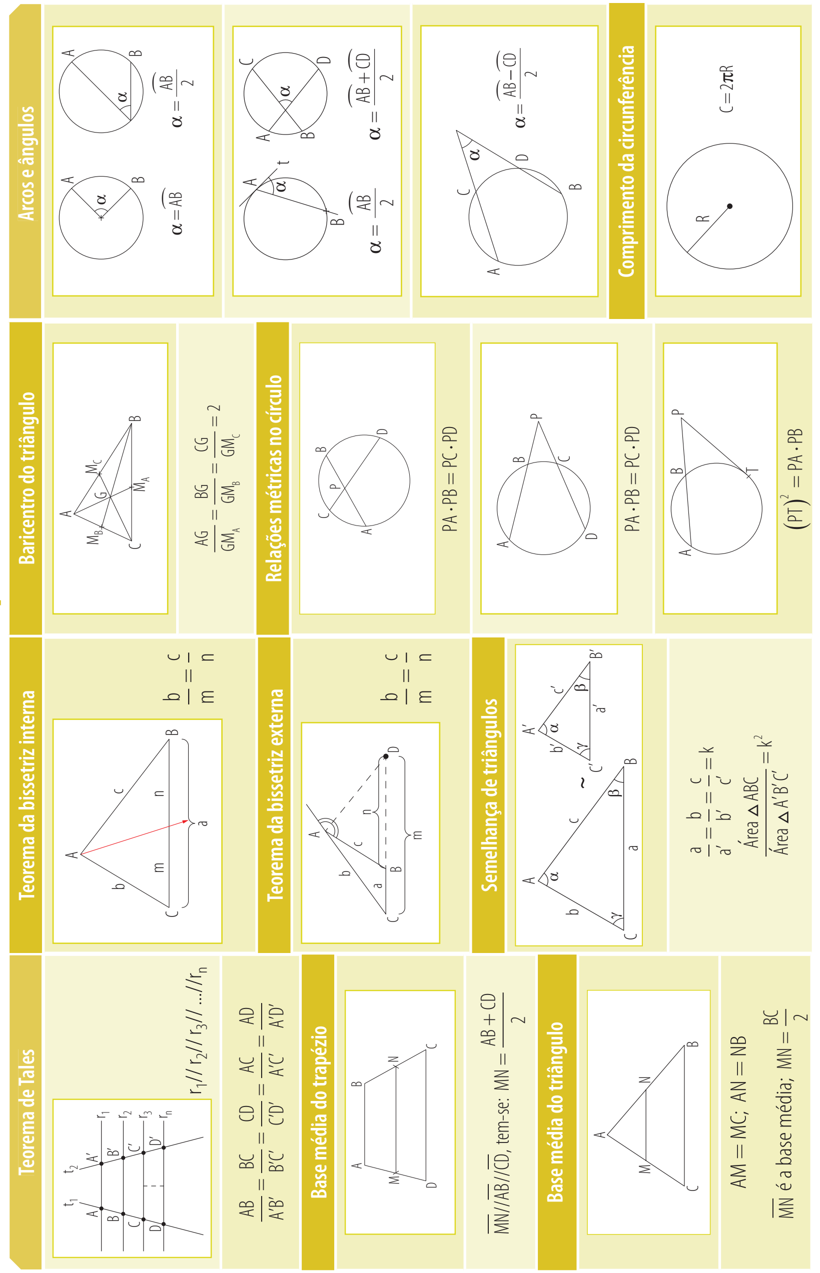
Matemática

Geometria plana



Teorema de Tales

$r_1 // r_2 // r_3 // \dots // r_n$

$$\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CD}{C'D'} = \frac{AC}{A'C'} = \frac{AD}{A'D'}$$

Base média do trapézio

$\overline{MN} // \overline{AB} // \overline{CD}$, tem-se: $MN = \frac{AB + CD}{2}$

