Dados $z_1 = 2 + 5i$, $z_2 = -6 + i$, $z_3 = 1 - 3i$ e $z_4 = -2 - i$, calcule:

1. $z_1 + z_2$

$z_1 + z_2 = (2+5i) + (-6+i)$

$z_1 + z_2 = (2-6) + (5i+i)$

$\boxed{z_1 + z_2 = -4 + 6i}$

2. $z_2 - z_3$

$z_2 - z_3 = (-6+i) - (1-3i)$

$z_2 - z_3 = (-6-1) + (i-(-3i))$

$\boxed{z_2 - z_3 = -7 + 4i}$

3. $z_1 + z_2 + z_3 + z_4$

$z_1 + z_2 + z_3 + z_4 = (2+5i) + (-6+i) + (1-3i) + (-2-i)$

$z_1 + z_2 + z_3 + z_4 = (2-6+1-2) + (5i+i-3i-i)$

$\boxed{z_1 + z_2 + z_3 + z_4 = -5 + 2i}$

4. $z_1 - z_2 + z_3 - z_4$

$z_1 - z_2 + z_3 - z_4 = (2+5i) - (-6+i) + (1-3i) - (-2-i)$

$z_1 - z_2 + z_3 - z_4 = (2-(-6)+1-(-2)) + (5i-i-3i-(-i))$

$\boxed{z_1 - z_2 + z_3 - z_4 = 11 + 2i}$

5. Calcule os reais x e y na igualdade:

$(x + 2yi) + (y - 2xi) - 2i = 11$

$x + 2yi + y - 2xi - 2i = 11 + 0.i$

Real $\begin{cases} x + y = 11 \end{cases}$
Imaginária $\begin{cases} 2yi - 2xi - 2i = 0 \end{cases}$
$\longrightarrow \begin{cases} x + y = 11 \\ -2xi + 2yi = 2i \end{cases}$

$2i(-x+y) = 2i.1$

$\cancel{x} + y = 11$
$-\cancel{x} + y = 1$
$0x + 2y = 12$
$y = \frac{12}{2}$ $\quad \boxed{y = 6}$

$x + y = 11$
$x + 6 = 11$
$x = 11 - 6$
$\boxed{x = 5}$

Dados $z_1 = 1 + i$, $z_2 = 3 + 2i$ e $z_3 = 6 - 4i$, calcule:

6. $z_1 \cdot z_2$

$z_1 \cdot z_2 = (1+i) \cdot (3+2i)$

$z_1 \cdot z_2 = 3 + 2i + 3i + 2i^{2 = -1}$

$z_1 \cdot z_2 = 3 + 5i + 2.(-1)$

$\boxed{z_1 \cdot z_2 = 1 + 5i}$

7. $z_2 \cdot z_3$

$z_2 \cdot z_3 = (3+2i).(6-4i)$

$z_2 \cdot z_3 = 18 - \cancel{12i} + \cancel{12i} \underbrace{-8i^2}$

$-8.(-1) = +8$

$\boxed{z_2 \cdot z_3 = 26}$

8. $z_1 \cdot z_3$

$z_1 \cdot z_3 = (1+i) \cdot (6-4i)$

$z_1 \cdot z_3 = 6 - 4i + 6i - 4i^{2=-1}$

$\boxed{z_1 \cdot z_3 = 10 + 2i}$

Calcule:

9. $\underbrace{[(2i + 1) - (3i - 1)]}_{1 + 2i - 3i + 1}\underbrace{[2i - (3 - i)]}_{2i - 3 - i}$

$2 - i \qquad 3i - 3$

$(2-i) \cdot (3i-3)$

$6i - 6 - 3i^{2 = -1} + 3i$

$\boxed{-3 + 9i}$

10. $(i + 2)(2i + 1) + i(i - 1) - 3(4 + 2i)(1 - 2i)$

$(i+2)(2i+1) \qquad i(i-1) \qquad 3(4+2i)\cdot(1-2i)$

$2i^2 + \cancel{1i} + 4i + 2 + i^2 - \cancel{1i} - 3(4 + 2i - 8i - 4i^2)$

$3i^2 + 4i + 2 - 3(-6i + 8)$

$3i^2 + 4i + 2 + 18i - 24$

$4i + 18i + 2 - 24 - 3$

$\boxed{22i - 25}$

Calcule os quocientes:

