{m}, m \neq 0$$

$m = 0 \Rightarrow 0 \cdot \text{cos } x - \text{sen } x = 0, \exists x, \forall m \in \mathbb{R}.$

b) $\text{sen } x = \frac{2t}{1+t^2}$ e $\text{cos } x = \frac{1-t^2}{1+t^2} \Rightarrow \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = m \Rightarrow$

$$\Rightarrow (-m-1)t^2 + 2t + 1 - m = 0 \Rightarrow \Delta = -4m^2 + 8 \geq 0 \Rightarrow$$

$$\Rightarrow -\sqrt{2} \leq m \leq \sqrt{2}$$

323. a) $2 \text{sen} \left(\frac{mx + nx}{2} \right) \cdot \text{cos} \left(\frac{mx - nx}{2} \right) = 0$. Então: $\text{sen} \frac{(m+n)x}{2} = 0 \Rightarrow$

$$\Rightarrow x = \frac{2k\pi}{m+n}$$

ou $\text{cos} \frac{(m-n)x}{2} = 0 \Rightarrow x = \frac{\pi}{m-n} + \frac{2k\pi}{m-n}$

b) $2 \text{cos} \frac{ax + bx}{2} \cdot \text{cos} \frac{ax - bx}{2} = 0$. Então: $\text{cos} \frac{(a+b)x}{2} = 0 \Rightarrow$

$$\Rightarrow x = \frac{\pi}{a+b} + \frac{2k\pi}{a+b}$$

ou $\text{cos} \frac{(a-b)x}{2} = 0 \Rightarrow x = \frac{\pi}{a-b} + \frac{2k\pi}{a-b}$

c) $\text{sen } 2x - \text{sen} \left(\frac{\pi}{4} - x \right) = 0 \Rightarrow 2 \text{sen} \left(\frac{3x}{2} - \frac{\pi}{8} \right) \cos \left(\frac{x}{2} + \frac{\pi}{8} \right) = 0$

Então:

$$\text{sen} \left(\frac{3x}{2} - \frac{\pi}{8} \right) = 0 \Rightarrow x = \frac{\pi}{12} + \frac{2k\pi}{3} \text{ ou } \cos \left(\frac{x}{2} + \frac{\pi}{8} \right) = 0 \Rightarrow$$

$$\Rightarrow x = \frac{3\pi}{4} + 2k\pi$$

325. a) $2 \text{sen } 3x \cdot \cos 2x - 2 \text{sen } 3x = 0 \Rightarrow 2 \text{sen } 3x (\cos 2x - 1) = 0.$

Então: $\text{sen } 3x = 0 \Rightarrow x = \frac{k\pi}{3}$ ou $\cos 2x = 1 \Rightarrow x = k\pi.$

b) $2 \cos (2x + a) \cos (x + a) + \cos (2x + a) = 0 \Rightarrow$

$\Rightarrow \cos (2x + a)[2 \cos (x + a) + 1] = 0.$ Então: $\cos (2x + a) = 0 \Rightarrow$

$\Rightarrow x = \frac{\pi}{4} - \frac{a}{2} + \frac{k\pi}{2}$

ou $2 \cos (x + a) + 1 = 0 \Rightarrow \cos (x + a) = -\frac{1}{2} \Rightarrow x = \frac{2\pi}{3} - a + 2k\pi$

ou $x = \frac{4\pi}{3} - a + 2k\pi$

c) $2 \text{sen } 4x \cdot \cos 3x - 2 \text{sen } 4x \cdot \text{sen} (-x) = 0 \Rightarrow$

$\Rightarrow 2 \text{sen } 4x (\cos 3x + \text{sen } x) = 0 \Rightarrow$

$\Rightarrow 2 \text{sen } 4x \left[\cos 3x + \cos \left(\frac{\pi}{2} - x \right) \right] = 0 \Rightarrow$

$\Rightarrow 4 \text{sen } 4x \cdot \cos \left(x + \frac{\pi}{4} \right) \cdot \cos \left(2x - \frac{\pi}{4} \right) = 0$

Então:

$\text{sen } 4x = 0 \Rightarrow x = \frac{k\pi}{4}$ ou $\cos \left(x + \frac{\pi}{4} \right) = 0 \Rightarrow$

$\Rightarrow x = \frac{\pi}{4} + k\pi$ ou $\cos \left(2x - \frac{\pi}{4} \right) = 0 \Rightarrow x = -\frac{3\pi}{8} + \frac{k\pi}{2}$

326. $\frac{1 + \cos (2x + 2a)}{2} + \frac{1 + \cos (2x - 2a)}{2} = 1 \Rightarrow$

$\Rightarrow \cos (2x + 2a) + \cos (2x - 2a) = 0 \Rightarrow 2 \cos (2x) \cdot \cos (2a) = 0 \Rightarrow$

$\Rightarrow \cos (2x) = 0 \Rightarrow x = \frac{\pi}{4} + k\pi$ ou $x = \frac{3\pi}{4} + k\pi$ (supondo $\cos 2a \neq 0$)

327. $(\text{sen } 3x - \text{sen } x) + (\cos 2x - \cos 0) = 0 \Rightarrow$

$\Rightarrow 2 \cdot \text{sen } x \cdot \cos 2x - 2 \cdot \text{sen}^2 x = 0 \Rightarrow 2 \cdot \text{sen } x \cdot (\cos 2x - \text{sen } x) = 0$

Então:

$\text{sen } x = 0 \Rightarrow x = k\pi$ ou

$\cos 2x = \text{sen } x \Rightarrow \cos 2x = \cos \left(\frac{\pi}{2} - x \right) \Rightarrow 2x = \pm \left(\frac{\pi}{2} - x \right) + 2k\pi \Rightarrow$

$\Rightarrow x = \frac{\pi}{6} + \frac{2k\pi}{3}$ ou $x = -\frac{\pi}{2} + 2k\pi$

328. $2 \cdot \operatorname{sen} x \cdot \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} \Rightarrow \operatorname{sen} x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} + 2k\pi$ ou $x = \frac{5\pi}{6} + 2k\pi$

329. a) Substituindo $x = 2\pi$ e $y = \frac{\pi}{2}$ na equação (1):

$$\operatorname{sen} \left(2\pi + \frac{\pi}{2} \right) + \operatorname{sen} \left(2\pi - \frac{\pi}{2} \right) = 1 - 1 \neq 2.$$

b) $\left. \begin{array}{l} 2 \cdot \operatorname{sen} x \cdot \cos y = 2 \\ \operatorname{sen} x + \cos y = 2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} \operatorname{sen} x \cdot \cos y = 1 \text{ (A)} \\ \operatorname{sen} x + \cos y = 2 \text{ (B)} \end{array} \right\}, \text{ (A) em (B)} \Rightarrow$

$$\Rightarrow \operatorname{sen}^2 x - 2 \operatorname{sen} x + 1 = 0$$

Então:

$$\operatorname{sen} x = 1 \text{ (C)} \Rightarrow x = \frac{\pi}{2} + 2k\pi, \text{ (C) em (A)} \Rightarrow \cos y = 1 \Rightarrow y = 2k\pi$$

330. $(\cos x + 1)(\operatorname{sen} x + 1) = 0$. Então: $\cos x + 1 = 0 \Rightarrow x = \pi + 2k\pi$ ou

$$\operatorname{sen} x + 1 = 0 \Rightarrow x = \frac{3\pi}{2} + 2k\pi$$

333. a) $(\cos^2 x + \operatorname{sen}^2 x)^2 - 2 \cdot \operatorname{sen}^2 x \cdot \cos^2 x = \frac{5}{8} \Rightarrow$

$$\Rightarrow 1 - 2 \cdot \operatorname{sen}^2 x \cdot \cos^2 x = \frac{5}{8} \Rightarrow 1 - \frac{1}{2} \operatorname{sen}^2 2x = \frac{5}{8} \Rightarrow$$

$$\Rightarrow \operatorname{sen}^2 2x = \frac{3}{4}$$

$$\text{Então: } \operatorname{sen} 2x = \frac{+\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{6} + k\pi \text{ ou } x = \frac{\pi}{3} + k\pi;$$

$$\operatorname{sen} 2x = \frac{-\sqrt{3}}{2} \Rightarrow x = \frac{2\pi}{3} + k\pi \text{ ou } x = \frac{5\pi}{6} + k\pi$$

b) $(\cos^2 x + \operatorname{sen}^2 x)(\cos^4 x - \cos^2 x \cdot \operatorname{sen}^2 x + \operatorname{sen}^4 x) = \frac{5}{8} \Rightarrow$

$$\Rightarrow (\operatorname{sen}^4 x + \cos^4 x) - \operatorname{sen}^2 x \cdot \cos^2 x = \frac{5}{8} \Rightarrow$$

$$\Rightarrow \left(1 - \frac{1}{2} \cdot \operatorname{sen}^2 2x \right) - \frac{1}{4} \cdot \operatorname{sen}^2 2x = \frac{5}{8} \Rightarrow 1 - \frac{3}{4} \cdot \operatorname{sen}^2 2x = \frac{5}{8}$$

$$\text{Então: } \operatorname{sen} 2x = \frac{\sqrt{2}}{2} \Rightarrow x = \frac{\pi}{8} + k\pi \text{ ou } x = \frac{3\pi}{8} + k\pi \text{ ou}$$

$$\operatorname{sen} 2x = \frac{-\sqrt{2}}{2} \Rightarrow x = \frac{5\pi}{8} + k\pi \text{ ou } x = \frac{7\pi}{8} + k\pi$$

c) $1 - \frac{1}{2} \cdot \operatorname{sen}^2 2x = \frac{1}{2}$. Então: $\operatorname{sen} 2x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi$

$$\text{ou } \operatorname{sen} 2x = -1 \Rightarrow x = \frac{3\pi}{4} + k\pi$$

d) $1 - \frac{3}{4} \cdot \operatorname{sen}^2 x = \frac{7}{16}$. Então: $\operatorname{sen} x = \pm \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3} + k\pi$ ou

$$x = \frac{2\pi}{3} + k\pi$$

e) $(\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cdot \cos x) = 1 \Rightarrow$
 $\Rightarrow (\sin x + \cos x)(1 - \sin x \cdot \cos x) = 1$. Fazendo $\sin x + \cos x = y$,
 temos $(\sin x + \cos x)^2 = y^2$ e daí: $\sin x \cdot \cos x = \frac{y^2 - 1}{2}$.

A equação fica $y \cdot \left(1 - \frac{y^2 - 1}{2}\right) = 1 \Rightarrow (y - 1)^2(y + 2) = 0 \Rightarrow$

$\Rightarrow y = 1$ ou $y = -2$ não serve, pois $-\sqrt{2} \leq y \leq \sqrt{2}$; então
 $\sin x + \cos x = 1 \Rightarrow$

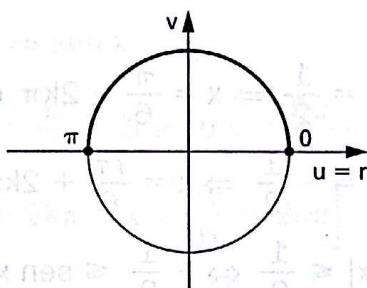
$\Rightarrow \sqrt{2} \cdot \cos\left(x - \frac{\pi}{4}\right) = 1 \Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \Rightarrow$

$\Rightarrow x = \frac{\pi}{2} + 2k\pi$ ou $x = 2k\pi$

CAPÍTULO XII — Inequações

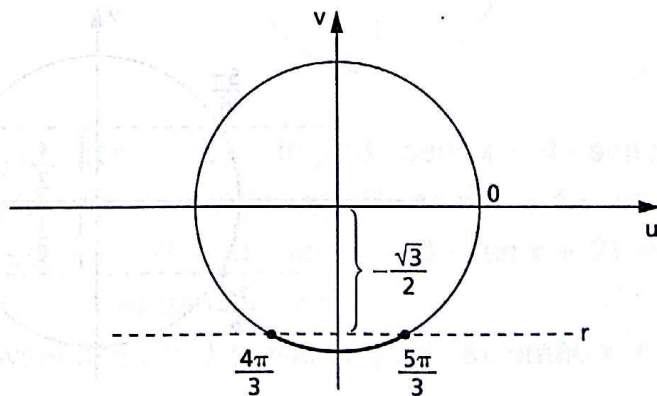
335. $\sin x = 0 \Rightarrow x = 2k\pi$ ou $x = \pi + 2k\pi$

$S = \{x \in \mathbb{R} \mid 2k\pi \leq x \leq \pi + 2k\pi\}$



336. $\sin x = -\frac{\sqrt{3}}{2} \Rightarrow x = \frac{4\pi}{3} + 2k\pi$ ou $x = \frac{5\pi}{3} + 2k\pi$

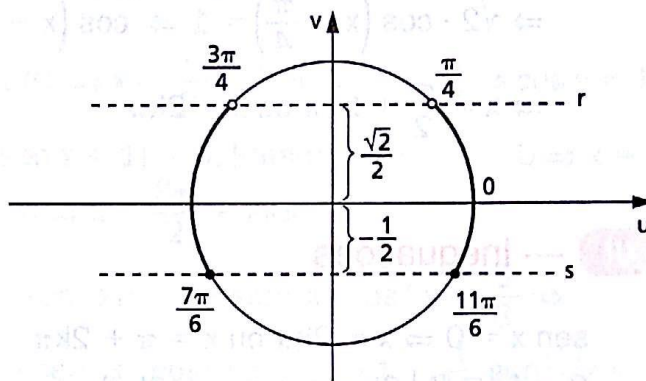
$S = \left\{x \in \mathbb{R} \mid \frac{4\pi}{3} + 2k\pi \leq x \leq \frac{5\pi}{3} + 2k\pi\right\}$



337. $\sin x = \frac{\sqrt{2}}{2} \Rightarrow x = \frac{\pi}{4} + 2k\pi$ ou $x = \frac{3\pi}{4} + 2k\pi$

$\sin x = -\frac{1}{2} \Rightarrow x = \frac{7\pi}{6} + 2k\pi$ ou $x = \frac{11\pi}{6} + 2k\pi$

$S = \left\{ x \in \mathbb{R} \mid 2k\pi \leq x < \frac{\pi}{4} + 2k\pi \text{ ou } \frac{3\pi}{4} + 2k\pi < x \leq \frac{7\pi}{6} + 2k\pi \right.$
 $\left. \text{ou } \frac{11\pi}{6} + 2k\pi \leq x < 2\pi + 2k\pi \right\}$

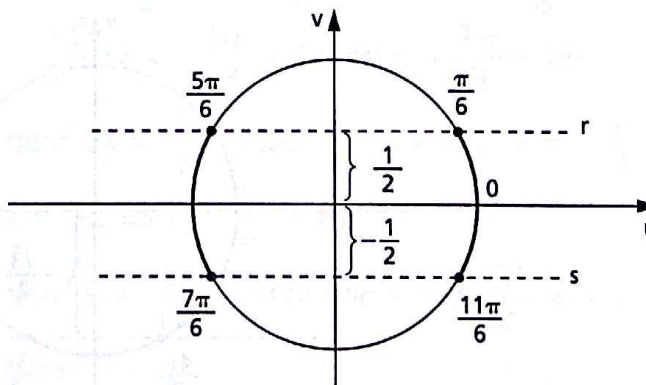


339. $\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} + 2k\pi$ ou $x = \frac{5\pi}{6} + 2k\pi$

$\sin x = -\frac{1}{2} \Rightarrow x = \frac{7\pi}{6} + 2k\pi$ ou $x = \frac{11\pi}{6} + 2k\pi$

$|\sin x| \leq \frac{1}{2} \Leftrightarrow -\frac{1}{2} \leq \sin x \leq \frac{1}{2}$

$S = \left\{ x \in \mathbb{R} \mid 2k\pi \leq x \leq \frac{\pi}{6} + 2k\pi \text{ ou } \frac{5\pi}{6} + 2k\pi \leq x \leq \frac{7\pi}{6} + 2k\pi \right.$
 $\left. \text{ou } \frac{11\pi}{6} + 2k\pi \leq x < 2\pi + 2k\pi \right\}$

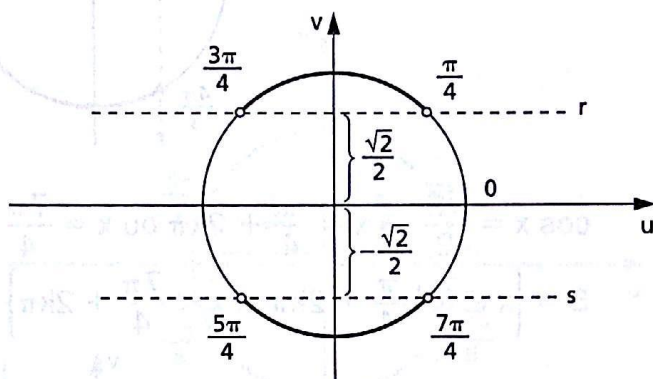


340. $|\sin x| > \frac{\sqrt{2}}{2} \Leftrightarrow \sin x > \frac{\sqrt{2}}{2} \text{ ou } \sin x < -\frac{\sqrt{2}}{2}$

$\sin x = \frac{\sqrt{2}}{2} \Rightarrow x = \frac{\pi}{4} + 2k\pi \text{ ou } x = \frac{3\pi}{4} + 2k\pi$

$\sin x = -\frac{\sqrt{2}}{2} \Rightarrow x = \frac{5\pi}{4} + 2k\pi \text{ ou } x = \frac{7\pi}{4} + 2k\pi$

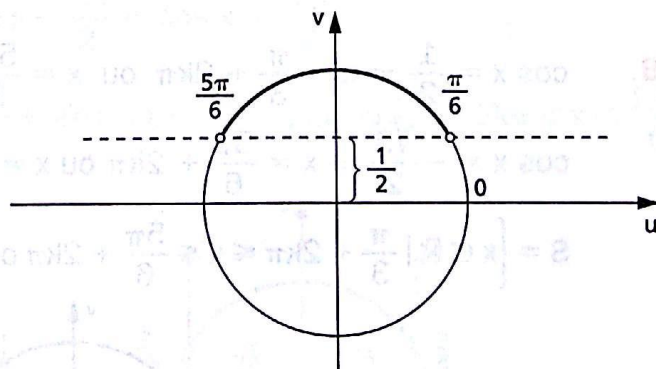
$S = \left\{ x \in \mathbb{R} \mid \frac{\pi}{4} + 2k\pi < x < \frac{3\pi}{4} + 2k\pi \text{ ou } \frac{5\pi}{4} + 2k\pi < x < \frac{7\pi}{4} + 2k\pi \right\}$



342. a) $2 \sin x - 1 > 0 \Rightarrow \sin x > \frac{1}{2}$

$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} + 2k\pi \text{ ou } x = \frac{5\pi}{6} + 2k\pi$

$S = \left\{ x \in \mathbb{R} \mid \frac{\pi}{6} + 2k\pi < x < \frac{5\pi}{6} + 2k\pi \right\}$

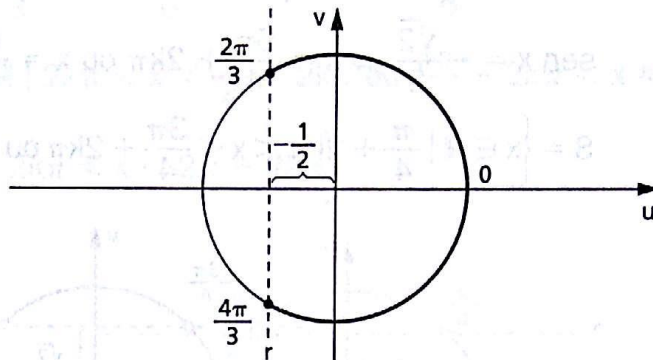


b) $2 \cdot \log_2 (2 \cdot \sin x - 1) = \log_2 (3 \cdot \sin^2 x - 4 \cdot \sin x + 2) \Rightarrow$
 $\Rightarrow \log_2 (2 \cdot \sin x - 1)^2 = \log_2 (3 \cdot \sin^2 x - 4 \cdot \sin x + 2) \Rightarrow$
 $\Rightarrow (2 \cdot \sin x - 1)^2 = (3 \cdot \sin^2 x - 4 \cdot \sin x + 2) \Rightarrow$
 $\Rightarrow \sin^2 x = 1 \Rightarrow \sin x = \pm 1$

Só convém $\sin x = 1$ (devido à parte a), então $x = \frac{\pi}{2} + 2k\pi$.