Base média do triângulo

$AM = MC$; $AN = NB$

$\overline{MN}$ é a base média; $MN = \frac{BC}{2}$

Teorema da bissetriz interna

$$\frac{b}{m} = \frac{c}{n}$$

Teorema da bissetriz externa

$$\frac{b}{m} = \frac{c}{n}$$

Semelhança de triângulos

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'} = k$$

$$\frac{\text{Área } \triangle ABC}{\text{Área } \triangle A'B'C'} = k^2$$

Baricentro do triângulo

$$\frac{AG}{GM_A} = \frac{BG}{GM_B} = \frac{CG}{GM_C} = 2$$

Relações métricas no círculo

$PA \cdot PB = PC \cdot PD$

$PA \cdot PB = PC \cdot PD$

$(PT)^2 = PA \cdot PB$

Arcos e ângulos

$\alpha = \overset{\frown}{AB}$ $\qquad \alpha = \frac{\overset{\frown}{AB}}{2}$

$\alpha = \frac{\overset{\frown}{AB}}{2}$ $\qquad \alpha = \frac{\overset{\frown}{AB} + \overset{\frown}{CD}}{2}$

$\alpha = \frac{\overset{\frown}{AB} - \overset{\frown}{CD}}{2}$

Comprimento da circunferência

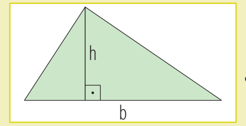
$C = 2\pi R$

Polígonos convexos

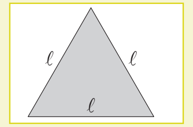
Sendo:	Tem-se:
n = número de lados d = número de diagonais S_i = soma dos ângulos internos S_e = soma dos ângulos externos	$d = \frac{n(n-3)}{2}$ $S_i = (n-2) \cdot 180°$ $S_e = 360°$

Áreas

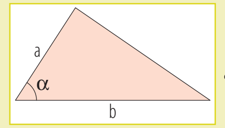
Triângulos



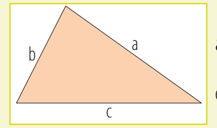
$\text{área} = \frac{bh}{2}$



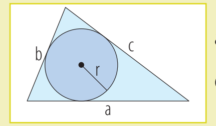
$\text{área} = \frac{\ell^2 \cdot \sqrt{3}}{4}$



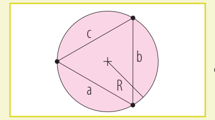
$\text{área} = \frac{1}{2} ab \cdot \text{sen}\,\alpha$



$\text{área} = \sqrt{p(p-a)\cdot(p-b)\cdot(p-c)}$, em que $p = \frac{a+b+c}{2}$

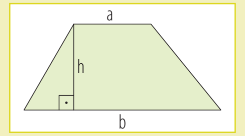


$\text{área} = pr$, em que $p = \frac{a+b+c}{2}$



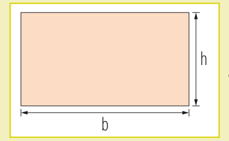
$\text{área} = \frac{abc}{4R}$

Trapézio



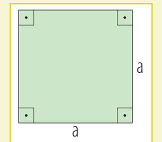
$\text{área} = \frac{(a+b)\cdot h}{2}$

Retângulo



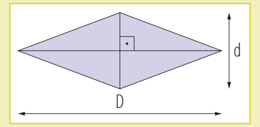
$\text{área} = b \cdot h$

Quadrado



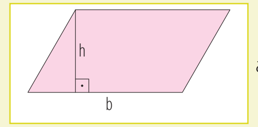
$\text{área} = a^2$

Losango



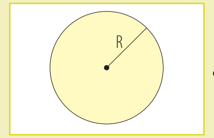
$\text{área} = \frac{D \cdot d}{2}$

Paralelogramo



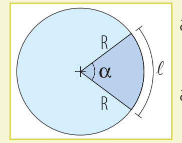
$\text{área} = b \cdot h$

Círculo



$\text{área} = \pi R^2$

Setor circular



$\text{área} = \frac{\alpha \cdot \pi \cdot R^2}{360°}$, α em graus

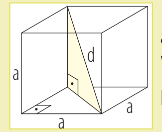
$\text{área} = \frac{\alpha \cdot R^2}{2}$, α em radianos

$\text{área} = \frac{\ell \cdot R}{2}$

Geometria espacial

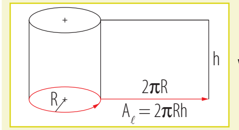
Sólidos

Cubo



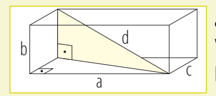
$\text{área} = 6a^2$
$\text{Volume} = a^3$
$\text{Diagonal} = d = a\sqrt{3}$

