Equações e Problemas

1) Equação do 2º Grau

$$ax^2 + bx + c = 0$$

$$\Delta = b^2 - 4ac$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$x_1 = \frac{-b + \sqrt{\Delta}}{2a} \qquad x_2 = \frac{-b - \sqrt{\Delta}}{2a}$$

2) $x^2 - 4x = 0$

$$x \cdot (x - 4) = 0$$

$x_1 = 0$ ou $x - 4 = 0$

$x_2 = 4$

$S = \{0, 4\}$

1) $2x^2 - 3x + 1 = 0$

$S = \{\frac{1}{2}, 1\}$

$\Delta = (-3)^2 - 4 \cdot 2 \cdot 1 \Rightarrow \Delta = 1$

$$x = \frac{-(-3) \pm \sqrt{1}}{2 \cdot 2} = \frac{3 \pm 1}{4}$$

$x_1 = 1$

$x_2 = \frac{2}{4} = \frac{1}{2}$

3) $x^2 - 8 = 0$

$S = \{-2\sqrt{2}, 2\sqrt{2}\}$

$x^2 = 8$

$x = \pm 2\sqrt{2}$

$x_1 = 2\sqrt{2}$

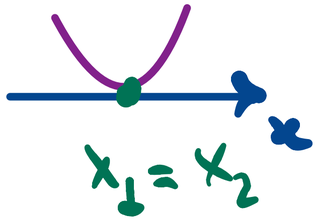
$x_2 = -2\sqrt{2}$

4) $x^2 - 6x + 9 = 0$

$\Delta = 36 - 36 \therefore \Delta = 0$

$x = \frac{6 \pm 0}{2}$ → $x_1 = 3$, $x_2 = 3$ → "RAIZ DUPLA"

$S = \{3\}$ → SOLUÇÃO ÚNICA



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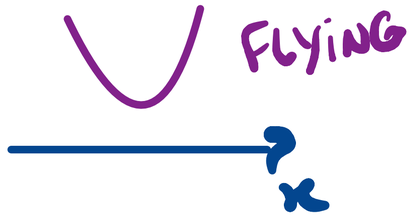
5) $x^2 - 2x + 2 = 0$

$\Delta = 4 - 8 \leftrightarrow \Delta = -4$

$x = \frac{2 \pm \sqrt{-4}}{2}$!!! NÃO EXISTE

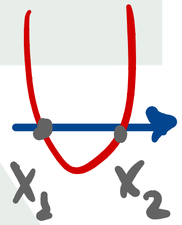
$x \in \mathbb{R}$

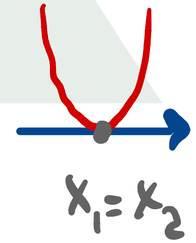
$S = \emptyset$



DELTA (Δ)

$\Delta > 0$	2 RAÍZES REAIS E DIFERENTES
$\Delta = 0$	2 RAÍZES REAIS IGUAIS (SOL. ÚNICA)
$\Delta < 0$	NÃO POSSUI RAIZ REAL





Ex: Para que $x^2 - 2x + m = 0$ tenha raízes reais

Devemos ter:

$$\Delta \geq 0$$

$$\Delta = 4 - 4m \geq 0$$

$$\times(-1) \quad -4m \geq -4$$

$$4m \leq 4$$

$$m \leq 1$$

a) $m \leq 1$

b) $m \geq 1$

c) $m \geq -1$

d) $m \leq -1$

② Relação entre coeficientes e raízes

$ax^2+bx+c=0 \rightarrow \Delta = b^2-4ac$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$x_1 = \frac{-b+\sqrt{\Delta}}{2a}$$

$$x_2 = \frac{-b-\sqrt{\Delta}}{2a}$$

$$S = x_1 + x_2 = -\frac{b}{a}$$

$$P = x_1 \cdot x_2 = \frac{c}{a}$$

Soma: $x_1 + x_2 = \frac{-b+\sqrt{\Delta}}{2a} + \frac{-b-\sqrt{\Delta}}{2a} = \frac{-2b}{2a} = -\frac{b}{a}$

Produto: $x_1 \cdot x_2 = \left(\frac{-b+\sqrt{\Delta}}{2a}\right)\left(\frac{-b-\sqrt{\Delta}}{2a}\right) = \frac{b^2+b\sqrt{\Delta}-b\sqrt{\Delta}-\Delta}{4a^2}$

$\rightarrow \Delta = b^2-4ac$

$$= \frac{b^2-b^2+4ac}{4a^2} = \frac{c}{a}$$

MESTRES DA MATEMÁTICA

Exemplo 1

$\Delta = 36 - 24 \Rightarrow \Delta = 12$

Sejam x_1 e x_2 as raízes de $2x^2 - 6x + 3 = 0$. Calcule:

$$\begin{cases} x_1 + x_2 = -\frac{(-6)}{2} = 3 \\ x_1 \cdot x_2 = \frac{3}{2} \end{cases}$$

A) $(x_1 + 3)(x_2 + 3)$

$x_1 \cdot x_2 + 3x_1 + 3x_2 + 9$

$x_1 \cdot x_2 + 3 \cdot (x_1 + x_2) + 9$

$\frac{3}{2} + 3 \cdot 3 + 9 = 19{,}5$

B) A soma dos inversos das raízes.

$$? = \frac{1}{x_1} + \frac{1}{x_2} = \frac{x_2 + x_1}{x_1 \cdot x_2} = \frac{3}{\frac{3}{2}} = 3 \cdot \frac{2}{3} = 2$$