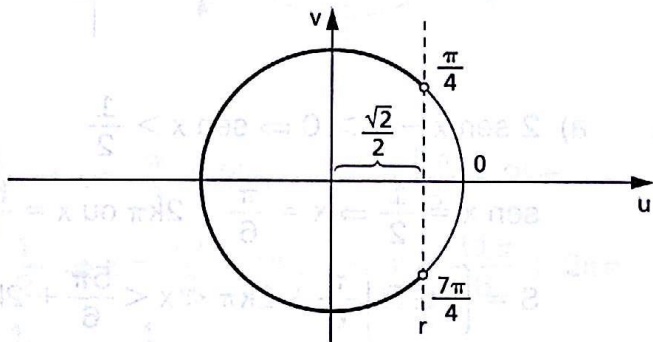
344. $\cos x = -\frac{1}{2} \Rightarrow x = \frac{2\pi}{3} + 2k\pi$ ou $x = \frac{4\pi}{3} + 2k\pi$

$$S = \left\{ x \in \mathbb{R} \mid 2k\pi \leq x \leq \frac{2\pi}{3} + 2k\pi \text{ ou } \frac{4\pi}{3} + 2k\pi \leq x < 2\pi + 2k\pi \right\}$$



345. $\cos x = \frac{\sqrt{2}}{2} \Rightarrow x = \frac{\pi}{4} + 2k\pi$ ou $x = \frac{7\pi}{4} + 2k\pi$

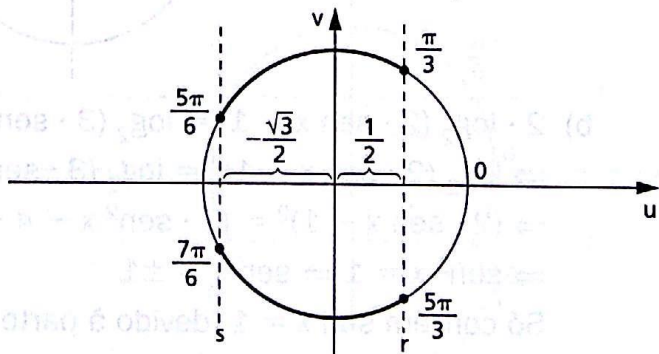
$$S = \left\{ x \in \mathbb{R} \mid \frac{\pi}{4} + 2k\pi < x < \frac{7\pi}{4} + 2k\pi \right\}$$



346. $\cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3} + 2k\pi$ ou $x = \frac{5\pi}{3} + 2k\pi$

$$\cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = \frac{5\pi}{6} + 2k\pi \text{ ou } x = \frac{7\pi}{6} + 2k\pi$$

$$S = \left\{ x \in \mathbb{R} \mid \frac{\pi}{3} + 2k\pi \leq x \leq \frac{5\pi}{6} + 2k\pi \text{ ou } \frac{7\pi}{6} + 2k\pi \leq x \leq \frac{5\pi}{3} + 2k\pi \right\}$$

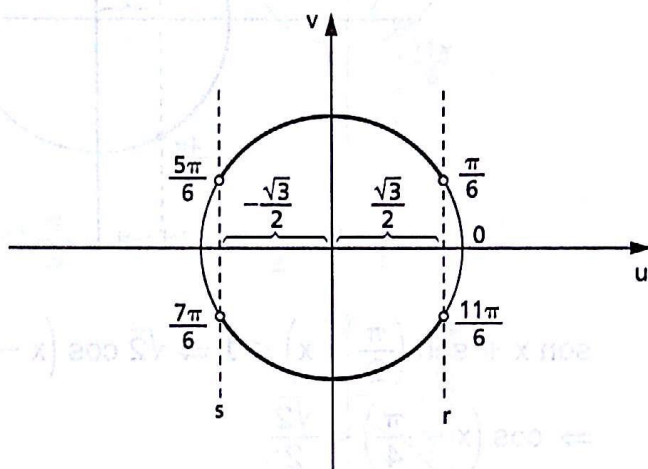


$$347. \quad |\cos x| < \frac{\sqrt{3}}{2} \Rightarrow -\frac{\sqrt{3}}{2} < \cos x < \frac{\sqrt{3}}{2}$$

$$\cos x = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{6} + 2k\pi \text{ ou } x = \frac{11\pi}{6} + 2k\pi$$

$$\cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = \frac{5\pi}{6} + 2k\pi \text{ ou } x = \frac{7\pi}{6} + 2k\pi$$

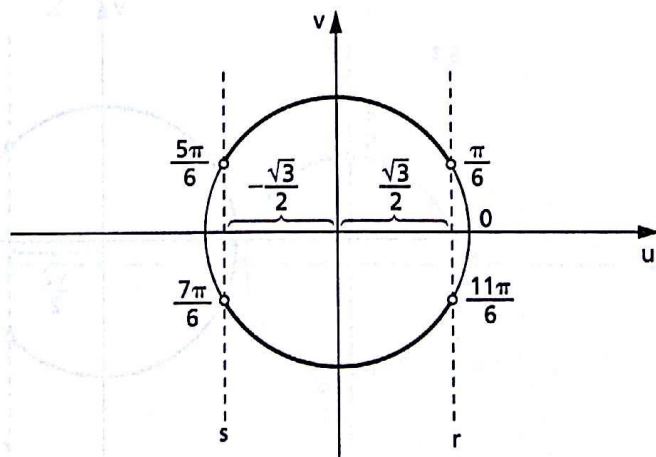
$$S = \left\{ x \in \mathbb{R} \mid \frac{\pi}{6} + 2k\pi < x < \frac{5\pi}{6} + 2k\pi \text{ ou } \frac{7\pi}{6} + 2k\pi < x < \frac{11\pi}{6} + 2k\pi \right\}$$



$$348. \quad |\cos x| > \frac{5}{3}; \text{ impossível, pois } -1 \leq \cos x \leq 1; S = \emptyset$$

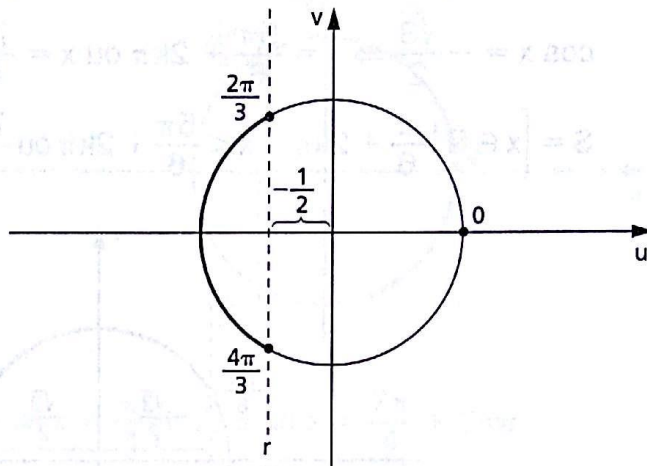
$$350. \quad \cos^2 x < \frac{3}{4} \Leftrightarrow -\frac{\sqrt{3}}{2} < \cos x < \frac{\sqrt{3}}{2}$$

$$S = \left\{ x \in \mathbb{R} \mid \frac{\pi}{6} + 2k\pi < x < \frac{5\pi}{6} + 2k\pi \text{ ou } \frac{7\pi}{6} + 2k\pi < x < \frac{11\pi}{6} + 2k\pi \right\}$$



351. $2 \cos^2 x - 1 - \cos x \geq 0 \Leftrightarrow \cos x \leq -\frac{1}{2}$ ou $\cos x = 1$

$$S = \left\{ x \in \mathbb{R} \mid \frac{2\pi}{3} + 2k\pi \leq x \leq \frac{4\pi}{3} + 2k\pi \text{ ou } x = 2k\pi \right\}$$



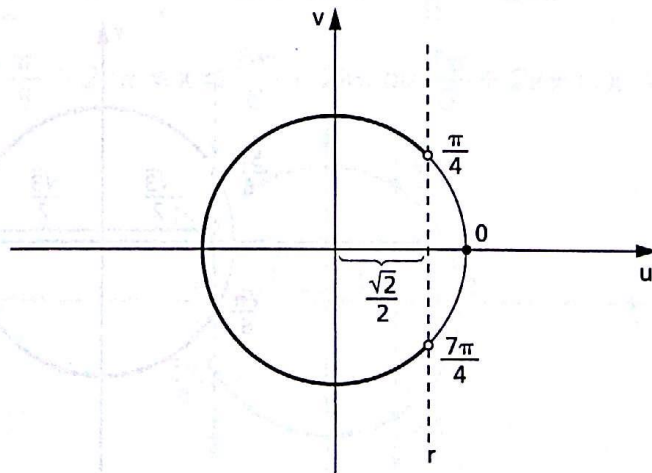
353. $\sin x + \sin \left(\frac{\pi}{2} - x \right) < 1 \Rightarrow \sqrt{2} \cos \left(x - \frac{\pi}{4} \right) < 1 \Rightarrow$
 $\Rightarrow \cos \left(x - \frac{\pi}{4} \right) < \frac{\sqrt{2}}{2}$

Fazendo $x - \frac{\pi}{4} = y$, temos:

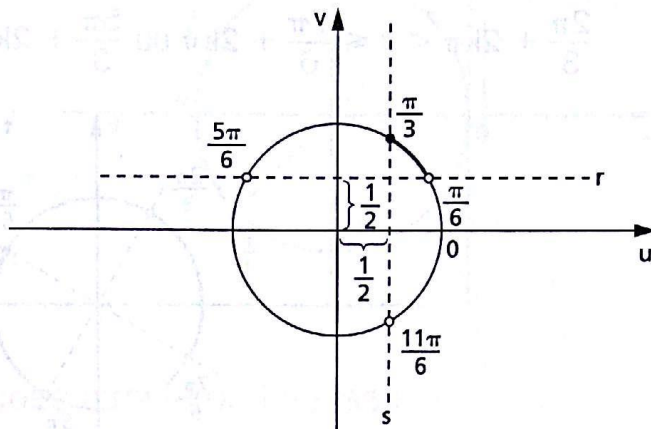
$$\frac{\pi}{4} + 2k\pi < y < \frac{7\pi}{4} + 2k\pi;$$

então $\frac{\pi}{4} + 2k\pi < x - \frac{\pi}{4} < \frac{7\pi}{4} + 2k\pi$

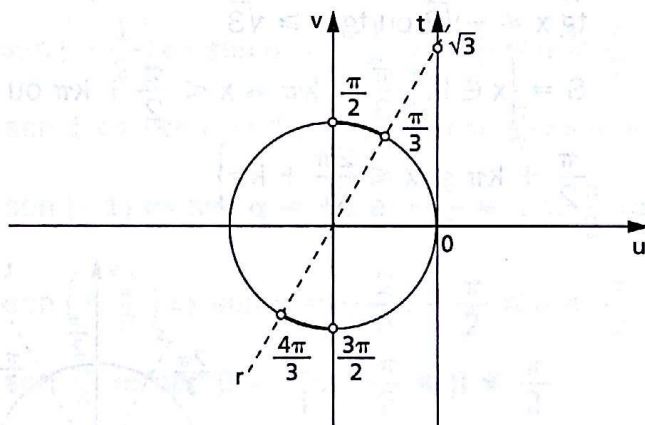
$$S = \left\{ x \in \mathbb{R} \mid \frac{\pi}{2} + 2k\pi < x < 2\pi + 2k\pi \right\}$$



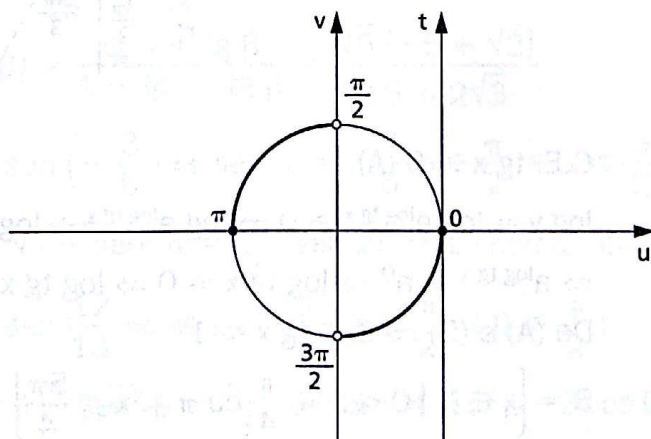
355. $S = \left\{ x \in \mathbb{R} \mid \frac{\pi}{6} + 2k\pi < x \leq \frac{\pi}{3} + 2k\pi \right\}$



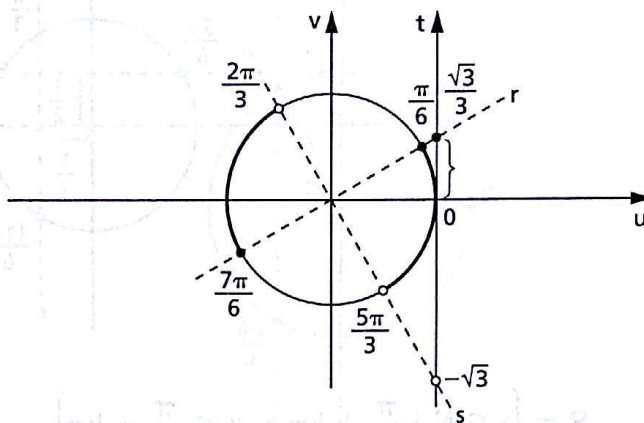
357. $S = \left\{ x \in \mathbb{R} \mid \frac{\pi}{3} + k\pi < x < \frac{\pi}{2} + k\pi \right\}$



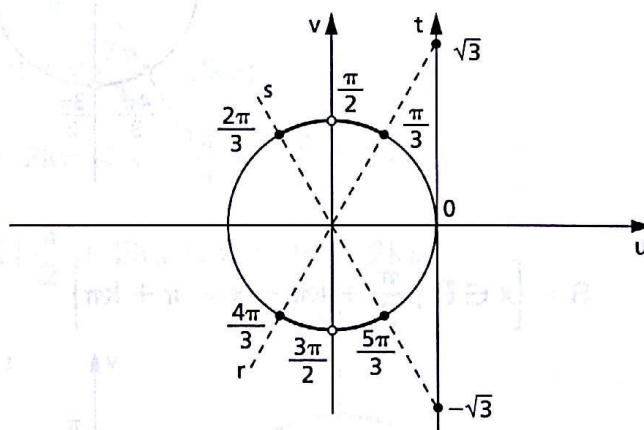
358. $S = \left\{ x \in \mathbb{R} \mid \frac{\pi}{2} + k\pi < x \leq \pi + k\pi \right\}$



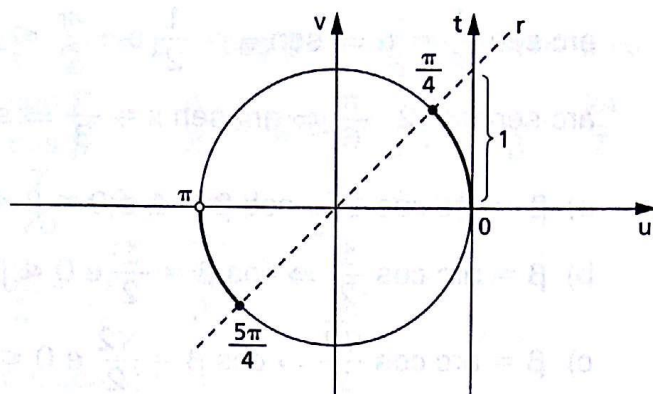
359. $S = \left\{ x \in \mathbb{R} \mid 2k\pi \leq x \leq \frac{\pi}{6} + 2k\pi \text{ ou } \frac{2\pi}{3} + 2k\pi < x \leq \frac{7\pi}{6} + 2k\pi \text{ ou } \frac{5\pi}{3} + 2k\pi < x \leq 2\pi + 2k\pi \right\}$



360. $\text{tg } x \leq -\sqrt{3} \text{ ou } \text{tg } x \geq \sqrt{3}$
 $S = \left\{ x \in \mathbb{R} \mid \frac{\pi}{3} + k\pi \leq x < \frac{\pi}{2} + k\pi \text{ ou } \frac{\pi}{2} + k\pi < x \leq \frac{2\pi}{3} + k\pi \right\}$



361. C.E. $\text{tg } x > 0$ (A)
 $\log y = \log a^{\log \text{tg } x} \geq 0 \Rightarrow \log a^{\log \text{tg } x} \geq \log 1 \Rightarrow \log a^{\log \text{tg } x} \geq \log a^0 \Rightarrow$
 $\Rightarrow a^{\log \text{tg } x} \geq a^0 \Rightarrow \log \text{tg } x \leq 0 \Rightarrow \log \text{tg } x \leq \log 1 \Rightarrow \text{tg } x \leq 1$ (B)
 De (A) e (B) $\Rightarrow 0 < \text{tg } x \leq 1$
 $S = \left\{ x \in \mathbb{R} \mid 0 < x \leq \frac{\pi}{4} \text{ ou } \pi < x \leq \frac{5\pi}{4} \right\}$


CAPÍTULO XIII — Funções circulares inversas

- 363.**
- $\alpha = \text{arc sen } 0 \Leftrightarrow \text{sen } \alpha = 0 \text{ e } -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2} \Rightarrow \alpha = 0$
 - $\alpha = \text{arc sen } \frac{\sqrt{3}}{2} \Leftrightarrow \text{sen } \alpha = \frac{\sqrt{3}}{2} \text{ e } -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{3}$
 - $\alpha = \text{arc sen } \left(-\frac{1}{2}\right) \Leftrightarrow \text{sen } \alpha = -\frac{1}{2} \text{ e } -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2} \Rightarrow \alpha = -\frac{\pi}{6}$
 - $\alpha = \text{arc sen } 1 \Leftrightarrow \text{sen } \alpha = 1 \text{ e } -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{2}$
 - $\alpha = \text{arc sen } (-1) \Leftrightarrow \text{sen } \alpha = -1 \text{ e } -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2} \Rightarrow \alpha = -\frac{\pi}{2}$

- 367.**
- $\alpha = \text{arc sen } \left(-\frac{2}{3}\right) \Rightarrow \text{sen } \alpha = -\frac{2}{3}, -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$
 $\beta = \text{arc sen } \frac{1}{4} \Rightarrow \text{sen } \beta = \frac{1}{4}, -\frac{\pi}{2} \leq \beta \leq \frac{\pi}{2}$
 $\text{sen}^2 \alpha = \frac{\text{tg}^2 \alpha}{1 + \text{tg}^2 \alpha} \Rightarrow \text{tg } \alpha = \frac{-2\sqrt{5}}{5}; \text{sen}^2 \beta = \frac{\text{tg}^2 \beta}{1 + \text{tg}^2 \beta} \Rightarrow$

$$\Rightarrow \text{tg } \beta = \frac{\sqrt{15}}{15}$$

$$\text{tg } (\alpha + \beta) = \frac{\text{tg } \alpha + \text{tg } \beta}{1 - \text{tg } \alpha \cdot \text{tg } \beta} = \frac{\sqrt{5}(-6 + \sqrt{3})}{15 + 2\sqrt{3}}$$

