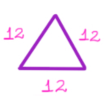
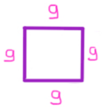
1. Com um arame de 36 m de comprimento construímos um triângulo equilátero e com o mesmo arame construímos depois um quadrado. Determine a razão entre o lado do triângulo e o lado do quadrado.

→ O perímetro dos dois quadriláteros é 36m !





→ O que os dois têm em comum?

Ambos possuem todos os lados de igual comprimento:

$$\ell_{quad} = \frac{P}{4} \Rightarrow \frac{36}{4} = 9m$$

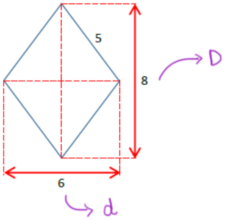
$$\ell_{triang} = \frac{P}{3} \Rightarrow \frac{36}{3} = 12m$$

$$\frac{\ell_{triang}}{\ell_{quad}} = \frac{12^{\,:3}}{9_{\,:3}} = \frac{4}{3}$$

4/3

Determine a área dos polígonos nos casos abaixo, sendo o metro a unidade das medidas indicadas.

2. Losango



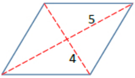
Lembre da fórmula da área do losango:

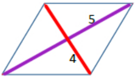
$$A = \frac{D \cdot d}{2}$$

$$A = \frac{8 \cdot 6}{2} = \frac{48}{2} = 24\,m^2$$

24 m²

3. Losango





$\rightarrow d = 2 \cdot 4 = 8\,m$

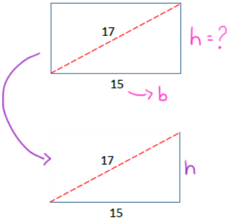
$\rightarrow D = 2 \cdot 5 = 10\,m$

$$A = \frac{10 \cdot 8}{2} = \frac{80}{2} = 40\,m^2$$

$\boxed{40\,m^2}$

---

4. Retângulo



Lembre da fórmula
da área do
retângulo:

$A = b \cdot h$

$$15^2 + h^2 = 17^2$$

$$h^2 = 17^2 - 15^2$$

$$h = \sqrt{64} = 8\,m$$

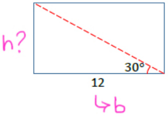
$$A = 15 \cdot 8 = 120\,m^2$$

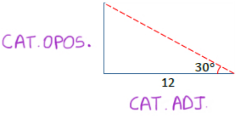
$\boxed{120\,m^2}$

---

5. Retângulo

Sabendo que $tg\,\alpha = \frac{CAT.\ OPOS.}{CAT.\ ADJ.}$





$$tg\,30° = \frac{h}{12}$$

$$h = 12 \cdot tg\,30°$$

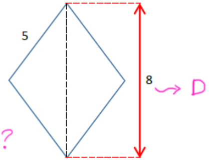
$$h = 12 \cdot \frac{\sqrt{3}}{3}$$

$$h = 4\sqrt{3}\,m$$

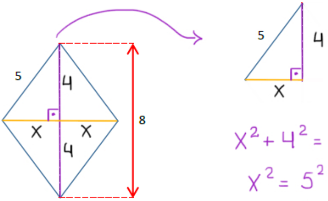
$$A = 12 \cdot 4\sqrt{3} = 48\sqrt{3}\,m^2$$

$\boxed{48\sqrt{3}\,m^2}$

6. Losango



d?



$X^2 + 4^2 = 5^2$

$X^2 = 5^2 - 4^2$

$X = \sqrt{9} = 3\,m$

$d = 2 \cdot X$

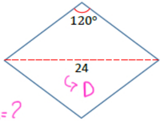
$d = 2 \cdot 3 = 6\,m$

$$A = \frac{8 \cdot 6}{2} = \frac{48}{2} = 24\,m^2$$

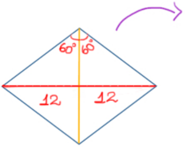
$\boxed{24\ m^2}$

---

7. Losango

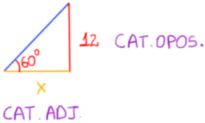


$d = ?$



Sabendo que:

$$tg\,\alpha = \frac{CAT.OPOS.}{CAT.ADJ.}$$



$d = 2 \cdot X$

$d = 2 \cdot 4\sqrt{3} = 8\sqrt{3}\,m$

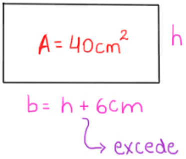
$$tg\,60° = \frac{12}{X}$$

$$X = \frac{12}{tg\,60°} = \frac{12}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 4\sqrt{3}\,m$$