Cilindro



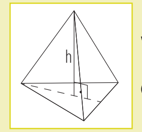
$\text{Volume} = \pi R^2 \cdot h$

Paralelepípedo reto-retângulo



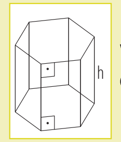
$\text{área} = 2\,(ab + ac + bc)$
$\text{Volume} = a \cdot b \cdot c$
$\text{Diagonal} = d = \sqrt{a^2 + b^2 + c^2}$

Pirâmide



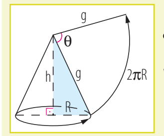
$\text{Volume} = \frac{1}{3} A_B \cdot h$, em que A_B = área da base

Prisma



$\text{Volume} = A_B \cdot h$, em que A_B = área da base

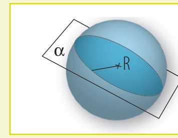
Cone



$\text{área lateral} = \pi R g$

$\text{Volume} = \frac{\pi R^2 \cdot h}{3}$ $\qquad \theta = \frac{360° \cdot R}{g}$

Esfera



$\text{área} = 4\pi R^2$

$\text{volume} = \frac{4}{3}\pi R^3$

Poliedros convexos

Relação de Euler

Sendo:	Tem-se:
V = nº de vértices A = nº de arestas F = nº de faces	$V - A + F = 2$

Geometria analítica

Equações da reta

Equação geral da reta: $ax + by + c = 0$

Equação reduzida da reta: $y = mx + n$

Feixes de retas

Por $(x_0; y_0)$: $y - y_0 = m\,(x - x_0)$, $m \in \mathbb{R}$

Paralelas: $y = mx + n$, m fixo e $n \in \mathbb{R}$

Posições relativas entre retas

$r // s \Leftrightarrow m_r = m_s$

$r \perp s \Leftrightarrow m_r \cdot m_s = -1$

Ângulo entre retas

$$\text{tg}\,\theta = \left| \frac{m_s - m_r}{1 + m_s \cdot m_r} \right|$$

Distâncias

Entre dois pontos: $d_{A;B} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

Entre ponto e reta: $d_{P;r} = \frac{|ax_P + by_P + c|}{\sqrt{a^2 + b^2}}$

Baricentro do triângulo ABC

$$G = \left(\frac{x_A + x_B + x_C}{3}; \frac{y_A + y_B + y_C}{3} \right)$$

Condição de alinhamento para três pontos

$$\begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix} = 0$$

Área do triângulo ABC

$$\text{área} = \frac{1}{2} \left\| \begin{matrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{matrix} \right\|$$

Equações da circunferência

Equação geral da circunferência:
$x^2 + y^2 - 2ax - 2by + a^2 + b^2 - r^2 = 0$

Equação reduzida da circunferência de centro (x_0, y_0):
$(x - x_0)^2 + (y - y_0)^2 = r^2$

Números complexos

$i^2 = -1$

$i^0 = 1; i^1 = i; i^2 = -1; i^3 = -i; i^4 = 1; \dots$

$z = a + bi$, $a \in \mathbb{R}$ e $b \in \mathbb{R}$

Conjugado de z

$z = a + bi \Rightarrow \bar{z} = a - bi$

Módulo de z

$|z| = \sqrt{a^2 + b^2}$

Forma trigonométrica de z

$z = |z|(\cos\theta + i \cdot \text{sen}\,\theta)$

Operações na forma trigonométrica

Sejam:

$z = |z|(\cos\theta + i \cdot \text{sen}\,\theta)$
$z_1 = |z_1|(\cos\theta_1 + i \cdot \text{sen}\,\theta_1)$
$z_2 = |z_2|(\cos\theta_2 + i \cdot \text{sen}\,\theta_2)$

$z_1 \cdot z_2 = |z_1| \cdot |z_2| \cdot [\cos(\theta_1 + \theta_2) + i \cdot \text{sen}(\theta_1 + \theta_2)]$

$\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|}[\cos(\theta_1 - \theta_2) + i \cdot \text{sen}(\theta_1 - \theta_2)]$

$z^n = |z|^n \cdot [\cos(n\theta) + i \cdot \text{sen}(n\theta)]$, $n \in \mathbb{N}$

$\sqrt[n]{z} = \sqrt[n]{|z|} \cdot \left[\cos\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right) + i \cdot \text{sen}\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right) \right]$,