- $\alpha = \text{arc sen } \left(-\frac{3}{5}\right) \Rightarrow \text{sen } \alpha = -\frac{3}{5} \text{ e } -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$

$$\cos \alpha = \sqrt{1 - \text{sen}^2 \alpha} = \frac{4}{5}; \text{sen } 2\alpha = 2 \cdot \text{sen } \alpha \cdot \cos \alpha = -\frac{24}{25}$$

- $\beta = \text{arc sen } \frac{12}{13} \Rightarrow \text{sen } \beta = \frac{12}{13} \text{ e } -\frac{\pi}{2} \leq \beta \leq \frac{\pi}{2};$

$$\cos \beta = \sqrt{1 - \text{sen}^2 \beta} = \frac{5}{13}; \cos 3\beta = 4 \cos^3 \beta - 3 \cos \beta = -\frac{2035}{2197}$$

368. $\text{arc sen } \frac{1}{2} = \alpha \Rightarrow \text{sen } \alpha = \frac{1}{2} \text{ e } -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{6}$
 $\text{arc sen } x = 2 \cdot \frac{\pi}{6} \Rightarrow \text{arc sen } x = \frac{\pi}{3} \Rightarrow \text{sen } \frac{\pi}{3} = x \Rightarrow x = \frac{\sqrt{3}}{2}$

370. a) $\beta = \text{arc cos } 1 \Rightarrow \text{cos } \beta = 1 \text{ e } 0 \leq \beta \leq \pi \Rightarrow \beta = 0$
 b) $\beta = \text{arc cos } \frac{1}{2} \Rightarrow \text{cos } \beta = \frac{1}{2} \text{ e } 0 \leq \beta \leq \pi \Rightarrow \beta = \frac{\pi}{3}$
 c) $\beta = \text{arc cos } \frac{\sqrt{2}}{2} \Rightarrow \text{cos } \beta = \frac{\sqrt{2}}{2} \text{ e } 0 \leq \beta \leq \pi \Rightarrow \beta = \frac{\pi}{4}$
 d) $\beta = \text{arc cos } 0 \Rightarrow \text{cos } \beta = 0 \text{ e } 0 \leq \beta \leq \pi \Rightarrow \beta = \frac{\pi}{2}$
 e) $\beta = \text{arc cos } (-1) \Rightarrow \text{cos } \beta = -1 \text{ e } 0 \leq \beta \leq \pi \Rightarrow \beta = \pi$

372. $\beta = \text{arc cos } \left(-\frac{3}{5}\right) \Rightarrow \text{cos } \beta = -\frac{3}{5} \text{ e } 0 \leq \beta \leq \pi$
 $\text{sen } \beta = \sqrt{1 - \text{cos}^2 \beta} \Rightarrow \text{sen } \beta = \frac{4}{5}$

373. $\beta = \text{arc cos } \frac{2}{7} \Rightarrow \text{cos } \beta = \frac{2}{7} \text{ e } 0 \leq \beta \leq \pi; \text{sen } \beta = \sqrt{1 - \text{cos}^2 \beta} =$
 $= \frac{3\sqrt{5}}{7}$
 $\text{cotg } \beta = \frac{\text{cos } \beta}{\text{sen } \beta} = \frac{2\sqrt{5}}{15}$

374. $\left. \begin{array}{l} \text{arc sen } x = A \Rightarrow \text{sen } A = x \\ \text{arc cos } x = B \Rightarrow \text{cos } B = x \end{array} \right\} \Rightarrow \text{sen } A = \text{cos } B \Rightarrow B = \frac{\pi}{2} - A = \text{arc cos } x$

376. a) $\text{arc cos } \frac{3}{5} = \beta \Rightarrow \text{cos } \beta = \frac{3}{5} \text{ e } 0 \leq \beta \leq \pi \Rightarrow$
 $\Rightarrow \text{sen } \beta = \sqrt{1 - \text{cos}^2 \beta} = \frac{4}{5}$
 $\text{arc cos } \frac{5}{13} = \alpha \Rightarrow \text{cos } \alpha = \frac{5}{13} \text{ e } 0 \leq \alpha \leq \pi \Rightarrow$
 $\Rightarrow \text{sen } \alpha = \sqrt{1 - \text{cos}^2 \alpha} = \frac{12}{13}$
 $\text{sen } (\beta - \alpha) = \text{sen } \beta \cdot \text{cos } \alpha - \text{sen } \alpha \cdot \text{cos } \beta = -\frac{16}{65}$
 b) $\text{arc sen } \frac{7}{25} = \beta \Rightarrow \text{sen } \beta = \frac{7}{25} \text{ e } -\frac{\pi}{2} \leq \beta \leq \frac{\pi}{2} \Rightarrow \text{cos } \beta = \frac{24}{25}$
 $\text{arc cos } \frac{12}{13} = \alpha \Rightarrow \text{cos } \alpha = \frac{12}{13} \text{ e } 0 \leq \alpha \leq \pi \Rightarrow \text{sen } \alpha = \frac{5}{13}$
 $\text{cos } (\beta - \alpha) = \text{cos } \beta \cdot \text{cos } \alpha + \text{sen } \beta \cdot \text{sen } \alpha = \frac{323}{325}$

c) $\arccos\left(-\frac{3}{5}\right) = \beta \Rightarrow \cos \beta = -\frac{3}{5}$ e $0 \leq \beta \leq \pi \Rightarrow \operatorname{sen} \beta = \frac{4}{5}$ e

$$\operatorname{tg} \beta = \frac{\operatorname{sen} \beta}{\cos \beta} = -\frac{4}{3} \Rightarrow \operatorname{tg} 2\beta = \frac{2 \operatorname{tg} \beta}{1 - \operatorname{tg}^2 \beta} = \frac{24}{7}$$

d) $\arccos \frac{7}{25} = \beta \Rightarrow \cos \beta = \frac{7}{25}$ e $0 \leq \beta \leq \pi \Rightarrow$

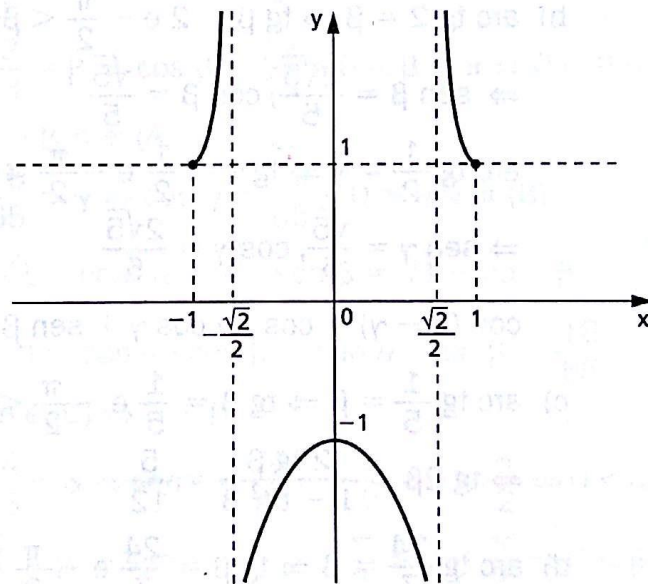
$$\Rightarrow \cos \frac{\beta}{2} = \sqrt{\frac{1 + \cos \beta}{2}} = \frac{4}{5}$$

377. a) $\cos(2 \arccos x) = 0$

$$\arccos x = \beta \Rightarrow \cos \beta = x \text{ e } 0 \leq \beta \leq \pi$$

$$\cos 2\beta = 0 \Rightarrow 2 \cos^2 \beta - 1 = 0 \Rightarrow 2x^2 - 1 = 0 \Rightarrow x = \pm \frac{\sqrt{2}}{2}$$

b) $g(x) = \frac{1}{f(x)} = \frac{1}{\cos 2\beta} = \frac{1}{2x^2 - 1}$, $-1 \leq x \leq 1$ e $x \neq \pm \frac{\sqrt{2}}{2}$ e
 $x \neq -\frac{\sqrt{2}}{2}$



378. $\beta = \arccos \sqrt{2} \Rightarrow \cos \beta = \sqrt{2}$, $\nexists \beta$ pois $-1 \leq \cos \beta \leq 1$, $\forall \beta \in \mathbb{R}$

380. $\arccos 0 = \alpha \Rightarrow \operatorname{tg} \alpha = 0$ e $-\frac{\pi}{2} < \alpha < \frac{\pi}{2} \Rightarrow \alpha = 0$

$$\arccos \sqrt{3} = \beta \Rightarrow \operatorname{tg} \beta = \sqrt{3} \text{ e } -\frac{\pi}{2} < \beta < \frac{\pi}{2} \Rightarrow \beta = \frac{\pi}{3}$$

$$\arccos(-1) = \alpha \Rightarrow \operatorname{tg} \alpha = -1 \text{ e } -\frac{\pi}{2} < \alpha < \frac{\pi}{2} \Rightarrow \alpha = -\frac{\pi}{4}$$

$$\arccos\left(-\frac{\sqrt{3}}{3}\right) = \beta \Rightarrow \operatorname{tg} \beta = -\frac{\sqrt{3}}{3} \text{ e } -\frac{\pi}{2} < \beta < \frac{\pi}{2} \Rightarrow \beta = -\frac{\pi}{6}$$

382. $\arctan\left(-\frac{4}{3}\right) = \beta \Rightarrow \tan \beta = -\frac{4}{3} \text{ e } -\frac{\pi}{2} < \beta < \frac{\pi}{2}$

$$\cos \beta = \sqrt{\frac{1}{1 + \tan^2 \beta}} = \frac{3}{5}$$

384. a) $\arctan 2 = \alpha \Rightarrow \tan \alpha = 2 \text{ e } -\frac{\pi}{2} < \alpha < \frac{\pi}{2}$

$\arctan 3 = \beta \Rightarrow \tan \beta = 3 \text{ e } -\frac{\pi}{2} < \beta < \frac{\pi}{2}$

$$\cos \alpha = \sqrt{\frac{1}{1 + \tan^2 \alpha}} = \frac{\sqrt{5}}{5}; \cos \beta = \sqrt{\frac{1}{1 + \tan^2 \beta}} = \frac{\sqrt{10}}{10}$$

$$\sin \alpha = \sqrt{\frac{\tan^2 \alpha}{1 + \tan^2 \alpha}} = \frac{2\sqrt{5}}{5}; \sin \beta = \sqrt{\frac{\tan^2 \beta}{1 + \tan^2 \beta}} = \frac{3\sqrt{10}}{10}$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \sin \beta \cdot \cos \alpha = \frac{\sqrt{2}}{2}$$

b) $\arctan 2 = \beta \Rightarrow \tan \beta = 2 \text{ e } -\frac{\pi}{2} < \beta < \frac{\pi}{2} \Rightarrow$

$$\Rightarrow \sin \beta = \frac{2\sqrt{5}}{5}, \cos \beta = \frac{\sqrt{5}}{5}$$

$\arctan \frac{1}{2} = \gamma \Rightarrow \tan \gamma = \frac{1}{2} \text{ e } -\frac{\pi}{2} < \gamma < \frac{\pi}{2} \Rightarrow$

$$\Rightarrow \sin \gamma = \frac{\sqrt{5}}{5}, \cos \gamma = \frac{2\sqrt{5}}{5}$$

$$\cos(\beta - \gamma) = \cos \beta \cdot \cos \gamma + \sin \beta \cdot \sin \gamma = \frac{4}{5}$$

c) $\arctan \frac{1}{5} = \beta \Rightarrow \tan \beta = \frac{1}{5} \text{ e } -\frac{\pi}{2} < \beta < \frac{\pi}{2} \Rightarrow$

$$\Rightarrow \tan 2\beta = \frac{2 \tan \beta}{1 - \tan^2 \beta} = \frac{5}{12}$$

d) $\arctan \frac{24}{7} = \beta \Rightarrow \tan \beta = \frac{24}{7} \text{ e } -\frac{\pi}{2} < \beta < \frac{\pi}{2} \Rightarrow$

$$\Rightarrow \cos \beta = \sqrt{\frac{1}{1 + \tan^2 \beta}} = \frac{7}{25}$$

$$\cos 3\beta = 4 \cdot \cos^3 \beta - 3 \cdot \cos \beta = -\frac{11753}{15625}$$

386. a) $\left. \begin{aligned} \arctan \frac{1}{2} = \beta &\Rightarrow \tan \beta = \frac{1}{2} \text{ e } -\frac{\pi}{2} < \beta < \frac{\pi}{2} \Rightarrow 0 < \beta < \frac{\pi}{2} \\ \arctan \frac{1}{3} = \alpha &\Rightarrow \tan \alpha = \frac{1}{3} \text{ e } -\frac{\pi}{2} < \alpha < \frac{\pi}{2} \Rightarrow 0 < \alpha < \frac{\pi}{2} \end{aligned} \right\} \Rightarrow$

$$\Rightarrow 0 < \beta + \alpha < \pi \text{ (A)}; \gamma = \frac{\pi}{4} \text{ (B)}$$

$$\operatorname{tg}(\beta + \alpha) = \frac{\operatorname{tg} \beta + \operatorname{tg} \alpha}{1 - \operatorname{tg} \beta \cdot \operatorname{tg} \alpha} = 1 = \operatorname{tg} \gamma \text{ (C)}$$

De (A), (B) e (C) $\Rightarrow \beta + \alpha = \gamma$.

$$\left. \begin{aligned} \text{b) } \operatorname{arc} \operatorname{sen} \frac{1}{\sqrt{5}} = \beta &\Rightarrow \operatorname{sen} \beta = \frac{1}{\sqrt{5}} \text{ e } -\frac{\pi}{2} \leq \beta \leq \frac{\pi}{2} \Rightarrow 0 < \beta < \frac{\pi}{2} \\ \operatorname{arc} \operatorname{cos} \frac{3}{\sqrt{10}} = \alpha &\Rightarrow \operatorname{cos} \alpha = \frac{3}{\sqrt{10}} \text{ e } 0 \leq \alpha \leq \pi \Rightarrow 0 < \alpha < \frac{\pi}{2} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow 0 < \beta + \alpha < \pi \text{ (A); } \gamma = \frac{\pi}{4} \text{ (B)}$$

$$\operatorname{sen}^2 \beta = \frac{\operatorname{tg}^2 \beta}{1 + \operatorname{tg}^2 \beta} \Rightarrow \operatorname{tg} \beta = \frac{1}{2}; \operatorname{cos}^2 \alpha = \frac{1}{1 + \operatorname{tg}^2 \alpha} \Rightarrow \operatorname{tg} \alpha = \frac{1}{3}$$

$$\operatorname{tg}(\beta + \alpha) = \frac{\operatorname{tg} \beta + \operatorname{tg} \alpha}{1 - \operatorname{tg} \beta \cdot \operatorname{tg} \alpha} = 1 = \operatorname{tg} \gamma \text{ (C)}$$

De (A), (B) e (C) $\Rightarrow \beta + \alpha = \gamma$.

$$\left. \begin{aligned} \text{c) } \operatorname{arc} \operatorname{cos} \frac{3}{5} = \alpha &\Rightarrow \operatorname{cos} \alpha = \frac{3}{5} \text{ e } 0 \leq \alpha \leq \pi \Rightarrow 0 < \alpha < \frac{\pi}{2} \\ \operatorname{arc} \operatorname{cos} \frac{12}{13} = \beta &\Rightarrow \operatorname{cos} \beta = \frac{12}{13} \text{ e } 0 \leq \beta \leq \pi \Rightarrow 0 < \beta < \frac{\pi}{2} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow 0 < \alpha + \beta < \pi \text{ (A)}$$

$$\operatorname{arc} \operatorname{cos} \frac{16}{65} = \gamma \Rightarrow \operatorname{cos} \gamma = \frac{16}{65} \text{ e } 0 \leq \gamma \leq \pi \text{ (B)}$$

$$\operatorname{sen} \alpha = \sqrt{1 - \operatorname{cos}^2 \alpha} = \frac{4}{5}; \operatorname{sen} \beta = \sqrt{1 - \operatorname{cos}^2 \beta} = \frac{5}{13}$$

$$\operatorname{cos}(\alpha + \beta) = \operatorname{cos} \alpha \cdot \operatorname{cos} \beta - \operatorname{sen} \alpha \cdot \operatorname{sen} \beta = \frac{16}{65} = \operatorname{cos} \gamma \text{ (C)}$$

De (A), (B) e (C) $\Rightarrow \alpha + \beta = \gamma$.

$$\left. \begin{aligned} \text{d) } \operatorname{arc} \operatorname{sen} \frac{24}{25} = \alpha &\Rightarrow \operatorname{sen} \alpha = \frac{24}{25} \text{ e } -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2} \Rightarrow 0 < \alpha < \frac{\pi}{2} \\ \operatorname{arc} \operatorname{sen} \frac{3}{5} = \beta &\Rightarrow \operatorname{sen} \beta = \frac{3}{5} \text{ e } -\frac{\pi}{2} \leq \beta \leq \frac{\pi}{2} \Rightarrow 0 < \beta < \frac{\pi}{2} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow 0 < \alpha + \beta < \pi \text{ (A)}$$

$$\operatorname{arc} \operatorname{tg} \frac{3}{4} = \gamma \Rightarrow \operatorname{tg} \gamma = \frac{3}{4} \text{ e } -\frac{\pi}{2} < \gamma < \frac{\pi}{2} \text{ (B)}$$

$$\operatorname{sen}^2 \alpha = \frac{\operatorname{tg}^2 \alpha}{1 + \operatorname{tg}^2 \alpha} \Rightarrow \operatorname{tg} \alpha = \frac{24}{7}; \operatorname{sen}^2 \beta = \frac{\operatorname{tg}^2 \beta}{1 + \operatorname{tg}^2 \beta} \Rightarrow \operatorname{tg} \beta = \frac{3}{4}$$

$$\operatorname{tg}(\alpha - \beta) = \frac{\frac{24}{7} - \frac{3}{4}}{1 + \frac{24}{7} \cdot \frac{3}{4}} = \frac{3}{4} = \operatorname{tg} \gamma \text{ (C)}$$

De (A), (B) e (C) $\Rightarrow \alpha - \beta = \gamma$.

