1. Qual é a tangente do ângulo agudo formado pelas retas $3x + 2y + 2 = 0$ e $-x + 2y + 5 = 0$?

$r: 3x + 2y + 2 = 0$

$2y = -2 - 3x$

$y = \frac{-2-3x}{2}$

$m_r = -\frac{3}{2}$

$s: -x + 2y + 5 = 0$

$2y = x - 5$

$y = \frac{x-5}{2}$

$m_s = \frac{1}{2}$

$tg\,\theta = \left|\frac{m_r - m_s}{1 + m_r . m_s}\right|$

$tg\,\theta = \left|\frac{-\frac{3}{2} - \frac{1}{2}}{1 + \frac{1}{2} \cdot -\frac{3}{2}}\right| \rightarrow tg\theta = \left|\frac{-4/2}{1/4}\right|$

$tg\theta = \left|\frac{-16}{2}\right| \rightarrow$ $tg\theta = 8$

2. Calcule a cotangente do ângulo agudo formado pelas retas $x = 3y + 7$ e $x = 13y + 9$.

$r: x = 3y + 7$

$\frac{x-7}{3} = y$

$y = \frac{1}{3}x - \frac{7}{3}$

$m_r = \frac{1}{3}$

$s: x = 13y + 9$

$\frac{x-9}{13} = y$

$y = \frac{1}{13}x - \frac{9}{13}$

$m_s = \frac{1}{13}$

$tg\theta \left|\frac{\frac{1}{3} - \frac{1}{13}}{1 + \frac{1}{3} \cdot \frac{1}{13}}\right| \rightarrow tg\theta = \left|\frac{10/39}{40/39}\right| \rightarrow tg\theta = 1/4$

$cotg\,\theta = \frac{1}{tg\theta} = \frac{1}{1/4} \rightarrow$ $cotg\,\theta = 4$

Calcule o ângulo agudo formado pelas seguintes retas:

3. $(r)\ x + 2y - 3 = 0$ e $(s)\ 2x + 3y - 5 = 0$

$r: x + 2y - 3 = 0$

$2y = -x + 3$

$y = \frac{-x+3}{2}$

$m_r = -1/2$

$s: 2x + 3y - 5 = 0$

$3y = 5 - 2x$

$y = -\frac{2x}{3} + \frac{5}{3}$

$m_s = -2/3$

$tg\theta \left|\frac{-\frac{1}{2} - (-\frac{2}{3})}{1 + (-\frac{1}{2}) \cdot (-\frac{2}{3})}\right|$

$tg\theta = \left|\frac{1/6}{4/3}\right| \rightarrow tg\theta = \frac{1}{8}$

$\theta = arc.tg\ \frac{1}{8}$

4. $(r)\ \frac{x}{3} + \frac{y}{5} = 1$ e $(s) \begin{cases} x = t + 1 \\ y = 2t \end{cases}$

$r: \frac{x}{3} + \frac{y}{5} = 1$

$\frac{y}{5} = -\frac{x}{3} + 1$

$y = -\frac{5x}{3} + 5$

$m_r = -5/3$

$s: \begin{cases} x = t + 1 \rightarrow t = x - 1 \\ y = 2t \rightarrow y = 2.(x-1) \end{cases}$