$$A = \frac{24 \cdot 8\sqrt{3}}{2} = \frac{192\sqrt{3}}{2} = 96\sqrt{3}\,m^2$$

$\boxed{96\sqrt{3}\ m^2}$

8. A área de um retângulo é 40 cm² e sua base excede em 6 cm sua altura. Determine a altura do retângulo.



$$A = b \cdot h$$
$$40 = (h + 6) \cdot h$$
$$40 = h^2 + 6h$$
$$h^2 + 6h - 40 = 0$$

Da Fórmula de Bhaskara temos:

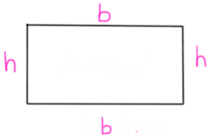
$h' = 4 \qquad h'' = -10$

Como a altura não pode ser negativa, $h = 4cm$

4 cm

---

9. Um retângulo tem 24 cm² de área e 20 cm de perímetro. Determine suas dimensões.



$$A = b \cdot h$$
$$24 = b \cdot h$$
$$b = \frac{24}{h}$$

$$P = b + b + h + h$$
$$20 = 2b + 2h$$
$$20 = 2 \cdot \frac{24}{h} + 2h$$
$$20 = \frac{48 + 2h^2}{h}$$
$$2h^2 - 20h + 48 = 0 \quad \div 2$$
$$h^2 - 10h + 24 = 0$$

Da Fórmula de Bhaskara, temos:

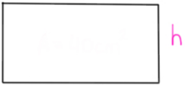
$h' = 4cm \qquad h'' = 6cm$

Assim:

$b' = \frac{24}{4} = 6cm \qquad b'' = \frac{24}{6} = 4cm$

4 cm; 6 cm

10. A base de um retângulo é o dobro de sua altura. Determine suas dimensões, sendo 72 cm² sua área.



$b = 2 \cdot h$

$b = 2 \cdot 6 = 12\,cm$

$A = b \cdot h$

$72 = (2 \cdot h) \cdot h$

$72 = 2 \cdot h^2$

$h^2 = \frac{72}{2}$

$h = \sqrt{36} = 6\,cm$

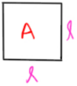
12 cm; 6 cm

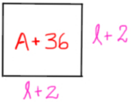
---

11. Determine o lado de um quadrado, sabendo que, se aumentarmos seu lado em 2 cm, sua área aumenta em 36 cm².

Lembre da Fórmula da área do quadrado:

$A = \ell^2$





$A = \ell^2$

$A + 36 = (\ell + 2)^2$

$A + 36 = \ell^2 + 4\ell + 4$

$\ell^2 + 36 = \ell^2 + 4\ell + 4$

$\cancel{\ell^2} + 36 = \cancel{\ell^2} + 4\ell + 4$

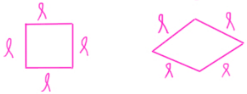
$4\ell = 36 - 4$

$\ell = \frac{32}{4} = 8\,cm$

8 cm

12. Um quadrado e um losango têm o mesmo perímetro. Determine a razão entre a área do quadrado e do losango, sabendo que as diagonais do losango estão entre si como 3/5 e que a diferença entre elas é igual a 40 cm.

→ Isso significa que o comprimento dos seus lados é igual !



$$\frac{d}{D} = \frac{3}{5} \Rightarrow d = \frac{3}{5} \cdot D \quad (1)$$

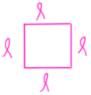
$$D - d = 40 \quad (2)$$

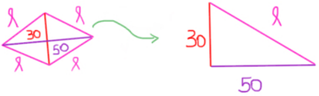
$$D - \frac{3}{5}D = 40 \qquad d = \frac{3 \cdot 100}{5} = 60\text{ cm}$$

$$\frac{5D - 3D}{5} = 40$$

$$2D = 200 \qquad A_{losango} = \frac{100 \cdot 60}{2} = \frac{6000}{2} = 3000\text{ cm}^2$$

$$D = \frac{200}{2} = 100\text{ cm}$$





$$\ell^2 = 30^2 + 50^2$$

$$\ell = \sqrt{3400}\text{ cm}$$

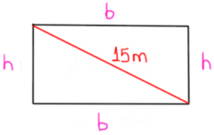
$$A_{quadrado} = \ell^2 = 3400\text{ cm}^2$$