387. a) $\text{arc tg } \frac{2}{3} = \alpha \Rightarrow \text{tg } \alpha = \frac{2}{3} \text{ e } -\frac{\pi}{2} < \alpha < \frac{\pi}{2} \Rightarrow 0 < \alpha < \frac{\pi}{4} \Rightarrow$
 $\Rightarrow 0 < 2\alpha < \frac{\pi}{2}$
 $\text{arc cos } \frac{12}{13} = \beta \Rightarrow \text{cos } \beta = \frac{12}{13} \text{ e } 0 \leq \beta \leq \pi \Rightarrow 0 < \beta < \frac{\pi}{2} \Rightarrow$
 $\Rightarrow 0 < 2\alpha + \beta < \pi \text{ (A)}$

$$\text{tg } 2\alpha = \frac{2 \cdot \frac{2}{3}}{1 - \left(\frac{2}{3}\right)^2} = \frac{12}{5}; \text{cos } 2\alpha = \sqrt{\frac{1}{1 + \left(\frac{12}{5}\right)^2}} = \frac{5}{13};$$

$$\text{sen } 2\alpha = \sqrt{1 - \left(\frac{5}{13}\right)^2} = \frac{12}{13}$$

$$\text{sen } \beta = \sqrt{1 - \left(\frac{12}{13}\right)^2} = \frac{5}{13}; \gamma = \frac{\pi}{2} \text{ (B)}$$

$$\text{cos } (2\alpha + \beta) = \frac{5}{13} \cdot \frac{12}{13} - \frac{12}{13} \cdot \frac{5}{13} = 0 = \text{cos } \gamma \text{ (C)}$$

De (A), (B) e (C) $\Rightarrow 2\alpha + \beta = \gamma$.

b) $\text{arc sen } \frac{1}{4} = \alpha \Rightarrow \text{sen } \alpha = \frac{1}{4} \text{ e } -\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2} \Rightarrow 0 < \alpha < \frac{\pi}{6} \Rightarrow$
 $\Rightarrow 0 < 3\alpha < \frac{\pi}{2}$
 $\text{arc cos } \frac{11}{16} = \beta \Rightarrow \text{cos } \beta = \frac{11}{16} \text{ e } 0 \leq \beta \leq \pi \Rightarrow 0 < \beta < \frac{\pi}{2}$
 $\Rightarrow 0 < 3\alpha + \beta < \pi \text{ (A); } \gamma = \frac{\pi}{2} \text{ (B)}$

$$\text{cos } \alpha = \sqrt{1 - \frac{1}{16}} = \frac{\sqrt{15}}{4}, \text{sen } \beta = \sqrt{1 - \frac{121}{256}} = \frac{3\sqrt{15}}{16};$$

$$\text{cos } 3\alpha = 4 \cdot \left(\frac{\sqrt{15}}{4}\right)^3 - \frac{3\sqrt{15}}{4} = \frac{3\sqrt{15}}{16}$$

$$\text{sen } 3\alpha = 3 \cdot \frac{1}{4} - 4\left(\frac{1}{4}\right)^3 = \frac{11}{16}, \text{cos } (3\alpha + \beta) =$$

$$= \frac{3\sqrt{15}}{16} \cdot \frac{11}{16} - \frac{11}{16} \cdot \frac{3\sqrt{15}}{16} = 0 = \text{cos } \gamma \text{ (C)}$$

De (A), (B) e (C) $\Rightarrow 3\alpha + \beta = \gamma$.

388. $\text{arc tg } \left(\frac{1 + e^x}{2}\right) = \alpha \Rightarrow \text{tg } \alpha = \frac{1 + e^x}{2} \text{ e } -\frac{\pi}{2} < \alpha < \frac{\pi}{2}$

$\text{arc tg } \left(\frac{1 - e^x}{2}\right) = \beta \Rightarrow \text{tg } \beta = \frac{1 - e^x}{2} \text{ e } -\frac{\pi}{2} < \beta < \frac{\pi}{2}$

$$\alpha + \beta = \frac{\pi}{4} \Rightarrow \operatorname{tg}(\alpha + \beta) = \frac{\left(\frac{1+e^x}{2}\right) + \left(\frac{1-e^x}{2}\right)}{1 - \left(\frac{1+e^x}{2}\right)\left(\frac{1-e^x}{2}\right)} =$$

$$= \frac{1}{1 - \frac{1-e^{2x}}{4}} = \frac{4}{3+e^{2x}} \Rightarrow \frac{4}{3+e^{2x}} = 1 \Rightarrow e^{2x} = 1 \Rightarrow x = 0, S = \{0\}$$

389. $\alpha = \operatorname{arc\,tg}(7x - 1) \Rightarrow \operatorname{tg} \alpha = 7x - 1$ e $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ (A)
 $\beta = \operatorname{arc\,sec}(2x + 1) \Rightarrow \operatorname{sec} \beta = 2x + 1$ e $0 \leq \beta \leq \pi$ (B)
 $\alpha = \beta \Rightarrow \operatorname{tg} \alpha = \operatorname{tg} \beta \Rightarrow \operatorname{tg}^2 \alpha = \operatorname{tg}^2 \beta \Rightarrow \operatorname{tg}^2 \alpha = \operatorname{sec}^2 \beta - 1 \Rightarrow$
 $\Rightarrow (7x - 1)^2 = (2x + 1)^2 - 1 \Rightarrow 45x^2 - 18x + 1 = 0 \Rightarrow$
 $\Rightarrow x = \frac{1}{3}$ ou $x = \frac{1}{15}$ (não satisfaz $\operatorname{tg} \alpha = -\operatorname{tg} \beta$) $\therefore x = \frac{1}{3}$

390. $\alpha = \operatorname{arc\,sen}\left(\frac{4}{5}\right) \Rightarrow \operatorname{sen} \alpha = \frac{4}{5}$ e $\frac{\pi}{2} < \alpha < \pi$
 $\beta = \operatorname{arc\,tg}\left(-\frac{4}{3}\right) \Rightarrow \operatorname{tg} \beta = -\frac{4}{3}$ e $\frac{3\pi}{2} < \beta \leq 2\pi$
 $\cos \alpha = -\sqrt{1 - \frac{16}{25}} = -\frac{3}{5}; \operatorname{sen} \beta = -\sqrt{\frac{\frac{16}{9}}{1 + \frac{16}{9}}} = -\frac{4}{5};$
 $\cos \beta = \sqrt{1 - \left(-\frac{4}{5}\right)^2} = \frac{3}{5}$
 $25 \cdot \cos(\alpha + \beta) = 25 \cdot \left[-\frac{3}{5} \cdot \frac{3}{5} - \frac{4}{5} \left(-\frac{4}{5}\right)\right] = 7$

Apêndice A — Resolução de equações e inequações em intervalos determinados

393. $\operatorname{sen} 3x = \operatorname{sen} \frac{\pi}{6} \Rightarrow \begin{cases} 3x = \frac{\pi}{6} + 2k\pi \Rightarrow x = \frac{\pi}{18} + 2k\frac{\pi}{3} \\ \text{ou} \\ 3x = \pi - \frac{\pi}{6} + 2k\pi \Rightarrow x = \frac{5\pi}{18} + 2k\frac{\pi}{3} \end{cases}$
 $k = 0 \Rightarrow x = \frac{\pi}{18}$ ou $x = \frac{5\pi}{18}; k = 1 \Rightarrow x = \frac{13\pi}{18}$ ou $x = \frac{17\pi}{18}$
 $S = \left\{ \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18} \right\}$

395. $3x = 2x + 2k\pi \Rightarrow x = 2k\pi$ ou $3x = \pi - 2x + 2k\pi \Rightarrow x = \frac{\pi}{5} + \frac{2k\pi}{5}$
 $S = \left\{ 0, \frac{\pi}{5}, \frac{3\pi}{5}, \pi \right\}$

397. $3x = 2x + 2k\pi \Rightarrow x = 2k\pi$ ou $3x = -2x + 2k\pi \Rightarrow x = \frac{2k\pi}{5}$
 $S = \left\{ 0, \frac{2\pi}{5}, \frac{4\pi}{5} \right\}$

398. $\text{sen } x \cdot (4 \text{sen}^2 x - 1) = 0$. Então: $\text{sen } x = 0 \Rightarrow x = k\pi$ ou $\text{sen } x = \frac{1}{2} \Rightarrow$
 $\Rightarrow x = \frac{\pi}{6} + 2k\pi$ ou $x = \frac{5\pi}{6} + 2k\pi$ ou $\text{sen } x = -\frac{1}{2} \Rightarrow$
 $\Rightarrow x = \frac{7\pi}{6} + 2k\pi$ ou $x = -\frac{\pi}{6} + 2k\pi \Rightarrow$
 $\Rightarrow S = \left\{ 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi \right\}$

400. $\text{tg } x + \frac{1}{\text{tg } x} = 2 \Rightarrow \text{tg}^2 x - 2 \text{tg } x + 1 = 0 \Rightarrow$
 $\Rightarrow \text{tg } x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi \Rightarrow S = \left\{ \frac{\pi}{4}, \frac{5\pi}{4} \right\}$

401. $\sqrt{\text{sen}^2 x} = \sqrt{\text{cos}^2 x} \Rightarrow \text{sen}^2 x = \text{cos}^2 x \Rightarrow \text{cos}^2 x - \text{sen}^2 x = 0 \Rightarrow$
 $\Rightarrow \text{cos } 2x = 0 \Rightarrow 2x = \frac{\pi}{2} + k\pi \Rightarrow x = \frac{\pi}{4} + \frac{k\pi}{2} \Rightarrow S = \left\{ \frac{\pi}{4}, \frac{3\pi}{4} \right\}$

402. $\text{sen } x + \text{sen } y = 2 \text{sen} \left(\frac{x+y}{2} \right) \cdot \text{cos} \left(\frac{x-y}{2} \right) = 1 \Rightarrow \text{cos} \left(\frac{\frac{\pi}{3} - 2y}{2} \right) = 1 \Rightarrow$
 $\Rightarrow \text{cos} \left(\frac{\pi}{6} - y \right) = \text{cos } 0 \Rightarrow \frac{\pi}{6} - y = 2k\pi \Rightarrow y = \frac{\pi}{6} - 2k\pi$
 $x + y = \frac{\pi}{3} \Rightarrow x = \frac{\pi}{6} + 2k\pi$

403. a) $\text{cos } 2x = \text{cos } \frac{\pi}{6} \Rightarrow \begin{cases} 2x = \frac{\pi}{6} + 2k\pi \Rightarrow x = \frac{\pi}{12} + k\pi \\ \text{ou} & \text{ou} \\ 2x = -\frac{\pi}{6} + 2k\pi \Rightarrow x = -\frac{\pi}{12} + k\pi \end{cases}$

$S = \left\{ \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12} \right\}$

b) $2x = x + 2k\pi \Rightarrow x = 2k\pi$ ou $2x = -x + 2k\pi \Rightarrow x = \frac{2k\pi}{3}$

$S = \left\{ 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi \right\}$

c) $\text{cos} \left(x + \frac{\pi}{6} \right) = \text{cos } \frac{\pi}{2} \Rightarrow x = \frac{\pi}{3} + k\pi \Rightarrow S = \left\{ \frac{\pi}{3}, \frac{4\pi}{3} \right\}$

- 404.** a) $3x = x + 2k\pi \Rightarrow x = k\pi$ ou $3x = -x + 2k\pi \Rightarrow x = \frac{k\pi}{2}$
 $S = \left\{ 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi \right\}$
- b) $5x = x + \frac{\pi}{3} + 2k\pi \Rightarrow x = \frac{\pi}{12} + \frac{k\pi}{2}$ ou $5x = -x - \frac{\pi}{3} + 2k\pi \Rightarrow$
 $\Rightarrow x = -\frac{\pi}{18} + \frac{k\pi}{3}$
 $S = \left\{ \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}, \frac{5\pi}{18}, \frac{11\pi}{18}, \frac{17\pi}{18}, \frac{23\pi}{18}, \frac{29\pi}{18}, \frac{35\pi}{18} \right\}$
- 405.** $\cos^2 x - \frac{2}{\cos^2 x} = 1 \Rightarrow \cos^4 x - \cos^2 x - 2 = 0 \Rightarrow \cos^2 x = 2$ ou
 $\cos^2 x = -1, S = \emptyset$
- 406.** O 1º membro é a soma dos 10 termos da PG., com $a_1 = 1$ e $q = \cos x$;
então sua soma é $\frac{1 \cdot \cos^{10} x - 1}{\cos x - 1} = 0$, e daí $\cos x = -1$; a equação
tem uma única solução.
- 407.** a) $\operatorname{tg} 2x = \operatorname{tg} \frac{\pi}{3} \Rightarrow x = \frac{\pi}{6} + \frac{k\pi}{2} \Rightarrow S = \left\{ \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3} \right\}$
- b) $2x = x + k\pi \Rightarrow x = k\pi \Rightarrow S = \{0, \pi, 2\pi\}$
- c) $\operatorname{tg} 3x = \operatorname{tg} \frac{\pi}{4} \Rightarrow 3x = \frac{\pi}{4} + k\pi \Rightarrow x = \frac{\pi}{12} + \frac{k\pi}{3} \Rightarrow$
 $\Rightarrow S = \left\{ \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{21\pi}{12} \right\}$
- d) $\operatorname{tg} 3x = \operatorname{tg} 2x \Rightarrow 3x = 2x + k\pi \Rightarrow x = k\pi \Rightarrow S = \{0, \pi, 2\pi\}$
- e) $2x = x + \frac{\pi}{4} + k\pi \Rightarrow x = \frac{\pi}{4} + k\pi$, mas $x + \frac{\pi}{4} \neq \frac{\pi}{2} + k\pi$, então $S = \emptyset$
- f) $\operatorname{tg} 4x = \operatorname{tg} \frac{\pi}{4} \Rightarrow 4x = \frac{\pi}{4} + k\pi \Rightarrow x = \frac{\pi}{16} + \frac{k\pi}{4} \Rightarrow$
 $\Rightarrow S = \left\{ \frac{\pi}{16}, \frac{5\pi}{16}, \frac{9\pi}{16}, \frac{13\pi}{16}, \frac{17\pi}{16}, \frac{21\pi}{16}, \frac{25\pi}{16}, \frac{29\pi}{16} \right\}$
- g) $2x = x + \frac{\pi}{4} + k\pi \Rightarrow x = \frac{\pi}{4} + k\pi \Rightarrow S = \left\{ \frac{\pi}{4}, \frac{5\pi}{4} \right\}$
- h) $\operatorname{tg} 2x = \operatorname{tg} \frac{\pi}{3} \Rightarrow 2x = \frac{\pi}{3} + k\pi \Rightarrow x = \frac{\pi}{6} + \frac{k\pi}{2}$
ou
 $\operatorname{tg} 2x = \operatorname{tg} \frac{2\pi}{3} \Rightarrow 2x = \frac{2\pi}{3} + k\pi \Rightarrow x = \frac{\pi}{3} + \frac{k\pi}{2} \Rightarrow$
 $\Rightarrow S = \left\{ \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{6}, \frac{5\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6} \right\}$

408. a) $1 + \operatorname{tg}^2 x - 2 \operatorname{tg} x = 0 \Rightarrow \operatorname{tg} x = 1 \Rightarrow x = \frac{\pi}{4} + k\pi \Rightarrow S = \left\{ \frac{\pi}{4}, \frac{5\pi}{4} \right\}$

b) $1 = \operatorname{sen}^2 x - \operatorname{sen} x \cdot \cos x \Rightarrow \operatorname{sen}^2 x + \cos^2 x = \operatorname{sen}^2 x - \operatorname{sen} x \cdot \cos x \Rightarrow$
 $\Rightarrow \cos x (\operatorname{sen} x + \cos x) = 0$

Então:

$\cos x = 0 \Rightarrow x = \frac{\pi}{2} + k\pi$ ou $\operatorname{sen} x + \cos x = 0 \Rightarrow \operatorname{sen} x = -\cos x \Rightarrow$

$\Rightarrow \operatorname{tg} x = -1 \Rightarrow x = \frac{3\pi}{4} + k\pi \Rightarrow S = \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4} \right\}$

c) $\operatorname{sen} 2x \cdot \cos \left(x + \frac{\pi}{4} \right) - \cos 2x \cdot \operatorname{sen} \left(x + \frac{\pi}{4} \right) = 0 \Rightarrow$

$\Rightarrow \operatorname{sen} \left[2x - \left(x + \frac{\pi}{4} \right) \right] = 0 \Rightarrow \operatorname{sen} \left(x - \frac{\pi}{4} \right) = 0 \Rightarrow$

$\Rightarrow x - \frac{\pi}{4} = k\pi \Rightarrow S = \left\{ \frac{\pi}{4}, \frac{5\pi}{4} \right\}$

d) $1 + \operatorname{sen} 2x - \operatorname{tg} x - \operatorname{tg} x \operatorname{sen} 2x = 1 + \operatorname{tg} x \Rightarrow \operatorname{sen} 2x(1 - \operatorname{tg} x) = 2 \operatorname{tg} x \Rightarrow$

$\Rightarrow \frac{2 \operatorname{tg} x}{1 + \operatorname{tg}^2 x} = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg} x} \Rightarrow \operatorname{tg} x(\operatorname{tg} x + 1) = 0$. Então: $\operatorname{tg} x = 0 \Rightarrow$

$\Rightarrow x = k\pi$ ou

$\operatorname{tg} x = -1 \Rightarrow x = \frac{3\pi}{4} + k\pi$ e daí $S = \left\{ 0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi \right\}$

e) $\sec x(3 \sec x - 2) = 0 \Rightarrow \sec x = 0$ ou $\sec x = \frac{2}{3}$ e daí $S = \emptyset$

f) $2 \operatorname{sen}^2 x = \frac{\operatorname{sen} x}{\cos x} \cdot \cos x \Rightarrow \operatorname{sen} x(2 \operatorname{sen} x - 1) = 0$. Então: $\operatorname{sen} x = 0 \Rightarrow$

$\Rightarrow x = k\pi$ ou $\operatorname{sen} x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} + 2k\pi$ ou $x = \frac{5\pi}{6} + 2k\pi \Rightarrow$

$\Rightarrow S = \left\{ 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, 2\pi \right\}$

409. $\frac{\pi}{2} p = \frac{\pi}{4} + \frac{k\pi}{2} \Rightarrow p = \frac{1}{2} + k$

$\operatorname{tg} \frac{\pi}{2} p = \frac{1}{\operatorname{tg} \frac{\pi}{2} p} \Rightarrow \operatorname{tg} \frac{\pi}{2} p = \pm 1$

410. $\operatorname{sen} x = \operatorname{tg} x \Rightarrow \cos x = 1 \Rightarrow x = 2k\pi \Rightarrow \nexists x \mid 0 < x < \pi \Rightarrow$ nenhum ponto

411. $1 + \operatorname{tg}^2 x - \operatorname{tg} x = 1 \Rightarrow \operatorname{tg} x(\operatorname{tg} x - 1) = 0 \Rightarrow \operatorname{tg} x = 0$ ou $\operatorname{tg} x = 1 \Rightarrow$
 $\Rightarrow x = k\pi$ ou $x = \frac{\pi}{4} + k\pi \Rightarrow S = \left\{ 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi \right\}$

412. $6x = 2x + k\pi \Rightarrow x = \frac{k\pi}{4} \Rightarrow x \in \left\{0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi\right\}$

$x = \frac{\pi}{4} \Rightarrow \cancel{\nexists} \operatorname{tg} 2x; x = \frac{3\pi}{4} \Rightarrow \cancel{\nexists} \operatorname{tg} 2x \Rightarrow S = \left\{0, \frac{\pi}{2}, \pi\right\}$