$n \in \mathbb{N}$, $k \in \mathbb{N}$ e $0 \le k < n$

Matemática financeira básica

Porcentagem

$x\% = \frac{x}{100}$

$x\% \text{ do total} = \frac{x}{100} \cdot \text{total}$

Juros simples

$J = C \cdot i \cdot t$

$M = C + J = C \cdot (1 + it)$

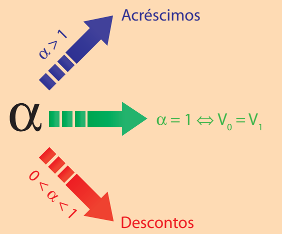
Juros compostos

$M = C \cdot (1 + i)^t$

Acréscimos e descontos

Valor inicial $\times$ fator de correção = Valor final

$V_0 \times \alpha = V_1$



Lucro e prejuízo

Lucro = Venda – Custo

Prejuízo = Custo – Venda

Potenciação e radiciação

Potenciação

$a^n = \underbrace{a \cdot a \cdot a \cdot a \cdot a \cdot \dots \cdot a}_{n \text{ fatores}}$

$a^0 = 1$

1. $a^m \cdot a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
3. $(a^m)^n = a^{m \cdot n}$
4. $(a \cdot b)^n = a^n \cdot b^n$
5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$
6. $a^{-n} = \frac{1}{a^n}, a \neq 0$

Radiciação

$\sqrt[n]{b} = a \Rightarrow a^n = b$

1. $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
2. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$
3. $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$
4. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[n \cdot m]{a}$
5. $\sqrt[n]{a^m} = a^{\frac{m}{n}}$
6. $\sqrt[n \cdot p]{a^{m \cdot p}} = \sqrt[n]{a^m}$

Produtos notáveis e fatoração

$(a+b)\cdot(a-b) = a^2 - b^2$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2\cdot(ab + ac + bc)$

$ab + ac = a\cdot(b+c)$

$ab + ac + db + dc = a\cdot(b+c) + d\cdot(b+c) = (b+c)\cdot(a+d)$

$ax^2 + bx + c = a\cdot(x - a_1)\cdot(x - a_2)$, em que a_1 e a_2 são raízes de $ax^2 + bx + c = 0$

$a^3 + b^3 = (a+b)\cdot(a^2 - ab + b^2)$

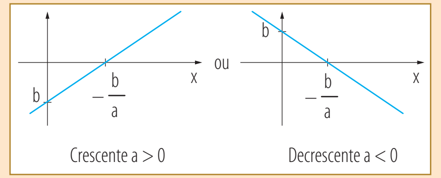
$a^3 - b^3 = (a-b)\cdot(a^2 + ab + b^2)$

ISBN 978-85-7901-619-6

POLIEDRO SISTEMA DE ENSINO

Função do 1º grau

$f(x) = ax+b \qquad (a \neq 0)$



Função do 2º grau

$f(x) = ax^2 + bx + c \qquad (a \neq 0)$

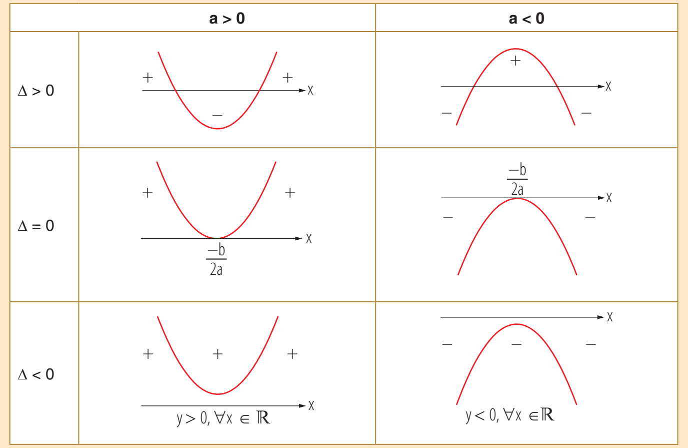
Raízes: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Soma das raízes: $\frac{-b}{a}$

Produto das raízes: $\frac{c}{a}$

Discriminante: $\Delta = b^2 - 4ac$

- $\Delta > 0$: duas raízes reais diferentes
- $\Delta = 0$: duas raízes reais iguais
- $\Delta < 0$: $\nexists$ raízes reais



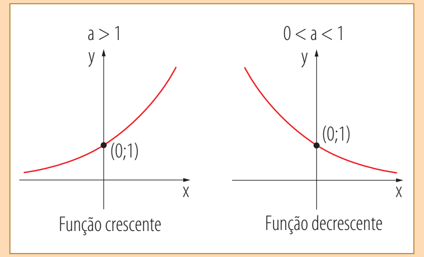
Vértice: $x_v = \frac{-b}{2a}$ e $y_v = \frac{-\Delta}{4a}$

