

# Equações e Problemas

## 1) Equação do 2º Grau

$$ax^2 + bx + c = 0$$

$$\Delta = b^2 - 4ac$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$x_1 = \frac{-b + \sqrt{\Delta}}{2a}$$

$$x_2 = \frac{-b - \sqrt{\Delta}}{2a}$$

$$2) x^2 - 4x = 0$$

$$x \cdot (x - 4) = 0$$

$$x_1 = 0$$

$$\text{ou } x - 4 = 0$$

$$x_2 = 4$$

$$S = \{0, 4\}$$

$$1) 2x^2 - 3x + 1 = 0$$

$$\Delta = (-3)^2 - 4 \cdot 2 \cdot 1 \Rightarrow \Delta = 1$$

$$x = \frac{-(-3) \pm \sqrt{1}}{2 \cdot 2} = \frac{3 \pm 1}{4}$$

$$x_1 = 1$$

$$x_2 = \frac{2}{4} = \frac{1}{2}$$

$$S = \left\{ \frac{1}{2}, 1 \right\}$$

$$3) x^2 - 8 = 0$$

$$x^2 = 8$$

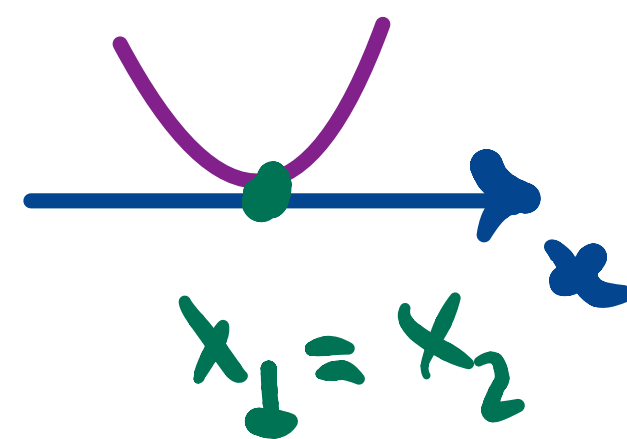
$$x = \pm 2\sqrt{2}$$

$$x_1 = 2\sqrt{2}$$

$$x_2 = -2\sqrt{2}$$

$$S = \{-2\sqrt{2}, 2\sqrt{2}\}$$

$$4) x^2 - 6x + 9 = 0$$

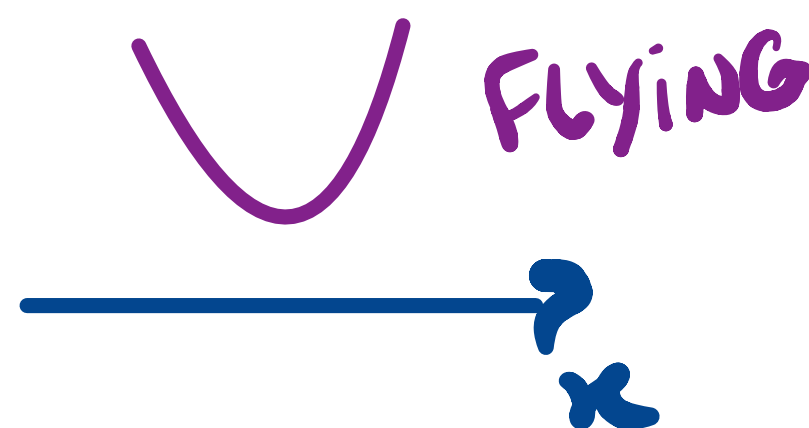


$$\Delta = 36 - 36 \quad \therefore \Delta = 0$$

$$x = \frac{6 \pm 0}{2} \rightarrow \begin{cases} x_1 = 3 \\ x_2 = 3 \end{cases} \rightarrow \text{"RAÍZ DUPLA"}$$