413. a) $(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x = \frac{5}{8} \Rightarrow \sin 2x = \pm \frac{\sqrt{3}}{2} \Rightarrow$
 $\Rightarrow \left(x = \frac{\pi}{6} + k\pi \text{ ou } x = \frac{\pi}{3} + k\pi \text{ ou } x = \frac{2\pi}{3} + k\pi \text{ ou } x = \frac{5\pi}{6} + k\pi\right) \Rightarrow$
 $\Rightarrow S = \left\{\frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}\right\}$

b) $(\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x) = \frac{5}{8} \Rightarrow$
 $\Rightarrow \sin 2x = \pm \frac{\sqrt{2}}{2} \Rightarrow \left(x = \frac{\pi}{8} + k\pi \text{ ou } x = \frac{3\pi}{8} + k\pi \text{ ou } x = \frac{5\pi}{8} + k\pi\right.$
 $\left.\text{ ou } x = \frac{7\pi}{8} + k\pi\right) \Rightarrow S = \left\{\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}\right\}$

c) $\sin^2 2x = 1 \Rightarrow \sin 2x = \pm 1 \Rightarrow \left(x = \frac{\pi}{4} + k\pi \text{ ou } x = \frac{3\pi}{4} + k\pi\right) \Rightarrow$
 $\Rightarrow S = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

d) $\sin^2 x = \frac{3}{4} \Rightarrow \sin x = \pm \frac{\sqrt{3}}{2} \Rightarrow \left(x = \frac{\pi}{3} + 2k\pi \text{ ou } x = \frac{4\pi}{3} + 2k\pi \text{ ou } x = \frac{2\pi}{3} + 2k\pi \text{ ou } x = \frac{5\pi}{3} + 2k\pi\right) \Rightarrow S = \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

e) $(\sin x + \cos x)(\sin^2 x - \sin x \cdot \cos x + \cos^2 x) = 1 \Rightarrow$
 $\Rightarrow (\sin x + \cos x)(1 - \sin x \cdot \cos x) = 1; \text{ fazendo } \sin x + \cos x = y$
 $\text{ e } \sin x \cdot \cos x = \frac{y^2 - 1}{2}, \text{ vem } y \cdot \left(1 - \frac{y^2 - 1}{2}\right) = 1 \Rightarrow$
 $\Rightarrow y = 1 \text{ ou } y = -2 \text{ (n\~ao serve, pois } -\sqrt{2} \leq y \leq \sqrt{2})$
 $\sin x + \cos x = 1 \Rightarrow \sqrt{2} \cos\left(x - \frac{\pi}{4}\right) = 1 \Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \Rightarrow$
 $\Rightarrow \left(x = \frac{\pi}{2} + 2k\pi \text{ ou } x = 2k\pi\right) \Rightarrow S = \left\{0, \frac{\pi}{2}, 2\pi\right\}$

414. $2x = x + 2k\pi \Rightarrow x = 2k\pi \text{ ou } 2x = \pi - x + 2k\pi \Rightarrow x = \frac{\pi}{3} + \frac{2k\pi}{3} \Rightarrow$

$\Rightarrow S = \left\{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\} \Rightarrow \text{quatro solu\~coes}$

415. $\frac{3 \operatorname{sen}^2 x}{\cos^2 x} + 5 = \frac{7}{\cos x} \Rightarrow 3(1 - \cos^2 x) + 5 \cos^2 x - 7 \cos x = 0 \Rightarrow$
 $\Rightarrow 2 \cos^2 x - 7 \cos x + 3 = 0 \Rightarrow \left(\cos x = 3 \text{ impossível ou } \cos x = \frac{1}{2} \right) \Rightarrow$
 $\Rightarrow \left(x = \frac{\pi}{3} + 2k\pi \text{ ou } x = -\frac{\pi}{3} + 2k\pi \right) \Rightarrow S = \left\{ -\frac{\pi}{3}, \frac{\pi}{3} \right\}$

416. $\operatorname{sen} \pi x = -\cos \pi x \Rightarrow \operatorname{sen} \pi x = \operatorname{sen} \left(\frac{3\pi}{2} - \pi x \right) \Rightarrow x = \frac{3}{4} + k \Rightarrow$
 $\Rightarrow S = \left\{ \frac{3}{4}, \frac{7}{4} \right\}$

417. $1 + \operatorname{tg}^2 x = \operatorname{tg} x + 1 \Rightarrow \operatorname{tg} x (\operatorname{tg} x - 1) = 0 \Rightarrow (\operatorname{tg} x = 0 \text{ ou } \operatorname{tg} x = 1) \Rightarrow$
 $\Rightarrow \left(x = k\pi \text{ ou } x = \frac{\pi}{4} + k\pi \right) \Rightarrow S = \left\{ 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi \right\}$

418. a) $2(1 - \cos^2 x) - 3 \cos x - 3 = 0 \Rightarrow 2 \cos^2 x + 3 \cos x + 1 = 0 \Rightarrow$
 $\Rightarrow \left(\cos x = -\frac{1}{2} \text{ ou } \cos x = -1 \right) \Rightarrow$
 $\Rightarrow \left(x = \frac{2\pi}{3} + 2k\pi \text{ ou } x = \frac{4\pi}{3} + 2k\pi \text{ ou } x = \pi + 2k\pi \right) \Rightarrow$
 $\Rightarrow S = \left\{ \frac{2\pi}{3}, \pi, \frac{4\pi}{3} \right\}$

b) $\cos^2 x - 2 \cos x + 1 = 0 \Rightarrow \cos x = 1 \Rightarrow x = 2k\pi \Rightarrow S = \{0, 2\pi\}$

c) $2 \cos^2 x + 5 \cos x - 3 = 0 \Rightarrow \cos x = \frac{1}{2} \Rightarrow$
 $\Rightarrow \left(x = \frac{\pi}{3} + 2k\pi \text{ ou } x = -\frac{\pi}{3} + 2k\pi \right) \Rightarrow S = \left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$

d) $4 \cos^2 x - 8 \cos x + 3 = 0 \Rightarrow \cos x = \frac{1}{2} \Rightarrow$
 $\Rightarrow \left(x = \frac{\pi}{3} + 2k\pi \text{ ou } x = -\frac{\pi}{3} + 2k\pi \right) \Rightarrow S = \left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$

419. $\cos x = \pm \frac{1}{2} \Rightarrow \left(x = \frac{\pi}{3} + 2k\pi \text{ ou } x = \frac{5\pi}{3} + 2k\pi \text{ ou } x = \frac{2\pi}{3} + 2k\pi \right.$
 $\left. \text{ ou } x = \frac{4\pi}{3} + 2k\pi \right) \Rightarrow S = \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\} \Rightarrow$
 $\Rightarrow \text{soma} = \frac{\pi + 2\pi + 4\pi + 5\pi}{3} = 4\pi$

420. $2 \cos^2 x + 3(1 - \cos^2 x) - 5 - 3 \cos x = 0 \Rightarrow \cos^2 x + 3 \cos x + 2 = 0 \Rightarrow$
 $\Rightarrow \cos x = -1 \Rightarrow x = \pi + 2k\pi \Rightarrow S = \{\pi\} \Rightarrow \text{uma solução}$

421. a) $\sin(x+y) + \sin(x-y) = \sin\frac{5\pi}{2} + \sin\left(\frac{3\pi}{2}\right) = 1 - 1 = 0 \neq 2$

b) $\left. \begin{array}{l} 2 \sin x \cos y = 2 \\ \sin x + \cos y = 2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} \sin x \cos y = 1 \quad (A) \\ \sin x + \cos y = 2 \quad (B) \end{array} \right\} (A) \text{ em } (B) \Rightarrow$

$\Rightarrow \sin^2 x - 2 \sin x + 1 = 0 \Rightarrow \sin x = 1 \quad (C) \Rightarrow x = \frac{\pi}{2} + 2k\pi$

$(C) \text{ em } (A) \Rightarrow \cos y = 1 \Rightarrow y = 2k\pi$

$k = \{0, 1\} \Rightarrow S = \left\{ \left(\frac{\pi}{2}, 0 \right) \right\} \text{ ou } \left\{ \left(\frac{\pi}{2}, 2\pi \right) \right\}$

422. $\left\{ \begin{array}{l} \sin a + \cos b = 1 \quad (A) \\ \sin a + \sin b = 1 \quad (B) \end{array} \right. (A) \text{ em } (B) \Rightarrow \sin b - \cos b = 0 \Rightarrow$

$\Rightarrow \sin\left(b - \frac{\pi}{4}\right) = 0 \Rightarrow b = \frac{\pi}{4} + k\pi \Rightarrow b = \frac{\pi}{4} \quad (C)$

$(C) \text{ em } (A) \Rightarrow \sin a = \frac{2 - \sqrt{2}}{2} \Rightarrow a = \arcsin \frac{2 - \sqrt{2}}{2}$

$S = \left\{ \left(\arcsin \frac{2 - \sqrt{2}}{2}, \frac{\pi}{4} \right) \right\}$

423. 1º caso: $\sin x \geq 0$

$|\sin x| = \sin x \Rightarrow 2 \sin^2 x + \sin x - 1 = 0 \Rightarrow \sin x = \frac{1}{2} \Rightarrow$

$\Rightarrow x = \frac{\pi}{6} + 2k\pi \text{ ou } x = \frac{5\pi}{6} + 2k\pi$

2º caso: $\sin x < 0$

$|\sin x| = -\sin x \Rightarrow 2 \sin^2 x - \sin x - 1 = 0 \Rightarrow \sin x = -\frac{1}{2} \Rightarrow$

$\Rightarrow x = \frac{7\pi}{6} + 2k\pi \text{ ou } x = \frac{11\pi}{6} + 2k\pi$

Então: $S = \left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$.

424. $\cos x = \pm \frac{1}{2} \Rightarrow \left(x = \frac{\pi}{3} + 2k\pi \text{ ou } x = \frac{5\pi}{3} + 2k\pi \text{ ou } x = \frac{2\pi}{3} + 2k\pi \right.$

$\left. \text{ ou } x = \frac{4\pi}{3} + 2k\pi \right) \Rightarrow S = \left\{ \frac{\pi}{3}, \frac{2\pi}{3} \right\} \Rightarrow \text{a soma é } \pi$

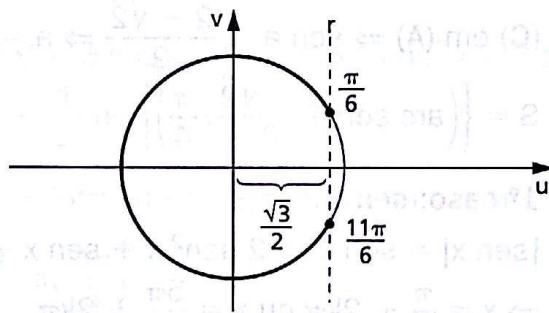
425. $\log 2 \sin^2 x = 0 \Rightarrow 2 \sin^2 x = 1 \Rightarrow \sin x = \pm \frac{\sqrt{2}}{2} \Rightarrow$

$\Rightarrow \left(x = \frac{\pi}{4} + 2k\pi \text{ ou } x = \frac{3\pi}{4} + 2k\pi \text{ ou } x = \frac{5\pi}{4} + 2k\pi \text{ ou } x = \frac{7\pi}{4} + 2k\pi \right) \Rightarrow$

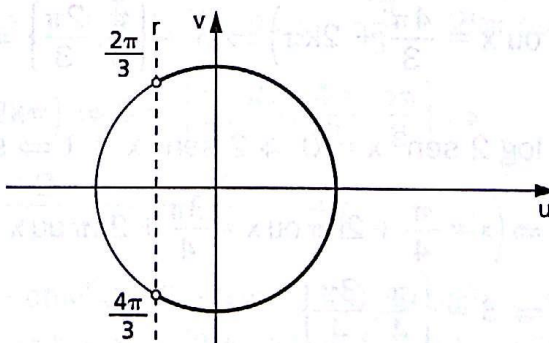
$\Rightarrow S = \left\{ \frac{\pi}{4}, \frac{3\pi}{4} \right\}$

426. $\text{sen } x = y \Rightarrow 2y^2 - 5y + 2 = 0 \Rightarrow y = 2$ não serve ou $y = \frac{1}{2} \Rightarrow$
 $\Rightarrow \text{sen } x = \frac{1}{2} \Rightarrow \left(x = \frac{\pi}{6} + 2k\pi \text{ ou } x = \frac{5\pi}{6} + 2k\pi \right) \Rightarrow$
 $\Rightarrow S = \left\{ \frac{\pi}{6}, \frac{5\pi}{6} \right\} \Rightarrow$ a soma é π

428. $2x = t \Rightarrow \cos t \leq \frac{\sqrt{3}}{2}$
 $\frac{\pi}{6} + 2k\pi \leq t \leq \frac{11\pi}{6} + 2k\pi$
 $\frac{\pi}{12} + k\pi \leq x \leq \frac{11\pi}{12} + k\pi$
 $S = \left\{ x \in \mathbb{R} \mid \frac{\pi}{12} \leq x \leq \frac{11\pi}{12} \text{ ou } \frac{13\pi}{12} \leq x \leq \frac{23\pi}{12} \right\}$



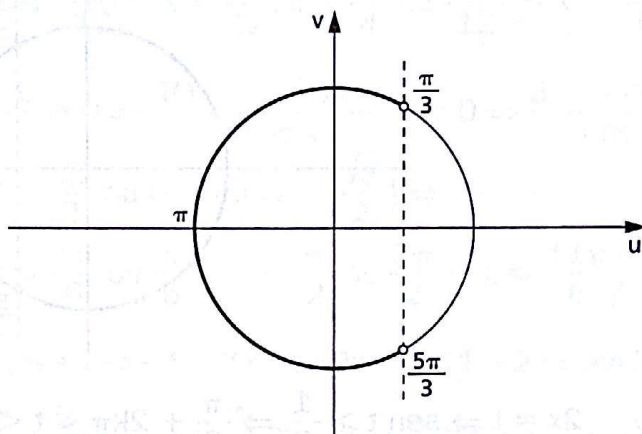
429. $4x = t \Rightarrow \cos t > -\frac{1}{2}$
 $2k\pi \leq t < \frac{2\pi}{3} + 2k\pi \text{ ou } \frac{4\pi}{3} + 2k\pi < t \leq 2\pi + 2k\pi$
 $\frac{k\pi}{2} \leq x < \frac{\pi}{6} + \frac{k\pi}{2} \text{ ou } \frac{\pi}{3} + \frac{k\pi}{2} < x \leq \frac{\pi}{2} + \frac{k\pi}{2}$
 $k = \{0, 1, 2\} \Rightarrow S = \left\{ x \in \mathbb{R} \mid 0 \leq x < \frac{\pi}{6} \text{ ou } \frac{\pi}{3} < x < \frac{2\pi}{3} \text{ ou } \right.$
 $\left. \frac{5\pi}{6} < x < \frac{7\pi}{6} \text{ ou } \frac{4\pi}{3} < x < \frac{5\pi}{3} \text{ ou } \frac{11\pi}{6} < x \leq 2\pi \right\}$



431. $\cos x = y \Rightarrow \frac{2y^2 + y - 1}{y - 1} > 0 \Rightarrow -1 < y < \frac{1}{2} \text{ ou } y > 1$

$-1 < \cos x < \frac{1}{2} \text{ ou } \cos x > 1 \text{ impossível}$

$S = \left\{ x \in \mathbb{R} \mid \frac{\pi}{3} < x < \pi \right\}$



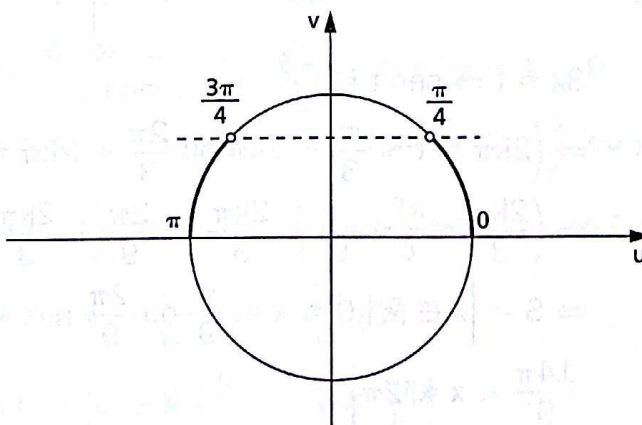
432. $\frac{1 - 2 \operatorname{sen}^2 x + \operatorname{sen} x + 1}{1 - 2 \operatorname{sen}^2 x} - 2 \geq 0 \Rightarrow$

$\Rightarrow \frac{2 \operatorname{sen}^2 x + \operatorname{sen} x}{1 - 2 \operatorname{sen}^2 x} \geq 0, \operatorname{sen} x = y \Rightarrow$

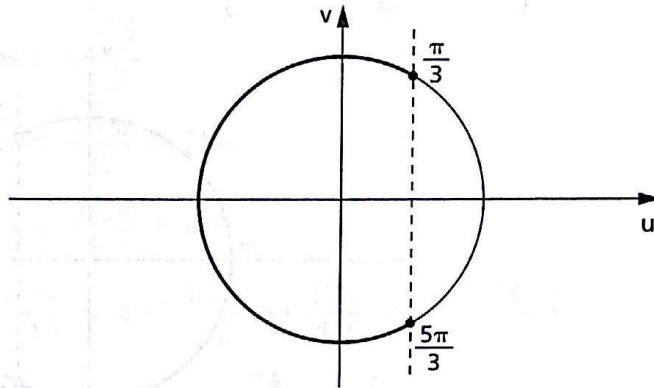
$\Rightarrow \frac{2y^2 + y}{1 - 2y^2} \geq 0 \Rightarrow -\frac{\sqrt{2}}{2} < y \leq -\frac{1}{2} \text{ ou } 0 \leq y < \frac{\sqrt{2}}{2} \Rightarrow$

$\Rightarrow -\frac{\sqrt{2}}{2} < \operatorname{sen} x \leq -\frac{1}{2} \text{ ou } 0 \leq \operatorname{sen} x < \frac{\sqrt{2}}{2} \Rightarrow$

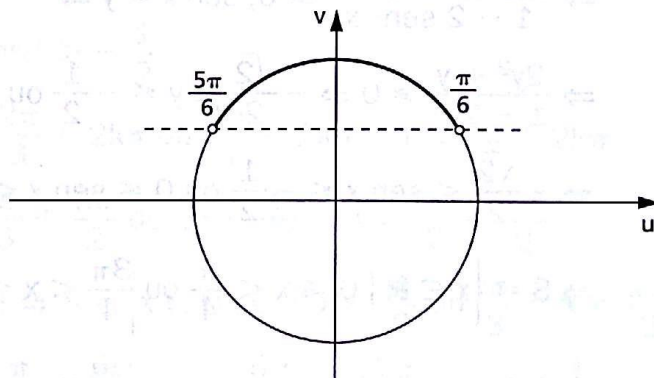
$\Rightarrow S = \left\{ x \in \mathbb{R} \mid 0 \leq x < \frac{\pi}{4} \text{ ou } \frac{3\pi}{4} < x \leq \pi \right\}$



433. $2^{\cos 2x} \leq 2^{\frac{1}{2}} \Rightarrow \cos 2x \leq \frac{1}{2}, 2x = t \Rightarrow \cos t \leq \frac{1}{2} \Rightarrow$
 $\Rightarrow \frac{\pi}{3} + 2k\pi \leq t \leq \frac{5\pi}{3} + 2k\pi \Rightarrow \frac{\pi}{6} + k\pi \leq x \leq \frac{5\pi}{6} + k\pi \Rightarrow$
 $\Rightarrow S = \left\{ x \in \mathbb{R} \mid \frac{\pi}{6} \leq x \leq \frac{5\pi}{6} \right\}$



435. $2x = t \Rightarrow \text{sen } t > \frac{1}{2} \Rightarrow \frac{\pi}{6} + 2k\pi < t < \frac{5\pi}{6} + 2k\pi \Rightarrow$
 $\Rightarrow \frac{\pi}{12} + k\pi < x < \frac{5\pi}{12} + k\pi$
 $S = \left\{ x \in \mathbb{R} \mid \frac{\pi}{12} < x < \frac{5\pi}{12} \text{ ou } \frac{13\pi}{12} < x < \frac{17\pi}{12} \right\}$