Imagem: $\left\{ y \in \mathbb{R} \,\middle|\, y \geq \frac{-\Delta}{4a} \text{ se } a>0 \text{ ou } y \leq \frac{-\Delta}{4a} \text{ se } a<0 \right\}$ **Domínio:** $\mathbb{R}$

Função exponencial

$f(x) = a^x \quad (a > 0 \text{ e } a \neq 1)$

$Im_f = \mathbb{R}^*_+ \quad D_f = \mathbb{R}$



- $a^{f(x)} = a^{g(x)} \Leftrightarrow f(x) = g(x)$
- Se $a > 1$; $a^{f(x)} > a^{g(x)} \Leftrightarrow f(x) > g(x)$
- Se $0 < a < 1$; $a^{f(x)} > a^{g(x)} \Leftrightarrow f(x) < g(x)$

Função logarítmica

$\log_b a = x \Leftrightarrow a = b^x \quad (a > 0 \text{ e } 0 < b \neq 1)$

$\log_c(a \cdot b) = \log_c a + \log_c b;\ a > 0, b > 0, 0 < c \neq 1$

$\log_c\left(\frac{a}{b}\right) = \log_c a - \log_c b;\ a > 0,\ b > 0,\ 0 < c \neq 1$

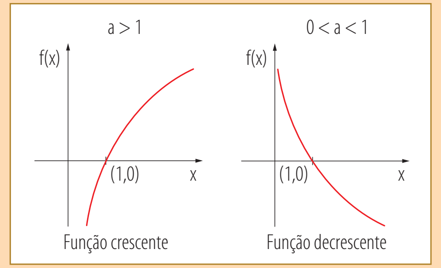
$\log_c a^m = m \cdot \log_c a;\ a > 0, 0 < c \neq 1 \text{ e } m \in \mathbb{R}$

$\log_{c^m} a = \frac{1}{m} \cdot \log_c a;\ a > 0,\ 0 < c \neq 1 \text{ e } m \in \mathbb{R}^*$

$\log_c a = \frac{\log_d a}{\log_d c};\ a > 0,\ 0 < c \neq 1 \text{ e } 0 < d \neq 1$

$f(x) = \log_a x \quad (a > 0 \text{ e } a \neq 1)$

$Im_f = \mathbb{R} \qquad D_f = \mathbb{R}^*_+$



PA e PG

PA

$(a_1, a_2, a_3, a_4, \ldots, a_{n-1}, a_n),\ n \in \mathbb{N}$

$a_n = a_{n-1} + r$

- PA (..., a, b, c, ...)
  $a + c = 2b$
- PA $(a_1, a_2, \ldots, a_{n-1}, a_n)$
  $a_1 + a_n = a_2 + a_{n-1} = \ldots$

Termo geral: $a_n = a_1 + (n-1)r$

Soma dos n termos: $S_n = \frac{(a_1 + a_n)n}{2}$

PG

$(a_1, a_2, a_3, a_4, \ldots, a_{n-1}, a_n),\ n \in \mathbb{N}$

$a_n = a_{n-1} \cdot q$

- PG (..., a, b, c, ...)
  $a \cdot c = b^2$
- PG $(a_1, a_2, \ldots, a_{n-1}, a_n)$
  $a_1 \cdot a_n = a_2 \cdot a_{n-1} = \ldots$

Termo geral: $a_n = a_1 \cdot q^{n-1}$

Soma dos n primeiros termos: $S_n = \frac{a_1(1 - q^n)}{1 - q}$

$S_\infty = \frac{a_1}{1 - q},\ -1 < q < 1$

Análise combinatória

Fatorial

$0! = 1;\ 1! = 1$

$n! = n \cdot (n-1)!,\ n \in \mathbb{N} \text{ e } n > 1$

Permutação

Simples: $P_n = n!$

Com repetição: $P_n^{\alpha, \beta, \gamma} = \frac{n!}{\alpha!\beta!\gamma!}$

Arranjo

Simples: $A_{n,p} = \frac{n!}{(n-p)!}$

Com repetição: $(AR)_{n,p} = n^p$

Combinação simples

$C_{n,p} = \frac{n!}{p!(n-p!)} = \binom{n}{p}$

Estatística

Médias

$M_a = \bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$

$M_{ap} = \frac{x_1 \cdot p_1 + x_2 \cdot p_2 + \ldots + x_n \cdot p_n}{p_1 + p_2 + \ldots + p_n}$

$M_g = \sqrt[n]{x_1 \cdot x_2 \cdot \ldots \cdot x_n}$

$M_h = \frac{1}{\left(\frac{\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}}{n}\right)}$

$M_q = \sqrt[n]{\frac{x_1^2 + x_2^2 + \ldots + x_n^2}{n}}$

Moda

É o valor que ocorre com a maior frequência.