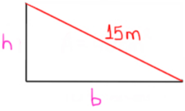
$$\frac{A_{quadrado}}{A_{losango}} = \frac{3400}{3000} = \frac{34}{30} = \frac{17}{15}$$

$$\boxed{17/15}$$

---

13. Determine a área de um retângulo de diagonal 15 m e perímetro 42 m.





$$b^2 + h^2 = 15^2$$

$$P = 2b + 2h$$

$$42 = 2b + 2h$$

$$b = \frac{42 - 2h}{2}$$

$$b = 21 - h$$

$$(21-h)^2 + h^2 = 15^2$$

$$h^2 - 42h + 441 + h^2 = 225$$

$$2h^2 - 42h + 441 - 225 = 0$$

$$2h^2 - 42h + 216 = 0 \quad \div 2$$

$$h^2 - 21h + 108 = 0$$

Da Fórmula de Bhaskara temos:

$$h' = 9 \qquad h'' = 12$$

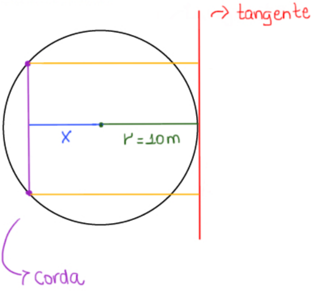
Já que $b = 21 - h \Rightarrow b' = 12 \qquad b'' = 9$

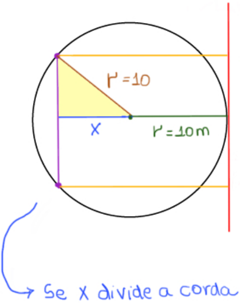
$$A = 12 \cdot 9 = 108\ m^2$$

$$\boxed{108\ m^2}$$

---

14. Um lado de um quadrado é corda de uma circunferência e o lado oposto é tangente a ela. Determine a área do quadrado, sendo 10 m o raio do círculo.

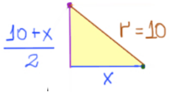




↪ Veja que o lado do quadrado possui comprimento de 10 + x

→ Se x divide a corda pela metade, e a corda tem comprimento ℓ:

$$\ell/2 = \frac{10 + x}{2}$$



$$\left(\frac{10+x}{2}\right)^2 + x^2 = 10^2$$

$$\frac{x^2 + 20x + 100}{4} + x^2 = 100$$

$$\frac{x^2 + 20x + 100 + 4x^2}{4} = 100$$

$$5x^2 + 20x - 300 = 0 \quad \div 5$$

$$x^2 + 4x - 60 = 0$$

Da fórmula de Bhaskara:

$x' = -10 \qquad x'' = 6$ //

$\ell = 10 + x$

$\ell = 10 + 6 = 16$ m

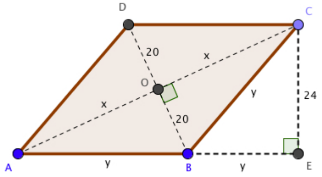
$A = \ell^2 = 16^2$

$A = 256$ m²

$\boxed{256 \text{ m}^2}$

---

15. Uma diagonal de um losango mede 40 m e a sua altura 24 m. Determine a área desse losango.



→ Veja que os triângulos AEC e AOB são semelhantes, então:

$$\frac{20}{24} = \frac{y}{2x}$$

$$40x = 24y$$

$$y = \frac{40^{\,:8}}{24_{\,:8}}x = \frac{5}{3}x$$

Aplicando o teorema de pitágoras no triângulo AOB:

$$x^2 + 20^2 = y^2$$

$$x^2 + 400 = \left(\frac{5}{3}x\right)^2$$

$$\underline{25}\,x^2 - x^2 = 400$$

Assim sabemos que D = 40 m e que:

$d = 2 \cdot x$

$d = 2 \cdot 15$

$d = 30$ m

$$9$$

$$\frac{25x^2 - 9x^2}{9} = 400$$

$$16x^2 = 3600$$

$$x^2 = \frac{3600}{16}$$

$$X = \sqrt{225} = 15\ m$$

$$A = \frac{40 \cdot 30}{2} = \frac{1200}{2}$$

$$A = 600\ m^2$$

$600\ m^2$