436. $3x = t \Rightarrow \text{sen } t \leq \frac{\sqrt{3}}{2} \Rightarrow$
 $\Rightarrow \left(2k\pi \leq t \leq \frac{\pi}{3} + 2k\pi \text{ ou } \frac{2\pi}{3} + 2k\pi \leq t \leq 2\pi + 2k\pi \right) \Rightarrow$
 $\Rightarrow \left(\frac{2k\pi}{3} \leq x \leq \frac{\pi}{9} + \frac{2k\pi}{3} \text{ ou } \frac{2\pi}{9} + \frac{2k\pi}{3} \leq x \leq \frac{2\pi}{3} + \frac{2k\pi}{3} \right) \Rightarrow$
 $\Rightarrow S = \left\{ x \in \mathbb{R} \mid 0 \leq x \leq \frac{\pi}{9} \text{ ou } \frac{2\pi}{9} \leq x \leq \frac{7\pi}{9} \text{ ou } \frac{8\pi}{9} \leq x \leq \frac{13\pi}{9} \text{ ou } \right.$
 $\left. \frac{14\pi}{9} \leq x \leq 2\pi \right\}$

437. $\frac{1}{4} \leq \frac{1}{2} \operatorname{sen} 2x < \frac{1}{2} \Rightarrow \frac{1}{2} \leq \operatorname{sen} t < 1, t = 2x,$
 $\frac{\pi}{6} + 2k\pi \leq t \leq \frac{5\pi}{6} + 2k\pi, t \neq \frac{\pi}{2} + 2k\pi \Rightarrow \frac{\pi}{12} + k\pi \leq x \leq \frac{5\pi}{12} + k\pi,$
 $x \neq \frac{\pi}{4} + k\pi \Rightarrow$
 $\Rightarrow S = \left\{ x \in \mathbb{R} \mid \frac{\pi}{12} \leq x \leq \frac{5\pi}{12}, x \neq \frac{\pi}{4} \text{ ou } \frac{13\pi}{12} \leq x \leq \frac{17\pi}{12}, x \neq \frac{5\pi}{4} \right\}$

438. $\frac{4 \operatorname{sen}^2 x - 1}{\cos x} \geq 0 \Rightarrow \frac{4(1 - \cos^2 x) - 1}{\cos x} \geq 0 \Rightarrow \frac{3 - 4 \cos^2 x}{\cos x} \geq 0 \Rightarrow$
 $\Rightarrow \left(\cos x \leq -\frac{\sqrt{3}}{2} \text{ ou } 0 < \cos x \leq \frac{\sqrt{3}}{2} \right) \Rightarrow$
 $\Rightarrow \left(\frac{5\pi}{6} \leq x \leq \frac{7\pi}{6} \text{ ou } \frac{\pi}{6} \leq x < \frac{\pi}{2} \text{ ou } \frac{3\pi}{2} < x \leq \frac{11\pi}{6} \right)$

439. $-1 \leq \operatorname{sen} 2x \leq +1 \Rightarrow -1 - 2 \leq \operatorname{sen} 2x - 2 \leq 1 - 2 \Rightarrow \operatorname{sen} 2x - 2 < 0 \text{ (A)}$
 $\frac{\operatorname{sen} 2x - 2}{\cos 2x + 3 \cos x - 1} \geq 0 \stackrel{(A)}{\Rightarrow} \cos 2x + 3 \cos x - 1 < 0 \Rightarrow$
 $\Rightarrow 2 \cos^2 x + 3 \cos x - 2 < 0 \Rightarrow -2 < \cos x < \frac{1}{2} \Rightarrow \frac{\pi}{3} < x < \frac{5\pi}{3}$

440. a) $\Delta \geq 0 \Rightarrow [-(4 \cos \alpha)]^2 - 4(2 \cos^2 \alpha)(4 \cos^2 \alpha - 1) \geq 0 \Rightarrow$
 $\Rightarrow -32 \cos^4 \alpha + 24 \cos^2 \alpha \geq 0$ fazendo $\cos \alpha = t \Rightarrow$
 $\Rightarrow -32t^4 + 24t^2 \geq 0 \Rightarrow$
 $\Rightarrow -8t^2(4t^2 - 3) \geq 0 \Rightarrow -\frac{\sqrt{3}}{2} \leq t \leq \frac{\sqrt{3}}{2} \Rightarrow -\frac{\sqrt{3}}{2} \leq \cos \alpha \leq \frac{\sqrt{3}}{2} \Rightarrow$
 $\Rightarrow S = \left\{ \alpha \in \mathbb{R} \mid \frac{\pi}{6} \leq \alpha \leq \frac{5\pi}{6} \right\}$

b) $-\frac{b}{a} < 0 \Rightarrow \frac{4 \cos \alpha}{2 \cos^2 \alpha} < 0 \Rightarrow \frac{2}{\cos \alpha} < 0 \Rightarrow \cos \alpha < 0 \Rightarrow \frac{\pi}{2} < \alpha \leq \pi \text{ (A)}$

$\frac{c}{a} > 0 \Rightarrow \frac{4 \cos^2 \alpha - 1}{2 \cos^2 \alpha} > 0 \Rightarrow 4 \cos^2 \alpha - 1 > 0$

Fazendo $\cos \alpha = y \Rightarrow 4y^2 - 1 > 0 \Rightarrow \left(y < -\frac{1}{2} \text{ ou } y > \frac{1}{2} \right) \Rightarrow$

$\Rightarrow \left(\cos \alpha < -\frac{1}{2} \text{ ou } \cos \alpha > \frac{1}{2} \right) \Rightarrow \left(\frac{2\pi}{3} < \alpha \leq \pi \text{ ou } 0 \leq \alpha < \frac{\pi}{3} \right) \text{ (B)}$

Soluções reais $\Rightarrow \frac{\pi}{6} \leq \alpha \leq \frac{5\pi}{6} \text{ (C)},$

$A \cap B \cap C = \left\{ \alpha \in \mathbb{R} \mid \frac{2\pi}{3} < \alpha \leq \frac{5\pi}{6} \right\}$

441. $(\cos x > 0 \text{ e } 2 \cdot \cos x - 1 > 0 \text{ e } 1 + \cos x > 0) \Rightarrow \cos x > \frac{1}{2}$ (A)

$$\log_{\cos x} (2 \cos x - 1)(1 + \cos x) > \log_{\cos x} \cos x \Rightarrow$$

$$\Rightarrow 2 \cos^2 x + \cos x - \cos x - 1 < 0 \Rightarrow$$

$$\Rightarrow 2 \cdot \cos^2 x - 1 < 0 \Rightarrow -\frac{\sqrt{2}}{2} < \cos x < \frac{\sqrt{2}}{2}$$
 (B).

De (A) e (B) vem $\frac{1}{2} < \cos x < \frac{\sqrt{2}}{2}$, então:

$$S = \left\{ x \in \mathbb{R} \mid \frac{\pi}{4} < x < \frac{\pi}{3} \text{ ou } \frac{5\pi}{3} < x < \frac{7\pi}{4} \right\}$$

442. Uma condição necessária é $\sin x > 0$. Então: $(\sqrt{1 - \cos x})^2 < \sin^2 x \Rightarrow$
 $\Rightarrow 1 - \cos x < 1 - \cos^2 x \Rightarrow \cos^2 x - \cos x < 0 \Rightarrow 0 < \cos x < 1 \Rightarrow$
 $\Rightarrow 0 < x < \frac{\pi}{2}$

444. Fazendo $2x = y \Rightarrow \operatorname{tg} y \geq -\sqrt{3} \Rightarrow \left(2k\pi \leq y < \frac{\pi}{2} + 2k\pi \text{ ou } \right.$
 $\left. \frac{2\pi}{3} + 2k\pi \leq y < \frac{3\pi}{2} + 2k\pi \text{ ou } \frac{5\pi}{3} + 2k\pi \leq y \leq 2\pi + 2k\pi \right) \Rightarrow$
 $\Rightarrow \left(k\pi \leq x < \frac{\pi}{4} + k\pi \text{ ou } \frac{2\pi}{6} + k\pi \leq x < \frac{3\pi}{4} + k\pi \text{ ou } \right.$
 $\left. \frac{5\pi}{6} + k\pi \leq x \leq \pi + k\pi \right) \Rightarrow$
 $\Rightarrow S = \left\{ x \in \mathbb{R} \mid 0 \leq x < \frac{\pi}{4} \text{ ou } \frac{\pi}{3} \leq x < \frac{3\pi}{4} \text{ ou } \frac{5\pi}{6} \leq x < \frac{5\pi}{4} \text{ ou } \right.$
 $\left. \frac{4\pi}{3} \leq x < \frac{7\pi}{4} \text{ ou } \frac{11\pi}{6} \leq x \leq 2\pi \right\}$

445. $\operatorname{tg}^2 2x - \operatorname{tg} 2x \leq 0 \Rightarrow 0 \leq \operatorname{tg} 2x \leq 1 \Rightarrow$
 $\Rightarrow k\pi \leq 2x \leq \frac{\pi}{4} + k\pi \Rightarrow \frac{k\pi}{2} \leq x \leq \frac{\pi}{8} + \frac{k\pi}{2} \Rightarrow$
 $\Rightarrow S = \left\{ x \in \mathbb{R} \mid 0 \leq x \leq \frac{\pi}{8} \text{ ou } \frac{\pi}{2} \leq x \leq \frac{5\pi}{8} \text{ ou } \pi \leq x \leq \frac{9\pi}{8} \text{ ou } \right.$
 $\left. \frac{3\pi}{2} \leq x \leq \frac{13\pi}{8} \right\}$

446. Fazendo $\operatorname{tg} 2x = t \Rightarrow t^2 - 3 < 0 \Rightarrow -\sqrt{3} < t < \sqrt{3} \Rightarrow -\sqrt{3} < \operatorname{tg} 2x < \sqrt{3}$.
 Fazendo $2x = y \Rightarrow -\sqrt{3} < \operatorname{tg} y < \sqrt{3} \Rightarrow \frac{2\pi}{3} + 2k\pi < y < \frac{4\pi}{3} + 2k\pi$ ou
 $2k\pi \leq y < \frac{\pi}{3} + 2k\pi$ ou $\frac{5\pi}{3} + 2k\pi < y < 2\pi + 2k\pi \Rightarrow$
 $\Rightarrow \frac{\pi}{3} + k\pi < x < \frac{2\pi}{3} + k\pi$ ou
 $k\pi \leq x < \frac{\pi}{6} + k\pi$ ou $\frac{5\pi}{6} + k\pi < x < \pi + k\pi \Rightarrow$

$$\Rightarrow S = \left\{ x \in \mathbb{R} \mid 0 \leq x < \frac{\pi}{6} \text{ ou } \frac{\pi}{3} < x < \frac{2\pi}{3} \text{ ou } \frac{5\pi}{6} < x < \frac{7\pi}{6} \text{ ou } \frac{4\pi}{3} < x < \frac{5\pi}{3} \text{ ou } \frac{11\pi}{6} < x \leq 2\pi \right\}$$

447. $\sin x - \cos x > 0 \Rightarrow \sin x - \sin\left(\frac{\pi}{2} - x\right) > 0 \Rightarrow \sin\left(x - \frac{\pi}{4}\right) > 0 \Rightarrow$
 $\Rightarrow 0 < x - \frac{\pi}{4} < \pi \Rightarrow \frac{\pi}{4} < x < \frac{5\pi}{4}$

448. $\cos x + \frac{\sin \frac{\pi}{3}}{\cos \frac{\pi}{3}} \cdot \sin x > \sqrt{2} \Rightarrow$

$$\Rightarrow \cos x \cdot \cos \frac{\pi}{3} + \sin \frac{\pi}{3} \cdot \sin x > \sqrt{2} \cos \frac{\pi}{3} \Rightarrow$$

$$\Rightarrow \cos\left(x - \frac{\pi}{3}\right) > \frac{\sqrt{2}}{2} \text{ fazendo } x - \frac{\pi}{3} = t \Rightarrow \cot t > \frac{\sqrt{2}}{2} \Rightarrow$$

$$\Rightarrow \left(0 \leq t < \frac{\pi}{4} \text{ ou } -\frac{\pi}{4} < t \leq 0\right). \text{ E daí: } 0 \leq x - \frac{\pi}{3} < \frac{\pi}{4} \Rightarrow$$

$$\Rightarrow \frac{\pi}{3} < x < \frac{7\pi}{12} \text{ ou } -\frac{\pi}{4} < x - \frac{\pi}{3} \leq 0 \Rightarrow \frac{\pi}{12} < x \leq \frac{\pi}{3}; \text{ portanto,}$$

$$S = \left\{ x \in \mathbb{R} \mid \frac{\pi}{12} < x < \frac{7\pi}{12} \right\}$$

449. $\pi \leq x \leq 2\pi \Rightarrow \sin x \leq 0 \Rightarrow |\cos x| \geq \sin x.$

Se $\sin x \geq 0$, a inequação equivale a $\cos^2 x \geq \sin^2 x$ e daí

$$2 \cdot \sin^2 x - 1 \leq 0, \text{ portanto } -\frac{\sqrt{2}}{2} \leq \sin x \leq \frac{\sqrt{2}}{2}.$$

Tendo em vista a hipótese, temos $0 \leq \sin x \leq \frac{\sqrt{2}}{2}$, de onde vem

$$0 \leq x \leq \frac{\pi}{4} \text{ ou } \frac{3\pi}{4} \leq x \leq \pi.$$

$$S = \left\{ x \in \mathbb{R} \mid 0 \leq x \leq \frac{\pi}{4} \text{ ou } \frac{3\pi}{4} \leq x \leq 2\pi \right\}$$

450. $\frac{2 \operatorname{tg} x \left(1 + \operatorname{tg}^2 x - \frac{1}{3}\right)}{1 + \operatorname{tg}^2 x} \leq 0 \Rightarrow \operatorname{tg} x \leq 0 \Rightarrow$

$$\Rightarrow \frac{\pi}{2} < x \leq \pi \text{ ou } \frac{3\pi}{2} < x \leq 2\pi$$

451. $\text{sen}^2 x - \frac{1}{4} \geq 0 \Rightarrow \text{sen } x \geq \frac{1}{2} \text{ ou } \text{sen } x \leq -\frac{1}{2} \Rightarrow \frac{\pi}{6} \leq x \leq \frac{5\pi}{6} \text{ ou } \frac{7\pi}{6} \leq x \leq \frac{11\pi}{6}$

452. $3^{2 \text{sen } x - 1} \geq 3^0 \Rightarrow 2 \text{sen } x - 1 \geq 0 \Rightarrow \text{sen } x \geq \frac{1}{2} \Rightarrow \frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$

453. a) $2 \text{sen } x - 1 > 0 \Rightarrow \text{sen } x > \frac{1}{2} \Rightarrow \frac{\pi}{6} < x < \frac{5\pi}{6}$

b) $\log_2 (2 \cdot \text{sen } x - 1) = \frac{1}{2} \cdot \log_2 (3 \text{sen}^2 x - 4 \cdot \text{sen } x + 2)$

$\log_2 (2 \cdot \text{sen } x - 1)^2 = \log_2 (3 \cdot \text{sen}^2 x - 4 \cdot \text{sen } x + 2)$

Então:

$(2 \cdot \text{sen } x - 1)^2 = 3 \cdot \text{sen}^2 x - 4 \cdot \text{sen } x + 2 \Rightarrow$

$\Rightarrow \text{sen}^2 x = 1 \Rightarrow \text{sen } x = \pm 1$

Levando em conta a parte a), resulta $\text{sen } x = 1 \Rightarrow x = \frac{\pi}{2}$.

454. $\cos^2 x - \frac{3}{4} > 0 \Rightarrow -\frac{\sqrt{3}}{2} < \cos x < \frac{\sqrt{3}}{2} \Rightarrow \frac{\pi}{6} < x < \frac{5\pi}{6} \text{ ou } \frac{7\pi}{6} < x < \frac{11\pi}{6}$

455. Fazendo $\cos x = y \Rightarrow \frac{2y^2 + y - 1}{y - 1} > 0 \Rightarrow -1 < y < \frac{1}{2} \text{ ou } y > 1$
e daí $-1 < \cos x < \frac{1}{2} \Rightarrow \frac{\pi}{3} < x < \pi$.

456. $\frac{4 \text{sen}^2 x - 1}{\cos x} \geq 0 \Rightarrow \frac{4 - 4 \cos^2 x - 1}{\cos x} \geq 0$

Fazendo $\cos x = y \Rightarrow \frac{3 - 4y^2}{y} \geq 0 \Rightarrow \left(y \leq -\frac{\sqrt{3}}{2} \text{ ou } 0 < y \leq \frac{\sqrt{3}}{2} \right) \Rightarrow$

$\Rightarrow \left(\cos x \leq -\frac{\sqrt{3}}{2} \text{ ou } 0 < \cos x \leq \frac{\sqrt{3}}{2} \right) \Rightarrow$

$\Rightarrow \frac{\pi}{6} \leq x < \frac{\pi}{2} \text{ ou } \frac{5\pi}{6} \leq x \leq \frac{7\pi}{6} \text{ ou } \frac{3\pi}{2} < x \leq \frac{11\pi}{6}$

457. $x^2 + x + \left(\text{tg } \alpha - \frac{3}{4} \right) > 0, \forall x \Rightarrow \Delta < 0 \Rightarrow 1 - 4 \text{tg } \alpha + 3 < 0 \Rightarrow \text{tg } \alpha > 1 \Rightarrow \frac{\pi}{4} < \alpha < \frac{\pi}{2}$

458. $\text{sen}^2 x - 2 \text{sen } x < 0 \Rightarrow 0 < \text{sen } x < 2 \Rightarrow 0 < x < \pi$

459. $\text{sen}^2 x + \text{cos}^2 x + 2 \text{sen } x \text{cos } x > 1 \Rightarrow \text{sen } 2x > 0$

Fazendo $2x = t$, temos:

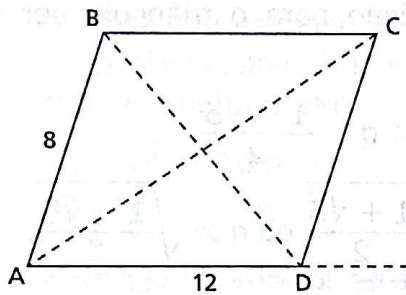
$\text{sen } t > 0 \Rightarrow 2k\pi < t < \pi + 2k\pi \Rightarrow k\pi < x < \frac{\pi}{2} + k\pi \Rightarrow$

$\Rightarrow 0 < x < \frac{\pi}{2} \text{ ou } \pi < x < \frac{3\pi}{2}$

Apêndice B — Trigonometria em triângulos quaisquer

461. $c^2 = 4^2 + (3\sqrt{2})^2 - 2 \cdot 4 \cdot 3\sqrt{2} \cdot \text{cos } 45^\circ \Rightarrow c = \sqrt{10}$

462.



$BD = a \text{ e } AC = b$

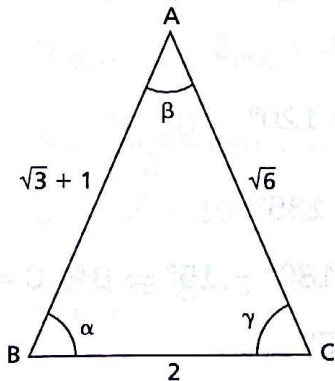
$a^2 = 8^2 + 12^2 - 2 \cdot 8 \cdot 12 \cdot \text{cos } 60^\circ \Rightarrow$

$\Rightarrow a = 4\sqrt{7} \text{ m}$

$b^2 = 8^2 + 12^2 - 2 \cdot 8 \cdot 12 \cdot \text{cos } 120^\circ \Rightarrow$

$\Rightarrow b = 4\sqrt{19} \text{ m}$

463.



