Equações e Manipulações Algébricas

#01 Equações do 1º grau e sistemas lineares

↳ equação linear

- ✓ Coisas iguais às mesmas coisas são iguais entre si.
- ✓ Coisas iguais adicionadas a coisas iguais formam coisas iguais.
- ✓ Coisas iguais subtraídas de coisas iguais formam coisas iguais.

Sistemas Lineares

exemplos:

#01

$$\begin{cases} 2x + y = 5 \quad (1) \\ x + y = 3 \quad (2) \end{cases}$$

$$\begin{cases} 2x + y = 5 \quad (1) \\ x + y = 3 \quad (2) \end{cases} \dashrightarrow \boxed{y = 5 - 2x}$$

$x + 5 - 2x = 3 \quad \therefore \quad -x + 5 = 3 \quad \therefore \quad -x = 3 - 5$

$-x = -2 \quad \therefore \quad \boxed{x = 2}$ $\quad y = 5 - 2 \cdot 2 \quad \therefore \boxed{y = 1}$

Outra forma:

$$\begin{matrix} 2x + y = 5 \\ (-) \quad x + y = 3 \end{matrix}$$

$2x + y - x - y = 5 - 3 \quad \therefore \quad x = 2 \quad \therefore \boxed{x = 2}$ $\quad \boxed{y = 1}$

Outra forma:

$$\begin{matrix} 2x + y = 5 \\ \times(-2) \quad x + y = 3 \end{matrix} \Bigg\} \begin{matrix} 2x + y = 5 \\ -2x - 2y = -6 \quad (+) \end{matrix}$$

$-y = -1$

$\boxed{y = 1}$

$\boxed{x = 2}$

#02

$$\begin{cases} 8x = y \\ 2x = 4 + y \end{cases}$$

1ª forma

$$\begin{array}{rl} & 8x = y \\ (-) & 2x = y + 4 \\ \hline & 6x = -4 \end{array}$$

$x = -4/6 \therefore \boxed{x = -\frac{2}{3}}$

$\boxed{y = -\frac{16}{3}}$

2ª forma

$$\begin{array}{rl} & 8x = y \\ (+) & -8x = -16 - 4y \\ \hline & 0 = -16 - 3y \end{array}$$

$3y = -16 \therefore \boxed{y = -16/3}$

$\boxed{x = -2/3}$

3ª forma

$$\frac{8x}{2x} = \frac{y}{y+4} \therefore 4 = \frac{y}{(y+4)} \therefore y = 4(y+4)$$

$y = 4y + 16 \therefore -3y = 16 \therefore \boxed{y = -\frac{16}{3}}$

$\boxed{x = -2/3}$

#02 Equação do 2º grau: Bhaskara

$$ax^2 + bx + c = 0 \qquad (a \neq 0)$$

↳ Ex.:

$$x^2 + x - 2 = 0 \begin{cases} \cdot a = 1 \\ \cdot b = 1 \\ \cdot c = -2 \end{cases}$$

Bhaskara: $x = \dfrac{-b \pm \sqrt{\Delta}}{4a}$

$$\Delta = b^2 - 4ac = 1^2 - 4 \cdot 1 \cdot (-2) = 1 + 8 = 9$$

$$\sqrt{9} = 3$$

$$x = \frac{-1 \pm 3}{2 \cdot 1} = \frac{-1 \pm 3}{2}$$

$x_1 = \dfrac{-1-3}{2} \therefore \boxed{x_1 = -2}$

$x_2 = \dfrac{-1+3}{2} \therefore \boxed{x_2 = 1}$

De onde vem o Bhaskara?

$$ax^2 + bx + c = 0 \quad \Big) \div (a)$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$(x + \square)^2 = x^2 + 2 \cdot x \cdot \square + \square^2$$

$$2 \cdot \square = b/a$$

$$\square = b/2a$$

$$\left(x + \frac{b}{2a}\right)^2 = x^2 + 2 \cdot x \cdot \frac{b}{2a} + \left(\frac{b}{2a}\right)^2$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} + \left(\frac{b}{2a}\right)^2 = 0 + \left(\frac{b}{2a}\right)^2$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = \left(\frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} = \left(\frac{b}{2a}\right)^2$$

$$\left(x + b/2a\right)^2 + c/a - c/a = \left(\frac{b}{2a}\right)^2 - c/a$$

$$\left(x + b/2a\right)^2 = \frac{b^2}{4a^2} - \frac{4a \cdot c}{4a \cdot a} = \frac{b^2 - 4ac}{4a^2}$$

$$\left(x + b/2a\right)^2 = \frac{b^2 - 4ac}{(2a)^2}$$

$$\boxed{\Delta = b^2 - 4ac}$$

$$\sqrt{\left(x + b/2a\right)^2} = \pm \frac{\sqrt{\Delta}}{\sqrt{(2a)^2}}$$

$$x + b/2a = \pm \frac{\sqrt{\Delta}}{2a} \qquad - \left(b/2a\right)$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{\Delta}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{\Delta}}{2a}}$$

#03 Outras equações

Exercícios

1) $\sqrt{x-5} - 4 = 0$ $\rightarrow$ $\sqrt{x-5} = 4$

$$\left(\sqrt{x-5}\right)^2 + 2 \cdot \sqrt{x-5} \cdot (-4) + 16 = 0$$

$$x - 5 - 8\underbrace{\sqrt{x-5}}_{4} + 16 = 0$$

$$x - 5 - 8 \cdot 4 + 16 = 0$$

$$x - 5 - 32 + 16 = 0$$

$$x - 21 = 0$$

$$\boxed{x = 21}$$

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$$\sqrt{x-5} - 4 + 4 = 0 + 4$$

$$\left(\sqrt{x-5}\right)^2 = 4^2$$

$$x - 5 = 16$$

$$\boxed{x = 21}$$

2) $\sqrt{x+4} - 2 = x \Rightarrow$ **x = 0**

$$\sqrt{x+4} = x+2$$

$$\left(\sqrt{x+4}\right)^2 = (x+2)^2$$

$$x+4 = x^2 + 2 \cdot x \cdot 2 + 2^2$$

$$x^2 + 4x - x = 0$$

$$x^2 + 3x = 0$$

$$\sqrt{x+4} - 2 = x$$

$$x \cdot (x+3) = 0$$

$x = 0$ ✓

$x + 3 = 0$

$x = -3$ ✗

i) $x = 0$ $\quad \sqrt{0+4} - 2 = 0 \quad \therefore \quad 2 - 2 = 0$ ✓

ii) $x = -3$ $\quad \sqrt{-3+4} - 2 = -3$

$$1 - 2 = -3$$

$$-1 = -3$$

$$1 = 3$$

$$\sqrt{x+4} - 2 = x$$

$$\sqrt{x+4} = x + 2$$

$$\left(\sqrt{x+4}\right)^2 = (x+2)^2$$

$$x + 4 = x^2 + 2 \cdot x \cdot 2 + 2^2$$

$$x + 4 = (x+2)^2$$

3) $\sqrt{x} = \sqrt{x-5} + 1$

$(\sqrt{x})^2 = (\sqrt{x-5} + 1)^2$

$x = (x-5) + 2\sqrt{x-5} \cdot 1 + 1^2$

$x - x = x - 5 + 2\sqrt{x-5} + 1 - x$

$0 = -4 + 2\sqrt{x-5}$

$4 = 2\sqrt{x-5}$

$2 = \sqrt{x-5}$

$2^2 = (\sqrt{x-5})^2$

$4 = x - 5$

$x = 4 + 5$

$x = 9$ ✓