$m_s = 2$

$tg\,\theta \left|\frac{-5/3 - 2}{1 - 5/3 . 2}\right|$

$tg\theta = \frac{11/3}{7/3}$ $\qquad \theta = arc.tg\left(\frac{11}{7}\right)$

5. $(r)\ \frac{x}{2} + \frac{y}{-3} = 1$ e $(s)\ 2x - 3 = 0$

$r: \frac{x}{2} + \frac{y}{-3} = 1$

$\frac{y}{-3} = -\frac{x}{2} + 1$

$y = \frac{3}{2}x - 3$

$m_r = 3/2$

$s: 2x - 3 = 0$

$2x = 3$

$x = 3/2$

$m_s \rightarrow \infty \rightarrow \nexists$

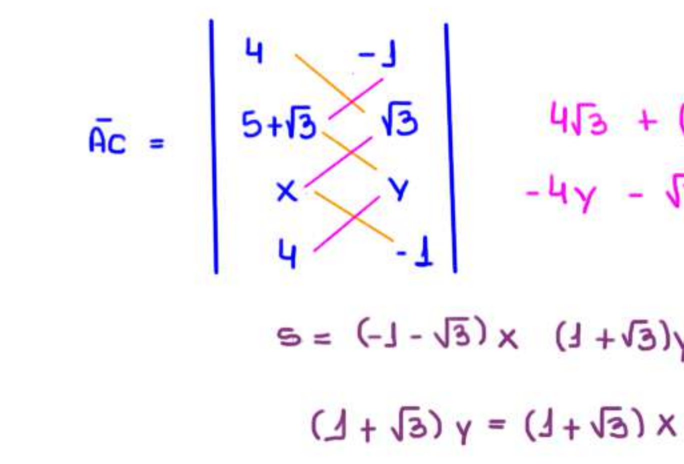
Calculamos uma reta t, ortogonal a m_r

$m_t = \frac{-1}{3/2}$ $\qquad m_t = -\frac{2}{3}$

$arctg\,\beta \left|\frac{0 - 2/3}{1 + 0 . 2/3}\right|$

$arctg\,\beta = \frac{2}{3}$

6. Dados os pontos A(4, -1), B(2, -1) e C($5+\sqrt{3}, \sqrt{3}$), calcule os ângulos internos do triângulo ABC.



Vamos calcular os coeficientes angulares.

$\overline{AC} = \begin{vmatrix} 4 & -1 \\ 5+\sqrt{3} & \sqrt{3} \\ x & y \\ 4 & -1 \end{vmatrix}$

$4\sqrt{3} + (5+\sqrt{3})y - x$
$-4y - \sqrt{3}x + (5+\sqrt{3}) = 0$

$s = (-1-\sqrt{3})x\ \ (1+\sqrt{3})y + (5 + 5\sqrt{3} = 0$

$(1+\sqrt{3})y = (1+\sqrt{3})x - (5 + 5\sqrt{3})$

$y = \frac{(1+\sqrt{3})x}{(1+\sqrt{3})} - \frac{5(1+\sqrt{3})}{1+\sqrt{3}}$ $\qquad y = x - 5$

$m_s = 1$

$\overline{AB} = \begin{vmatrix} 4 & -1 \\ 2 & -1 \\ x & y \\ 4 & -1 \end{vmatrix}$

$-4 + 2y - x - 4y + x + 2$

$t: -2y - 2 = 0$

$-2y = 2$

$y = -1$

$m_t = 0$, não tem coeficiente angular

$tg\theta = \left|\frac{m_t - m_s}{1 + m_t \cdot m_s}\right|$ $\qquad tg\theta = \left|\frac{0 - 1}{1 + 0 \cdot 1}\right|$ $\qquad tg\theta = -1$ ‖ $45°$

Como está no segundo quadrante, $tg\,135° = -tg45°$

→ Então, $\theta = 135°$ ou $\frac{3\pi}{4}$

$\overline{BC} = \begin{vmatrix} 2 & -1 \\ 5+\sqrt{3} & \sqrt{3} \\ x & y \\ 2 & -1 \end{vmatrix}$

$2\sqrt{3} + (5+\sqrt{3})y - x - 2y$
$-\sqrt{3}x + (5+\sqrt{3})$

$r = (-1-\sqrt{3})x + (3+\sqrt{3})y + (5 + 3\sqrt{3})$

$(3+\sqrt{3})y = (1+\sqrt{3})x - (5 + 3\sqrt{3})$

$y = \frac{(1+\sqrt{3})x}{(3+\sqrt{3})} - \frac{5-3\sqrt{3}}{3+\sqrt{3}}$

$m_r = \frac{1+\sqrt{3}}{3+\sqrt{3}} \cdot \frac{3-\sqrt{3}}{3-\sqrt{3}}$

$m_r = \frac{3 - \sqrt{3} + 3\sqrt{3} - \sqrt{3^2}}{9 - 3\sqrt{3} + 3\sqrt{3} - \sqrt{3^2}}$

$m_r = \frac{2\sqrt{3}}{6} \rightarrow m_r = \frac{\sqrt{3}}{3}$

$tg\,\alpha = \left|\frac{m_r - m_t}{1 + m_r \cdot m_t}\right|$ $\qquad tg\alpha = \left|\frac{\sqrt{3}/3 - 0}{1 + \sqrt{3}/3 \cdot 0}\right|$

$tg\alpha = \frac{\sqrt{3}}{3} \rightarrow \alpha = 30°$ ou $\frac{\pi}{6}$

Como a soma dos ângulos internos é 180°

$180° = \theta + \alpha + x$

$180° = 135° + 30° + x$

$\rightarrow x = 15°$ ou $\frac{\pi}{12}$

Logo, $\frac{3\pi}{4}, \frac{\pi}{6}$ e $\frac{\pi}{12}$