$2^2 = (\sqrt{6})^2 + (\sqrt{3} + 1)^2 - 2(\sqrt{6})(\sqrt{3} + 1) \text{cos } \beta \Rightarrow$

$\Rightarrow \text{cos } \beta = \frac{\sqrt{2}}{2} \Rightarrow \beta = 45^\circ$

$(\sqrt{6})^2 = (\sqrt{3} + 1)^2 + 2^2 - 2 \cdot 2(\sqrt{3} + 1) \text{cos } \alpha \Rightarrow$

$\Rightarrow \text{cos } \alpha = \frac{1}{2} \Rightarrow \alpha = 60^\circ$

$\alpha + \beta + \gamma = 180^\circ \Rightarrow \gamma = 75^\circ$

464. $a, b, c \in \mathbb{Q} \Rightarrow a^2, b^2, c^2 \in \mathbb{Q} \Rightarrow (a^2 + c^2 - b^2) \in \mathbb{Q}$

$a, c \in \mathbb{Q} \Rightarrow 2ac \in \mathbb{Q}$

$\frac{a^2 + c^2 - b^2}{2ac} \in \mathbb{Q} \text{ e } \text{cos } \beta = \frac{a^2 + c^2 - b^2}{2ac} \Rightarrow \text{cos } \beta \in \mathbb{Q}$

465. $(x^2 + x + 1)^2 = (x^2 - 1)^2 + (2x + 1)^2 - 2(x^2 - 1)(2x + 1) \cdot \text{cos } \beta \Rightarrow$

$\Rightarrow \text{cos } \beta = \frac{2x^3 + x^2 - 2x - 1}{-4x^3 - 2x^2 + 4x + 2} = \frac{1 \cdot (2x^3 + x^2 - 2x - 1)}{-2 \cdot (2x^3 + x^2 - 2x - 1)} = -\frac{1}{2} \Rightarrow$

$\Rightarrow \beta = 120^\circ$

466. $a^2 = c^2 + 1 - 2c \cos 120^\circ \Rightarrow a^2 - c^2 - c = 1 \Rightarrow (2c)^2 - c^2 - c = 1 \Rightarrow$
 $\Rightarrow c = \frac{1 + \sqrt{13}}{6}$

- 468.** a) $17^2 = 15^2 + 8^2 \Rightarrow$ O triângulo é retângulo.
 b) $10^2 > 5^2 + 6^2 \Rightarrow$ O triângulo é obtusângulo.
 c) $8^2 < 6^2 + 7^2 \Rightarrow$ O triângulo é acutângulo.

469. Chamando as medidas dos lados de a, aq, aq^2 , só falta impor duas condições:

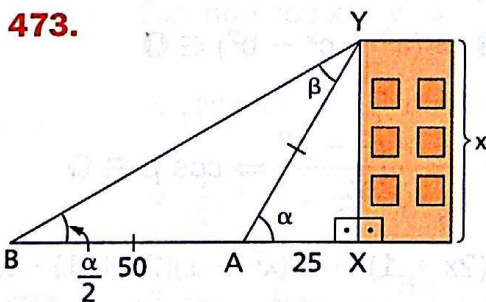
- (1) o maior lado é menor que a soma dos outros dois (condição para existência do triângulo): $aq^2 < a + aq$;
 (2) o quadrado do maior lado é maior que a soma dos quadrados dos outros dois (condição para o triângulo ser obtusângulo): $(aq^2)^2 > a^2 + (aq)^2$.

De (1) resulta $\frac{1 - \sqrt{5}}{2} < q < \frac{1 + \sqrt{5}}{2}$.

De (2) resulta $q < -\sqrt{\frac{1 + \sqrt{5}}{2}}$ ou $q > \sqrt{\frac{1 + \sqrt{5}}{2}}$.

Como $q > 0$, temos $\sqrt{\frac{1 + \sqrt{5}}{2}} < q < \frac{1 - \sqrt{5}}{2}$.

472. $\sin \hat{B} = \frac{\sqrt{3}}{2} \Rightarrow \hat{B} = 60^\circ$ ou $\hat{B} = 120^\circ$
 $\sin \hat{C} = \frac{\sqrt{2}}{2} \Rightarrow \hat{C} = 45^\circ$ ou $\hat{C} = 135^\circ$
 $A + B + C = 180^\circ \Rightarrow B + C = 180^\circ - 15^\circ \Rightarrow B + C = 165^\circ$
 $\Rightarrow \hat{B} = 120^\circ$ e $\hat{C} = 45^\circ$ e $\hat{A} = 15^\circ$



$\alpha = \frac{\alpha}{2} + \beta$ (ângulo externo ao $\triangle ABY$) \Rightarrow

$\Rightarrow \beta = \frac{\alpha}{2}$

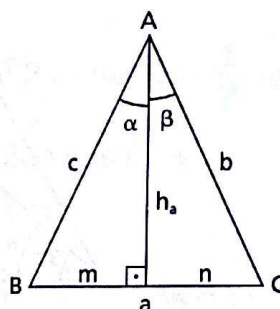
\therefore o triângulo ABY é isósceles \Rightarrow

$\Rightarrow BA = AY = 50$ m

No $\triangle AXY$ temos $XY^2 = AY^2 - AX^2 =$
 $= 50^2 - 25^2$

$XY = 25\sqrt{3}$ m

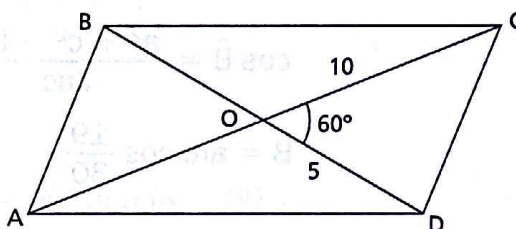
$$\begin{aligned}
 474. \quad & \left. \begin{aligned} \operatorname{tg} \alpha &= \frac{m}{h_a} \Rightarrow h_a = \frac{m}{\operatorname{tg} \alpha} \\ \operatorname{tg} \beta &= \frac{n}{h_a} \Rightarrow h_a = \frac{n}{\operatorname{tg} \beta} \end{aligned} \right\} \Rightarrow \frac{m}{\operatorname{tg} \alpha} = \frac{n}{\operatorname{tg} \beta} \Rightarrow \\
 & \Rightarrow \frac{m+n}{\operatorname{tg} \alpha + \operatorname{tg} \beta} = \frac{m}{\operatorname{tg} \alpha} \Rightarrow \frac{a}{\operatorname{tg} \alpha + \operatorname{tg} \beta} = \frac{m}{\operatorname{tg} \alpha} \Rightarrow \\
 & \Rightarrow a = \frac{m}{\operatorname{tg} \alpha} \cdot (\operatorname{tg} \alpha + \operatorname{tg} \beta) \Rightarrow a = h_a (\operatorname{tg} \alpha + \operatorname{tg} \beta)
 \end{aligned}$$



$$475. \quad S = \frac{4 \cdot 7 \operatorname{sen} 60^\circ}{2} \Rightarrow S = 7\sqrt{3} \text{ m}^2$$

476. Sabe-se que as diagonais de um paralelogramo dividem-se mutuamente ao meio, então:

$$\begin{aligned}
 \overline{AO} &= \overline{OC} = 10 \text{ m e} \\
 \overline{BO} &= \overline{OD} = 5 \text{ m}
 \end{aligned}$$



Além disso, as diagonais dividem o paralelogramo em quatro triângulos de áreas iguais, então:

$$S_{ABCD} = 4 \cdot S_{DOC} = 4 \cdot \frac{\overline{DO} \cdot \overline{OC} \cdot \operatorname{sen} \hat{D}OC}{2} = 4 \cdot \frac{5 \cdot 10 \cdot \sqrt{3}}{45} = 50\sqrt{3}$$

$$477. \quad S = \frac{8 \cdot 10}{2} \operatorname{sen} \alpha \Rightarrow \operatorname{sen} \alpha = \frac{1}{2} \Rightarrow \alpha = 30^\circ,$$

$$a^2 = 8^2 + 10^2 - 2 \cdot 8 \cdot 10 \cdot \cos 30^\circ \Rightarrow a = 2\sqrt{41 - 20\sqrt{3}},$$

$$\frac{a}{\operatorname{sen} \alpha} = 2R \Rightarrow \frac{2\sqrt{41 - 20\sqrt{3}}}{\frac{1}{2}} = 2R \Rightarrow R = 2\sqrt{41 - 20\sqrt{3}} \text{ (m)}$$

$$\begin{aligned}
 478. \quad & 7^2 = c^2 + 8^2 - 2 \cdot 8 \cdot c \cdot \cos 60^\circ \Rightarrow c^2 - 8c + 15 = 0 \Rightarrow c = 5 \text{ ou } c = 3 \\
 & c = 5 \text{ m} \Rightarrow S = \frac{5 \cdot 8}{2} \operatorname{sen} 60^\circ \Rightarrow S = 10\sqrt{3} \text{ m}^2 \text{ ou } c = 3 \text{ m} \Rightarrow S = 6\sqrt{3} \text{ m}^2
 \end{aligned}$$

$$479. \quad A + B + C = 180^\circ \Rightarrow A + B = 180^\circ - C \text{ ou } B + C = 180^\circ - A,$$

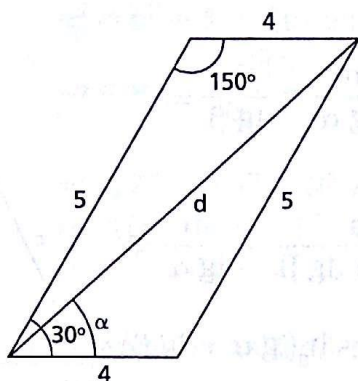
$$\cos(180^\circ - C) = -\cos C \Rightarrow \cos(A + B) = \cos(180^\circ - C) = \frac{1}{2} \Rightarrow$$

$$\Rightarrow \cos C = -\frac{1}{2} \Rightarrow C = 120^\circ$$

$$\operatorname{sen}(B + C) = \operatorname{sen}(180^\circ - A) = \operatorname{sen} A \Rightarrow \operatorname{sen} A = \frac{1}{2} \Rightarrow A = 30^\circ$$

$$B = 180^\circ - (A + C) = 30^\circ$$

480.



$$d^2 = 4^2 + 5^2 - 2 \cdot 4 \cdot 5 \cdot \cos 150^\circ = 41 + 20\sqrt{3}$$

$$\frac{5}{\sin \alpha} = \frac{\sqrt{41 + 20\sqrt{3}}}{\sin 150^\circ} \Rightarrow$$

$$\Rightarrow \sin \alpha = \frac{5}{2\sqrt{41 + 20\sqrt{3}}}$$

$$\alpha = \arcsin \frac{5}{2\sqrt{41 + 20\sqrt{3}}}$$

482.

$$\frac{a}{5} = \frac{b}{7} = \frac{c}{9} = k \Rightarrow a = 5k, b = 7k, c = 9k$$

$$\cos \hat{B} = \frac{a^2 + c^2 - b^2}{2ac} = \frac{25k^2 + 81k^2 - 49k^2}{2(5k)(9k)} = \frac{57}{90} = \frac{19}{30}$$

$$\hat{B} = \arccos \frac{19}{30}$$

483.

$$\sin 15^\circ = \sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \sin 30^\circ \cos 45^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\frac{1}{\sin 15^\circ} = \frac{\sqrt{3} + 1}{\sin \hat{B}} \Rightarrow \sin \hat{B} = \frac{\sqrt{2}}{2} \Rightarrow \hat{B} = 45^\circ \text{ ou } \hat{B} = 135^\circ$$

$$\hat{A} + \hat{B} + \hat{C} = 180^\circ \Rightarrow \hat{B} + \hat{C} = 180^\circ - 15^\circ = 165^\circ \Rightarrow$$

$$\Rightarrow (\hat{C} = 120^\circ \text{ e } \hat{B} = 45^\circ) \text{ ou } (\hat{C} = 30^\circ \text{ e } \hat{B} = 135^\circ)$$

484.

$$c^2 = (2b)^2 + b^2 - 2 \cdot 2b \cdot b \cdot \cos 60^\circ \Rightarrow c^2 = 3b^2 \Rightarrow c = b\sqrt{3}$$

$$\frac{c}{\sin 60^\circ} = \frac{b}{\sin \hat{B}} \Rightarrow \sin \hat{B} = \frac{1}{2} \Rightarrow \hat{B} = 30^\circ \text{ (pois } \hat{B} < 120^\circ)$$

$$\hat{A} = 180^\circ - (\hat{B} + \hat{C}) = 90^\circ$$

485.

$$\hat{B} = 180^\circ - (\hat{A} + \hat{C}) = 180^\circ - 3\hat{A} \Rightarrow \sin \hat{B} = \sin(180^\circ - 3\hat{A}) = \sin 3\hat{A}$$

$$\frac{b}{\sin \hat{B}} = \frac{c}{\sin \hat{C}} \Rightarrow \frac{b}{c} = \frac{\sin 3\hat{A}}{\sin 2\hat{A}} = \frac{3 \sin \hat{A} - 4 \sin^3 \hat{A}}{2 \sin \hat{A} \cdot \cos \hat{A}} = \frac{3 - 4 \sin^2 \hat{A}}{2 \cos \hat{A}} \Rightarrow$$

$$\Rightarrow \left(\frac{2}{\sqrt{3}}\right)^2 = \left(\frac{3 - 4 \sin^2 \hat{A}}{2 \cos \hat{A}}\right)^2 \Rightarrow 48 \sin^4 \hat{A} - 56 \sin^2 \hat{A} + 11 = 0 \Rightarrow$$

$$\Rightarrow \sin \hat{A} = \frac{1}{2} \Rightarrow \hat{A} = 30^\circ, \text{ então } \hat{C} = 60^\circ \text{ e } \hat{B} = 90^\circ$$

486. $\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}} \Rightarrow \frac{6}{\sin 3\hat{B}} = \frac{3}{\sin \hat{B}} \Rightarrow$
 $\Rightarrow 2 = \frac{\sin 3\hat{B}}{\sin \hat{B}} = \frac{3 \sin \hat{B} - 4 \sin^3 \hat{B}}{\sin \hat{B}} = 3 - 4 \sin^2 \hat{B} \Rightarrow \sin \hat{B} = \frac{1}{2} \Rightarrow$
 $\Rightarrow \hat{B} = 30^\circ, \hat{A} = 90^\circ \text{ e } \hat{C} = 60^\circ, c^2 = b^2 + a^2 - 2ab \cos \hat{C} \Rightarrow$
 $\Rightarrow c^2 = 3^2 + 6^2 - 2 \cdot 3 \cdot 6 \cdot \cos 60^\circ \Rightarrow c = 3\sqrt{3} \text{ m}$

487. $\hat{A} + \hat{B} + \hat{C} = 180^\circ \Rightarrow \hat{B} + \hat{C} = 180^\circ - \hat{A} \Rightarrow \text{tg}(\hat{B} + \hat{C}) = -\text{tg} \hat{A} \Rightarrow$
 $\Rightarrow \frac{\text{tg} \hat{B} + \text{tg} \hat{C}}{1 - \text{tg} \hat{B} \cdot \text{tg} \hat{C}} = -\text{tg} \hat{A} \Rightarrow \frac{2 \cdot \text{tg} \hat{A}}{1 - \text{tg} \hat{B} \cdot \text{tg} \hat{C}} = -\text{tg} \hat{A} \Rightarrow$
 $\Rightarrow 2 = -1 + \text{tg} \hat{B} \cdot \text{tg} \hat{C} \Rightarrow \text{tg} \hat{B} \cdot \text{tg} \hat{C} = 3$

488. $S = \frac{3 \cdot 4 \cdot \sin \alpha}{2} = 6 \cdot \sin \alpha$
 $S - 3 = \frac{3 \cdot 4 \cdot \sin(\alpha - 60^\circ)}{2} = 6 \cdot \sin(\alpha - 60^\circ)$

Então:

$$6 \cdot \sin \alpha - 3 = 6 \cdot \sin(\alpha - 60^\circ) \Rightarrow \sin \alpha - \sin(\alpha - 60^\circ) = \frac{1}{2} \Rightarrow$$

$$\Rightarrow 2 \cdot \sin 30^\circ \cdot \cos(\alpha - 30^\circ) = \frac{1}{2} \Rightarrow \cos(\alpha - 30^\circ) = \frac{1}{2} \Rightarrow$$

$$\Rightarrow \alpha - 30^\circ = 60^\circ \Rightarrow \alpha = 90^\circ \Rightarrow S = 6 \cdot \sin \alpha = 6 \text{ m}^2$$

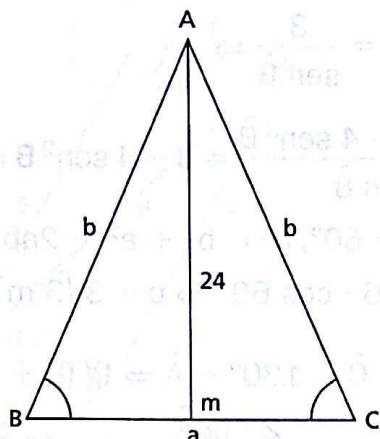
Apêndice C — Resolução de triângulos

490. $a^2 = b^2 + c^2 \Rightarrow a^2 - c^2 = 3^2 \Rightarrow (a + c)(a - c) = 9 \Rightarrow$
 $\Rightarrow (a + c)\sqrt{3} = 9 \Rightarrow a + c = 3\sqrt{3} \quad (1)$
 É dado que $a - c = \sqrt{3} \quad (2)$. De (1) e (2) resulta $a = 2\sqrt{3}$ e $c = \sqrt{3}$,
 então $\sin \hat{B} = \frac{b}{a} = \frac{\sqrt{3}}{2}$ e $\hat{B} = 60^\circ$.

491. Do sistema $a + b = 18$, $a + c = 25$ e $b^2 + c^2 = a^2$, resulta
 $(18 - a)^2 + (25 - a)^2 = a^2$ e daí $a = 13$, portanto
 $b = 18 - a = 5$ e $c = 25 - a = 12$.

Finalmente $\sin \hat{B} = \frac{b}{a} = \frac{5}{13}$ e $\sin \hat{C} = \frac{c}{a} = \frac{12}{13}$.