Mediana

É o valor central, ou a média aritmética dos dois valores centrais, do conjunto organizado em ordem crescente ou decrescente.

Amplitude total

É a diferença entre o maior e o menor valor do conjunto.

Desvio médio

$\text{Desvio médio} = \frac{\sum_{i=1}^{n} |x_i - \bar{x}|}{n}$

Desvio padrão

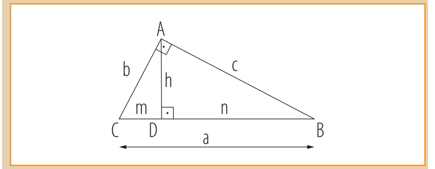
$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}}$

Variância

s^2

Trigonometria

Relações métricas no triângulo retângulo



$h^2 = m \cdot n$

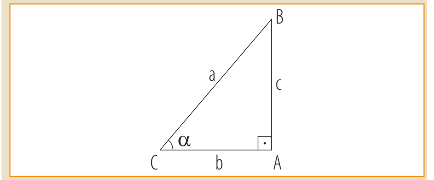
$b^2 = a \cdot m$

$c^2 = a \cdot n$

$b \cdot c = a \cdot h$

$a^2 = b^2 + c^2$ (Pitágoras)

Razões trigonométricas

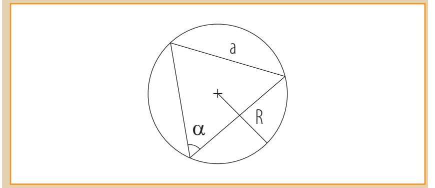


$\text{sen}\ \alpha = \frac{c}{a}$

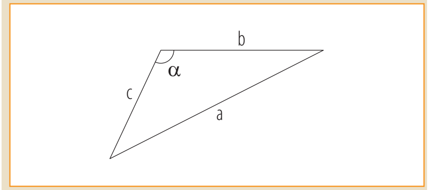
$\cos \alpha = \frac{b}{a}$

$\text{tg}\ \alpha = \frac{c}{b}$

Triângulo qualquer



$\frac{a}{\text{sen}\,\alpha} = 2R$



$a^2 = b^2 + c^2 - 2bc \cdot \cos \alpha$

Valores notáveis

	seno	cosseno	tangente
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	–

Relações fundamentais

$\text{sen}^2 x + \cos^2 x = 1 \ (\forall x \in \mathbb{R})$

$\text{tg}\, x = \frac{\text{sen}\, x}{\cos x} \left(x \neq \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\right)$

$\text{cotg}\, x = \frac{\cos x}{\text{sen}\, x} \ (x \neq k\pi,\ k \in \mathbb{Z})$

$\sec x = \frac{1}{\cos x} \left(x \neq \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\right)$

$\text{cossec}\, x = \frac{1}{\text{sen}\, x} \ (x \neq k\pi,\ k \in \mathbb{Z})$

Consequências $\left(x \neq \frac{k\pi}{2}\right)$

$\text{cotg}\, x = \frac{1}{\text{tg}\, x}$

$1 + \text{tg}^2 x = \sec^2 x$

$1 + \text{cotg}^2 x = \text{cossec}^2 x$

$\cos^2 x = \frac{1}{1 + \text{tg}^2 x}$

$\text{sen}^2 x = \frac{\text{tg}^2 x}{1 + \text{tg}^2 x}$

Funções circulares inversas

$y = \text{arcsen}\, x \Leftrightarrow \text{sen}\, y = x \text{ e } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$y = \text{arccos}\, x \Leftrightarrow \cos y = x \text{ e } 0 \leq y \leq \pi$

$y = \text{arctg}\, x \Leftrightarrow \text{tg}\, y = x \text{ e } -\frac{\pi}{2} < y < \frac{\pi}{2}$