11. $\frac{1 + i}{3 + i} \times \frac{3-i}{3-i}$

$\frac{(3 + 3i - i - i^2)}{3^2 + \cancel{3i} - \cancel{3i} - i^2} \rightarrow \frac{3 - (-1) + 2i}{9 - (-1)}$

$\frac{4 + 2i}{10} \rightarrow \boxed{\frac{2}{5} + \frac{i}{5}}$

12. $\frac{2 - i}{1 - 2i} \times \frac{1+2i}{1+2i}$

$\frac{2 - i + 4i - 2i^2}{1^2 - \cancel{2i} + \cancel{2i} - 4i^2} \rightarrow \frac{2 - 2.(-1) + 3i}{1 - (-4)}$

$\boxed{\frac{4}{5} + \frac{3i}{5}}$

13. $\frac{3 + 2i}{4i} \times \left(\frac{-4i}{-4i}\right)$

$\frac{-12i - 8i^2}{-16i^2}$

$\frac{-12i + 8}{16} \rightarrow \boxed{\frac{1}{2} - \frac{3i}{4}}$

14. $\frac{\sqrt{3} - i}{\sqrt{3}}$ não possui parte imaginária, portanto o conjugado é: $\sqrt{3}$

$\frac{\sqrt{3} - i}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}^2 - \sqrt{3}i}{\sqrt{3}^2}$

$\frac{3}{3} - \frac{\sqrt{3}i}{3} = \boxed{1 - \frac{\sqrt{3}i}{3}}$

Calcule o inverso de z em cada caso:

15. $z = 4 - 3i$

$\frac{1}{z} = \frac{1}{4-3i} \times \frac{4+3i}{4+3i}$

$\frac{4+3i}{4^2 - \cancel{12i} + \cancel{12i} - 9i^2} = \frac{4+3i}{25} \rightarrow \boxed{\frac{4}{25} + \frac{3i}{25}}$

16. $z = 12 + 5i$

$\frac{1}{z} = \frac{1}{12+5i} \times \frac{12-5i}{12-5i}$

$\frac{1}{z} = \frac{12 - 5i}{12^2 - \cancel{60i} + \cancel{60i} - 5i^2} \rightarrow \frac{12 - 5i}{144 + 25}$

$\boxed{\frac{1}{z} = \frac{12}{169} - \frac{5i}{169}}$

17. Para que valores reais de x o número $\frac{1+xi}{1-xi}$ é imaginário puro? E para quais é real?

$\frac{1+xi}{1-xi} \times \frac{1+xi}{1+xi} = \frac{1^2 + 2xi + x^2i^2}{1 - x^2i^2}$

$\frac{-x^2 + 2xi + 1}{x^2 + 1}$

Imaginário puro: $\frac{-x^2 + 2xi + 1}{x^2 + 1} = y.i \qquad \frac{0}{x^2+1} + \frac{2xi}{x^2+1}$ Im

logo: $-x^2 + 1 = 0$

$-x^2 = -1 \cdot (-1)$

$x^2 = 1$

$x = \sqrt{1} \rightarrow \boxed{x = \pm 1}$

Número real: $\frac{-x^2 + 2xi + 1}{x^2 + 1} = y + 0.i$

$\frac{-x^2 + 1}{x^2 + 1} + \frac{i}{x^2 + 1}$

logo, $2xi = 0$

$\boxed{x = 0}$

18. Calcule os reais x e y de modo que o número complexo $z = x + yi$ verifique a igualdade $\bar{z} + 2iz - 8 = i$.

Lembre que $i \times i = i^2 = -1$

$\bar{z} + 2iz - 8 = i$

conjugado

$(x - yi) + 2i(x + yi) - 8 = i$

$x - yi + 2xi - 2y - 8 = i + 0$

$\begin{cases} x - 2y - 8 = 0 \\ -yi + 2xi = i \end{cases} \rightarrow \begin{cases} x - 2y = 8 \\ 2x - y = 1 \end{cases} \times(-2)$

$\begin{cases} x - 2y = 8 \\ -4x + 2y = -2 \end{cases}$

$-3x + 0y = 6$

$-3x = 6$

$x = \frac{6}{-3} \rightarrow \boxed{x = -2}$

$x - 2y = 8 \rightarrow -2 - 2y = 8 \rightarrow -2y = 10^{\cdot(-1)} \rightarrow \boxed{y = -5}$