492.



$$2b + a = 64 \quad (1)$$

$$b^2 = \left(\frac{a}{2}\right)^2 + 24^2 \quad (2)$$

De (1) $a = 64 - 2b$, que substituído em (2) dá $b^2 = (32 - b)^2 + 576$ e daí $b = 25$.

$$(1) \quad a = 64 - 2b = 14$$

$$\cos \hat{B} = \cos \hat{C} = \frac{\frac{a}{2}}{b} = \frac{7}{25}$$

$$\sin \frac{\hat{A}}{2} = \frac{\frac{a}{2}}{b} = \frac{7}{25}$$

493.

$$b = 1, c = \operatorname{tg} \varphi \Rightarrow a^2 + b^2 = c^2 = 1 + \operatorname{tg}^2 \varphi = \sec^2 \varphi \Rightarrow a = \sec \varphi$$

$$\operatorname{tg} \hat{B} = \frac{b}{c} = \frac{1}{\operatorname{tg} \varphi} = \operatorname{cotg} \varphi$$

$$\operatorname{tg} \hat{C} = \frac{c}{b} = \operatorname{tg} \varphi$$

494.

$$\text{lad os: } a, b = a + 1, c = a + 2$$

$$\frac{a}{\operatorname{sen} \hat{A}} = \frac{c}{\operatorname{sen} \hat{C}} \Rightarrow \frac{a}{\operatorname{sen} \hat{A}} = \frac{a + 2}{\operatorname{sen} 2\hat{A}} \Rightarrow \cos \hat{A} = \frac{a + 2}{2a}$$

$$a^2 = (a + 1)^2 + (a + 2)^2 - 2(a + 1)(a + 2) \cdot \frac{a + 2}{2a} \Rightarrow a^2 - 3a - 4 = 0 \Rightarrow$$

$$\Rightarrow a = 4 \text{ e daí: } b = 5, c = 6, \cos \hat{A} = \frac{3}{4}, \hat{C} = 2 \cdot \operatorname{arc} \operatorname{sen} \frac{\sqrt{7}}{4}$$

495.

$$\frac{(a + b + c) \cdot r}{2} = \frac{a \cdot h}{2} \Rightarrow h = \frac{2r^2 + 2ra}{a} \Rightarrow h = \frac{12}{5}$$

$$(a + b + c) \cdot r = a \cdot h \Rightarrow b + c = 7 \quad (A), b \cdot c = a \cdot h \Rightarrow b \cdot c = 12 \quad (B)$$

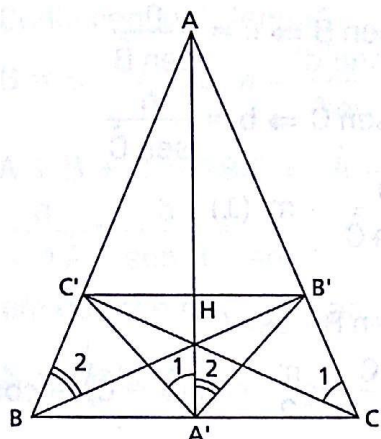
De (A) e (B) vem $b^2 - 7b + 12 = 0 \Rightarrow b = 4 \text{ e } c = 3 \text{ ou } b = 3 \text{ e } c = 4$.

$$\cos \hat{A} = \frac{9 + 16 - 25}{24} = 0 \Rightarrow \hat{A} = 90^\circ, \operatorname{sen} \hat{B} = \frac{3}{5} \Rightarrow \hat{B} = \operatorname{arc} \operatorname{sen} \frac{3}{5} \text{ e}$$

$$\cos \hat{C} = \frac{3}{5} \Rightarrow \hat{C} = \operatorname{arc} \cos \frac{3}{5} \text{ ou } \operatorname{sen} \hat{B} = \frac{4}{5} \Rightarrow \hat{B} = \operatorname{arc} \operatorname{sen} \frac{4}{5} \text{ e}$$

$$\cos \hat{C} = \frac{3}{4} \Rightarrow \hat{C} = \operatorname{arc} \operatorname{sen} \frac{3}{4}$$

496.



Seja H o ponto em que as alturas AA' , BB' e CC' se interceptam. Os quadriláteros $HA'CB'$ e $HA'BC'$ são inscritíveis porque têm dois ângulos opostos retos e, portanto, suplementares, então:

$$\hat{A}'_1 \equiv \hat{C}'_1 \equiv 90^\circ - \hat{A} \text{ e } \hat{A}'_2 \equiv \hat{B}'_2 \equiv 90^\circ - \hat{A}.$$

Chamando de \hat{A}' , \hat{B}' e \hat{C}' os ângulos do triângulo $A'B'C'$, obtemos:

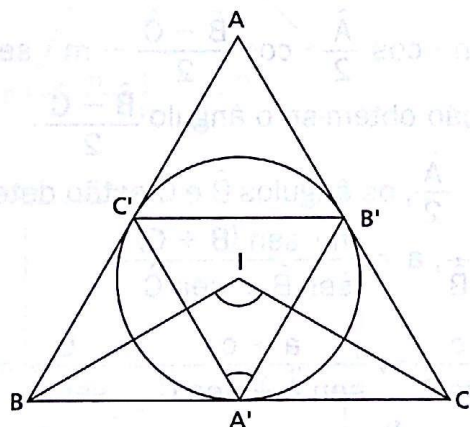
$$\hat{A}' \equiv \hat{A}'_1 + \hat{A}'_2 \equiv 180^\circ - 2\hat{A}, \hat{B}' \equiv 180^\circ - 2\hat{B} \text{ e } \hat{C}' \equiv 180^\circ - 2\hat{C}.$$

Aplicando a lei dos senos ao triângulo $A'B'C'$, temos:

$$\frac{A'B'}{\text{sen } \hat{C}} = \frac{B'C'}{\text{sen } \hat{A}'} = \frac{a \cos C}{\text{sen } A} \Rightarrow A'B' = \frac{a \text{ sen } C \cos C}{\text{sen } A}.$$

$$\text{Analogamente: } B'C' = \frac{c \text{ sen } \hat{A} \cos \hat{A}}{\text{sen } B} \text{ e } A'C' = \frac{b \text{ sen } \hat{B} \cos \hat{B}}{\text{sen } C}.$$

497.



Seja I o centro da circunferência inscrita em ABC . Ligando I com B e com C , temos:

$$\hat{A}' \equiv 180^\circ - \hat{B}IC \equiv \frac{\hat{B} + \hat{C}}{2}.$$

Analogamente:

$$\hat{B}' \equiv 180^\circ - \hat{A}IC \equiv \frac{\hat{A} + \hat{C}}{2};$$

$$\hat{C}' \equiv 180^\circ - \hat{A}IB \equiv \frac{\hat{A} + \hat{B}}{2}.$$

O triângulo $CA'B'$ é isósceles ($CA' \equiv CB'$), então

$$c' = A'B' = 2 \cdot CA' \cdot \text{sen } \frac{\hat{C}}{2} = 2(p - c) \text{ sen } \frac{\hat{C}}{2} \text{ em que } p = \frac{a + b + c}{2}.$$

Analogamente, temos:

$$b' = 2(p - b) \text{ sen } \frac{\hat{B}}{2} \text{ e } a' = 2(p - a) \text{ sen } \frac{\hat{A}}{2}.$$

498.

$$\cos \hat{A} = \sqrt{1 - \text{sen}^2 A} = \frac{41}{50}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos \hat{A} \Rightarrow 9 = b^2 + (10 - b)^2 - 2b(10 - b) \cdot \frac{41}{50} \Rightarrow b^2 - 10b + 25 = 0 \Rightarrow b = 5 \Rightarrow c = 5$$

Então $\hat{B} = \hat{C}$ e $\hat{A} + \hat{B} + \hat{C} = \pi$, portanto:

$$\hat{B} = \hat{C} = \frac{\pi}{2} - \frac{\hat{A}}{2} = \frac{\pi}{2} - \frac{1}{2} \cdot \text{arc sen } \frac{3\sqrt{91}}{50}.$$

499. $h_a = c \cdot \text{sen } \hat{B} \Rightarrow n = c \cdot \text{sen } \hat{B} \Rightarrow c = \frac{n}{\text{sen } \hat{B}}$

$h_a = b \cdot \text{sen } \hat{C} \Rightarrow n = b \cdot \text{sen } \hat{C} \Rightarrow b = \frac{n}{\text{sen } \hat{C}}$

$b + c = m \Rightarrow \frac{n}{\text{sen } \hat{B}} + \frac{n}{\text{sen } \hat{C}} = m \quad (1)$

De (1) vem:

$n(\text{sen } \hat{B} + \text{sen } \hat{C}) = m \cdot \text{sen } \hat{B} \cdot \text{sen } \hat{C}$

$2n \cdot \text{sen } \frac{\hat{B} + \hat{C}}{2} \cdot \cos \frac{\hat{B} - \hat{C}}{2} = \frac{m}{2} [\cos(\hat{B} - \hat{C}) - \cos(\hat{B} + \hat{C})] \quad (2)$

Notemos que:

$\cos(\hat{B} + \hat{C}) = -\cos \hat{A}$ (dado)

$\text{sen } \frac{\hat{B} + \hat{C}}{2} = \cos \frac{\hat{A}}{2}$ (calculável a partir de $\cos \hat{A}$)

$\cos(\hat{B} - \hat{C}) = 2 \cdot \cos^2 \frac{\hat{B} - \hat{C}}{2} - 1$

Então a equação (2) fica:

$m \cdot \cos^2 \frac{\hat{B} - \hat{C}}{2} - 2n \cdot \cos \frac{\hat{A}}{2} \cdot \cos \frac{\hat{B} - \hat{C}}{2} - m \cdot \text{sen}^2 \frac{\hat{A}}{2} = 0$

A partir dessa equação obtém-se o ângulo $\frac{\hat{B} - \hat{C}}{2}$.

Como $\frac{\hat{B} + \hat{C}}{2} = \frac{\pi}{2} - \frac{\hat{A}}{2}$, os ângulos \hat{B} e \hat{C} estão determinados e daí

$b = \frac{n}{\text{sen } \hat{C}}, c = \frac{n}{\text{sen } \hat{B}}, a = \frac{m \cdot \text{sen}(\hat{B} + \hat{C})}{\text{sen } \hat{B} + \text{sen } \hat{C}}$.

500. $\frac{a}{\text{sen } \hat{A}} = \frac{b}{\text{sen } \hat{B}} = \frac{c}{\text{sen } \hat{C}} \Rightarrow \frac{a + c}{\text{sen } \hat{A} + \text{sen } \hat{C}} = \frac{b}{\text{sen } \hat{B}} \Rightarrow$

$\Rightarrow \frac{b}{\text{sen}(\hat{A} + \hat{C})} = \frac{k}{\text{sen } \hat{A} + \text{sen } \hat{C}} \Rightarrow$

$\Rightarrow \frac{b}{2 \cdot \text{sen } \frac{\hat{A} + \hat{C}}{2} \cdot \cos \frac{\hat{A} + \hat{C}}{2}} = \frac{k}{2 \cdot \text{sen } \frac{\hat{A} + \hat{C}}{2} \cdot \cos \frac{\hat{A} - \hat{C}}{2}} \Rightarrow$

$\Rightarrow \frac{b}{\cos \frac{\hat{A} + \hat{C}}{2}} = \frac{k}{\cos \frac{\hat{A} - \hat{C}}{2}} \Rightarrow$

$\Rightarrow \frac{b + k}{\cos \frac{\hat{A} + \hat{C}}{2} + \cos \frac{\hat{A} - \hat{C}}{2}} = \frac{k - b}{\cos \frac{\hat{A} - \hat{C}}{2} - \cos \frac{\hat{A} + \hat{C}}{2}} \Rightarrow$

$\Rightarrow \frac{b + k}{2 \cdot \cos \frac{\hat{A}}{2} \cdot \cos \frac{\hat{C}}{2}} = \frac{k - b}{2 \cdot \text{sen } \frac{\hat{A}}{2} \cdot \text{sen } \frac{\hat{C}}{2}} \Rightarrow \cotg \frac{\hat{C}}{2} = \frac{b + k}{k - b} \cdot \text{tg } \frac{\hat{A}}{2}$

Conhecendo \hat{C} , temos:

$$B = \pi - \hat{A} - \hat{C}, a = \frac{b \operatorname{sen} \hat{A}}{\operatorname{sen} \hat{B}} \text{ e } c = \frac{b \operatorname{sen} \hat{C}}{\operatorname{sen} \hat{B}}.$$

501. $\hat{A} + \hat{B} + \hat{C} = 180^\circ \Rightarrow \hat{A} = 180^\circ - (\hat{B} + \hat{C})$

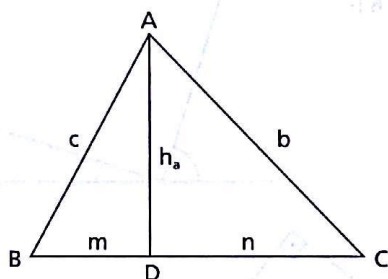
$$\frac{a}{\operatorname{sen} \hat{A}} = \frac{b}{\operatorname{sen} \hat{B}} = \frac{c}{\operatorname{sen} \hat{C}} = 2R \Rightarrow a = 2R \operatorname{sen} \hat{A}, b = 2R \operatorname{sen} \hat{B}, c = 2R \operatorname{sen} \hat{C}$$

em que R é calculado assim:

$$S = \frac{1}{2} bc \operatorname{sen} \hat{A} = \frac{1}{2} \cdot 4R^2 \cdot \operatorname{sen} \hat{A} \cdot \operatorname{sen} \hat{B} \cdot \operatorname{sen} \hat{C}, \text{ então:}$$

$$2R = \sqrt{\frac{2S}{\operatorname{sen} \hat{A} \cdot \operatorname{sen} \hat{B} \cdot \operatorname{sen} \hat{C}}}$$

502.



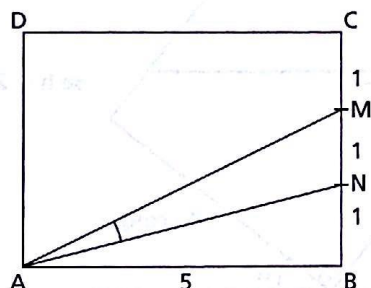
$$a = m + n = \frac{h_a}{\operatorname{tg} \hat{B}} + \frac{h_a}{\operatorname{tg} \hat{C}}$$

$$b = \frac{h_a}{\operatorname{sen} \hat{C}}$$

$$c = \frac{h_a}{\operatorname{sen} \hat{B}}$$

$$\hat{A} = 180^\circ - (\hat{B} + \hat{C})$$

503.

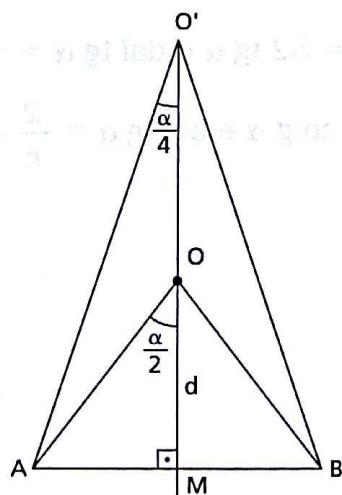


$$\operatorname{tg} \hat{MAB} = \frac{2}{5} \text{ e } \operatorname{tg} \hat{NAB} = \frac{1}{5}$$

$$\operatorname{tg} \hat{MAN} = \operatorname{tg} (\hat{MAB} - \hat{NAB}) =$$

$$= \frac{\frac{2}{5} - \frac{1}{5}}{1 + \frac{2}{5} \cdot \frac{1}{5}} = \frac{\frac{1}{5}}{\frac{25}{25}} = \frac{5}{27}$$

504.



$$\frac{AB}{2} = d \cdot \operatorname{tg} \frac{\alpha}{2} \text{ e}$$

$$\frac{AB}{2} = (d + OO') \cdot \operatorname{tg} \frac{\alpha}{4}$$

Então:

$$d \cdot \operatorname{tg} \frac{\alpha}{2} = (d + OO') \cdot \operatorname{tg} \frac{\alpha}{4}$$

$$OO' = \frac{d \left(\operatorname{tg} \frac{\alpha}{2} - \operatorname{tg} \frac{\alpha}{4} \right)}{\operatorname{tg} \frac{\alpha}{4}} = \frac{d}{\cos \frac{\alpha}{2}}$$

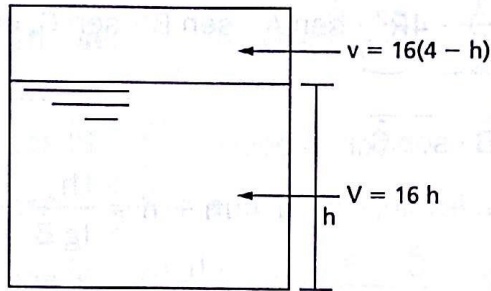
505. $\cos \theta = \frac{100 + 49 - 169}{140} = -\frac{1}{7} \Rightarrow \sec^2 \theta = 49 \Rightarrow 1 + \operatorname{tg}^2 \theta = 49 \Rightarrow$

$\Rightarrow \operatorname{tg} \theta = -4\sqrt{3} = \alpha$

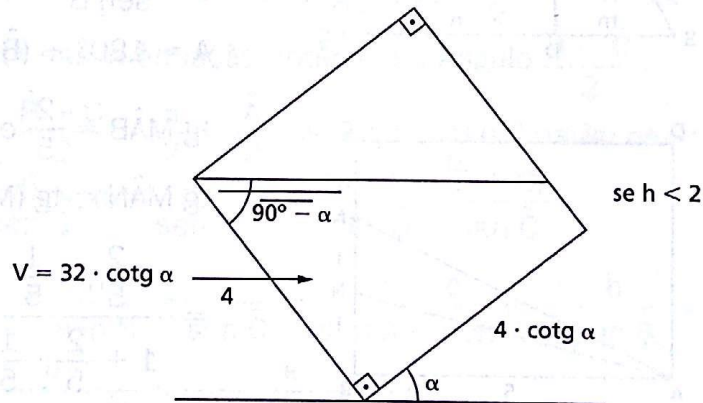
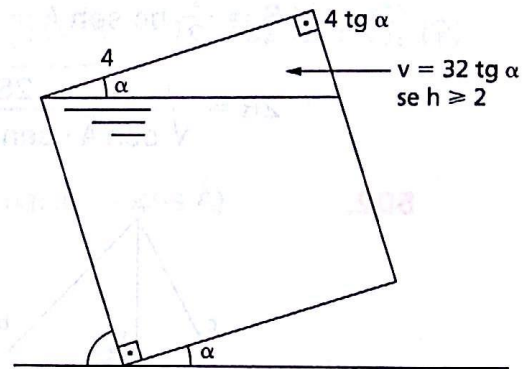
Então:

$|\sqrt{3}\alpha| = |\sqrt{3}(-4\sqrt{3})| = 12$

506. situação inicial



situação final



Se $h \geq 2$, então $16(4 - h) = 32 \operatorname{tg} \alpha$ e daí $\operatorname{tg} \alpha = \frac{4 - h}{2}$.

Se $h < 2$, então $16h = 32 \operatorname{cotg} \alpha$ e daí $\operatorname{tg} \alpha = \frac{2}{h}$.



FUNDAMENTOS DE MATEMÁTICA ELEMENTAR é uma coleção consagrada ao longo dos anos por oferecer ao estudante o mais completo conteúdo de Matemática elementar. Os volumes estão organizados da seguinte forma:

VOLUME 1	conjuntos, funções
VOLUME 2	logaritmos
VOLUME 3	trigonometria
VOLUME 4	sequências, matrizes, determinantes, sistemas
VOLUME 5	combinatória, probabilidade
VOLUME 6	complexos, polinômios, equações
VOLUME 7	geometria analítica
VOLUME 8	limites, derivadas, noções de integral
VOLUME 9	geometria plana
VOLUME 10	geometria espacial
VOLUME 11	matemática comercial, matemática financeira, estatística descritiva

A coleção atende a alunos do ensino médio que procuram uma formação mais aprofundada, estudantes em fase pré-vestibular e também universitários que necessitam rever a Matemática elementar.

Os volumes contêm teoria e exercícios de aplicação, além de uma seção de questões de vestibulares, acompanhadas de respostas. Há ainda uma série de artigos sobre história da Matemática relacionados aos temas abordados.

Na presente edição, a seção de questões de vestibulares foi atualizada, apresentando novos testes e questões dissertativas selecionados a partir dos melhores vestibulares do país.