Adição e subtração de arcos

$\cos(a+b) = \cos a \cdot \cos b - \text{sen}\, a \cdot \text{sen}\, b$

$\cos(a-b) = \cos a \cdot \cos b + \text{sen}\, a \cdot \text{sen}\, b$

$\text{sen}(a+b) = \text{sen}\, a \cdot \cos b + \text{sen}\, b \cdot \cos a$

$\text{sen}(a-b) = \text{sen}\, a \cdot \cos b - \text{sen}\, b \cdot \cos a$

$\text{tg}(a+b) = \frac{\text{tg}\, a + \text{tg}\, b}{1 - \text{tg}\, a \cdot \text{tg}\, b}$

$\text{tg}(a-b) = \frac{\text{tg} a - \text{tg} b}{1 + \text{tg} a \cdot \text{tg} b}$

Arco duplo

$\text{sen}\, 2a = 2 \cdot \text{sen}\, a \cdot \cos a$

$\cos 2a = \begin{cases} \cos^2 a - \text{sen}^2 a \\ \text{ou} \\ 2\cos^2 a - 1 \\ \text{ou} \\ 1 - 2\text{sen}^2 a \end{cases}$

$\text{tg}\, 2a = \frac{2\text{tg}\, a}{1 - \text{tg}^2 a}$

Arco triplo

$\text{sen}\, 3a = 3\text{sen}\, a - 4\text{sen}^3 a$

$\cos 3a = 4\cos^3 a - 3\cos a$

$\text{tg}\, 3a = \frac{3\text{tg}\, a - \text{tg}^3 a}{1 - 3\text{tg}^2 a}$

Arco metade

$\text{sen}\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$

$\cos\frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$

$\text{tg}\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}}$

Transformação em produto

$\cos p + \cos q = 2 \cdot \cos\frac{p+q}{2} \cdot \cos\frac{p-q}{2}$

$\cos p - \cos q = -2 \cdot \text{sen}\frac{p+q}{2} \cdot \text{sen}\frac{p-q}{2}$

$\text{sen}\, p + \text{sen}\, q = 2 \cdot \text{sen}\frac{p+q}{2} \cdot \cos\frac{p-q}{2}$

$\text{sen}\, p - \text{sen}\, q = 2 \cdot \text{sen}\frac{p-q}{2} \cdot \cos\frac{p+q}{2}$

$\text{tg}\, p + \text{tg}\, q = \frac{\text{sen}(p+q)}{\cos p \cdot \cos q}$

$\text{tg}\, p - \text{tg}\, q = \frac{\text{sen}(p-q)}{\cos p \cdot \cos q}$

Equações trigonométricas fundamentais

$\text{sen}\ \alpha = \text{sen}\ \beta \Rightarrow \alpha = \beta + 2k\pi \text{ ou } \alpha = (\pi - \beta) + 2k\pi$

$\cos \alpha = \cos \beta \Rightarrow \alpha = \pm\beta + 2k\pi$

$\text{tg}\ \alpha = \text{tg}\ \beta \Rightarrow \alpha = \beta + k\pi$

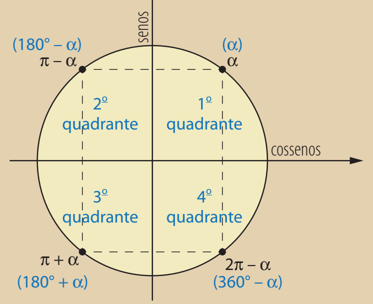
Simetrias no ciclo trigonométrico

$\begin{cases} \text{sen}(\pi - \alpha) = \text{sen}\alpha \\ \cos(\pi - \alpha) = -\cos\alpha \end{cases}$

$\begin{cases} \text{sen}(180° - \alpha) = \text{sen}\alpha \\ \cos(180° - \alpha) = -\cos\alpha \end{cases}$

$\begin{cases} \text{sen}(\pi + \alpha) = -\text{sen}\alpha \\ \cos(\pi + \alpha) = -\cos\alpha \end{cases}$

$\begin{cases} \text{sen}(180° + \alpha) = -\text{sen}\alpha \\ \cos(180° + \alpha) = -\cos\alpha \end{cases}$



$\begin{cases} \text{sen}(2\pi - \alpha) = -\text{sen}\alpha \\ \cos(2\pi - \alpha) = \cos\alpha \end{cases}$

$\begin{cases} \text{sen}(360° - \alpha) = -\text{sen}\alpha \\ \cos(360° - \alpha) = \cos\alpha \end{cases}$