$$S = \{3\} \rightarrow \text{SOLUÇÃO ÚNICA}$$

$$5) x^2 - 2x + 2 = 0$$



$$\Delta = 4 - 8 \rightarrow \Delta = -4$$

$$x = \frac{2 \pm \sqrt{-4}}{2} \quad \text{!!!} \quad \text{NÃO EXISTE } x \in \mathbb{R}$$

$$S = \emptyset$$

## DELTA ( $\Delta$ )

|              |                                    |  |
|--------------|------------------------------------|--|
| $\Delta > 0$ | 2 RAÍZES REAIS E DIFERENTES        | <p>A coordinate system with a horizontal x-axis. A red parabola opens upwards, intersecting the x-axis at two distinct points marked with grey dots and labeled <math>x_1</math> and <math>x_2</math>.</p> |
| $\Delta = 0$ | 2 RAÍZES REAIS IGUAIS (SOL. ÚNICA) | <p>A coordinate system with a horizontal x-axis. A red parabola opens upwards, touching the x-axis at a single point marked with a grey dot and labeled <math>x_1 = x_2</math>.</p>                        |
| $\Delta < 0$ | NÃO POSSUI RAÍZ REAL               | <p>A coordinate system with a horizontal x-axis. A red parabola opens upwards and is positioned entirely above the x-axis, not touching it.</p>  |

Ex: Para que  $x^2 - 2x + m = 0$  TENHA

**RAÍZES REAIS**

DEVEMOS TER:

$$\Delta = 4 - 4m \geq 0$$

**A)**  $m \leq 1$

$\times (-1)$   $-4m \geq -4$

B)  $m \geq 1$

$$4m \leq 4$$

C)  $m \geq -1$

**$m \leq 1$**

D)  $m \leq -1$

$\Delta \geq 0$

## 2) RELACÃO ENTRE COEFICIENTES E RAÍZES

$$ax^2 + bx + c = 0 \rightarrow \Delta = b^2 - 4ac$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$x_1 = \frac{-b + \sqrt{\Delta}}{2a}$$

$$x_2 = \frac{-b - \sqrt{\Delta}}{2a}$$

$$S = x_1 + x_2 = -\frac{b}{a}$$

$$P = x_1 \cdot x_2 = \frac{c}{a}$$

$$\text{SOMA: } x_1 + x_2 = \frac{\cancel{-b + \sqrt{\Delta}}}{2a} + \frac{\cancel{-b - \sqrt{\Delta}}}{2a} = \frac{-2b}{2a} = -\frac{b}{a}$$

$$\text{PRODUTO: } x_1 \cdot x_2 = \left( \frac{\cancel{-b + \sqrt{\Delta}}}{2a} \right) \left( \frac{\cancel{-b - \sqrt{\Delta}}}{2a} \right) = \frac{b^2 + \cancel{b\sqrt{\Delta}} - \cancel{b\sqrt{\Delta}} - \Delta}{4a^2}$$

$$= \frac{b^2 - b^2 + 4ac}{4a^2} = \frac{c}{a}$$

## Exemplo 1

$$\Delta = 36 - 24 \Rightarrow \Delta = 12$$

Sejam  $x_1$  e  $x_2$  as raízes de  $2x^2 - 6x + 3 = 0$ . Calcule:

A)  $(x_1 + 3)(x_2 + 3)$

$$x_1 \cdot x_2 + 3x_1 + 3x_2 + 9$$
$$x_1 \cdot x_2 + 3 \cdot (x_1 + x_2) + 9$$
$$\frac{3}{2} + 3 \cdot 3 + 9 = 19,5$$

$$\begin{cases} x_1 + x_2 = -\frac{(-6)}{2} = 3 \\ x_1 \cdot x_2 = \frac{3}{2} \end{cases}$$

B) A soma dos inversos das raízes.

$$? = \frac{1}{x_1} + \frac{1}{x_2} = \frac{x_2 + x_1}{x_1 \cdot x_2} = \frac{3}{\frac{3}{2}} = 3 \cdot \frac{2}{3} = 2$$