Importante:

- $sen\ 2x = 2\ sen\ x \cdot cos\ x$
- $cos\ 2x = cos^2 x - sen^2 x$
- $tg\ 2x = \frac{2 \cdot tg\,x}{1 - tg^2 x}$

1. Dados $sen\ 37° \cong 0{,}60$ e $cos\ 37° \cong 0{,}80$, calcule $sen\ 74°$ e $cos\ 74°$.

$sen\ 74 = sen\ (2 \cdot 37)$

$sen\ 74° = 2\ sen\ 37° \cdot cos\ 37°$

$sen\ 74° = 2 \cdot 0{,}6 \cdot 0{,}8$

$sen\ 74° \cong 0{,}96$

$cos\ 74° = cos\ (2 \cdot 37)$

$cos\ 74° = cos^2 37 - sen^2 37$

$cos\ 74° = 0{,}8^2 - 0{,}6^2$

$cos\ 74° = 0{,}64 - 0{,}36$

$cos\ 74° \cong 0{,}28$

2. Dado $cos\ x = -\frac{\sqrt{6}}{9}, \frac{\pi}{2} < x < \pi$, calcule $cos\ 2x$ e $sen\ 2x$.

$cos\ x = -\frac{\sqrt{6}}{9}$

$cos^2 x + sen^2 x = 1$

$\left(\frac{\sqrt{6}}{9}\right)^2 + sen^2 x = 1$

$sen^2 x = \frac{75}{81}$

$sen\ x = \pm\sqrt{\frac{75}{81}}$ → $\pi/2 < x < \pi$ sen x é positivo

$sen\ x = +\frac{\sqrt{75}}{9}$

$sen\ 2x = 2\ sen\ x \cdot cos\ x$

$sen\ 2x = 2 \cdot \left(-\frac{\sqrt{6}}{9}\right) \cdot \left(\frac{\sqrt{75}}{9}\right)$

$sen\ 2x = -\frac{2 \cdot 5 \cdot 3 \cdot \sqrt{2}}{81}$

$sen\ 2x = -\frac{10\sqrt{2}}{27}$

$cos\ 2x = cos^2 x - sen^2 x$

$cos\ 2x = \left(-\frac{\sqrt{6}}{9}\right)^2 - \left(\frac{\sqrt{75}}{9}\right)^2$

$cos\ 2x = \frac{6}{81} - \frac{75}{81}$

$cos\ 2x = -\frac{69}{81}$

$cos\ 2x = -\frac{23}{27}$

3. Se $cotg\ x = -4$, qual é o valor de $tg\ 2x$?

$\frac{1}{tg\,x} = -4$ $tg\,x = -\frac{1}{4}$

$tg\,2x = \frac{2 \cdot tg\,x}{1 - tg^2 x}$ → $tg\,2x = \frac{2 \cdot (-1/4)}{1 - (-1/4)^2}$

$tg\ 2x = \frac{-1/2}{1 - 1/16}$ → $tg\,2x = \frac{-1/2}{15/16}$

$tg\,2x = -\frac{16}{30}$ → $tg\,2x = -8/15$

4. Qual é o valor de $2sen\ 22°30' \cdot cos\ 22°30'$?

$sen\ 2x = 2 \cdot sen\,x \cdot cos\,x$

$= sen \cdot (2 \cdot (22°30'))$

$22°30' + 22°30' = 44°60' = 45°$ (+1°)

$sen\ (45°) = \frac{\sqrt{2}}{2}$

5. Calcule $sen\ 2x$, sabendo que: $tg\ x + cotg\ x = 3$.

$tg\,x + cotg\,x = 3$

$tg\,x + \frac{1}{tg\,x} = 3$ → $\frac{tg^2 x + 1}{tg\,x} = 3$

$\frac{\frac{sen^2 x}{cos^2 x} + 1}{\frac{sen\,x}{cos\,x}} = 3$ → $\frac{\frac{sen^2 x + cos^2 x}{cos^2 x}}{\frac{sen\,x}{cos\,x}} = 3$

$\frac{(sen^2 x + cos^2 x) \cdot cos\,x}{sen\,x \cdot cos^2 x} = 3$

$\frac{(sen^2 x + cos^2 x)}{sen\,x \cdot cos\,x} = 3 \cdot \frac{1}{2}$ (com $sen^2 x + cos^2 x = 1$)

$\frac{1}{2\,sen\,x \cdot cos\,x} = \frac{3}{2}$ (com $2\,sen\,x \cdot cos\,x = sen\,2x$)

$sen\ 2x = \frac{2}{3}$

6. Sendo $cotg\ x = \frac{12}{5}$ e $0 < x < \frac{\pi}{2}$, calcule $cos\ 2x$.

→ $tg\,x = \frac{5}{12}$ $0 < x < \pi/2$

$tg\,x^2 = \frac{sen^2 x}{cos^2 x} = \frac{25}{144}$

$sen^2 x = \frac{25}{144} \cdot cos^2 x$ $sen^2 x + cos^2 x = 1$

$\frac{25}{144} \cdot cos^2 x + cos^2 x = 1$

$\frac{169\ cos^2 x}{144} = 1$ → $cos^2 x = 144/169$

$sen^2 x = \frac{25}{144} \cdot \frac{144}{169}$ → $sen^2 x = 25/169$

$cos\ 2x = cos^2 x - sen^2 x$

$cos\ 2x = \frac{144}{169} - \frac{25}{169}$

$cos\ 2x = \frac{119}{169}$

7. Sabendo que $sen\ a = \frac{3}{5}$ e $cos\ a = \frac{4}{5}$, calcule $sen\ 2a + cos\ 2a$.

$sen\ a = \frac{3}{5}$ → $sen^2 a = \frac{9}{25}$

$cos\ a = \frac{4}{5}$ → $cos^2 a = \frac{16}{25}$

$sen\ 2a + cos\ 2a$

$2 \cdot sen\,a \cdot cos\,a + (cos^2 a - sen^2 a)$

$2 \cdot \frac{3}{5} \cdot \frac{4}{5} + \frac{16}{25} - \frac{9}{25}$

$\frac{24}{25} + \frac{16}{25} - \frac{9}{25} = \frac{31}{25}$