Matemática

Geometria Plana

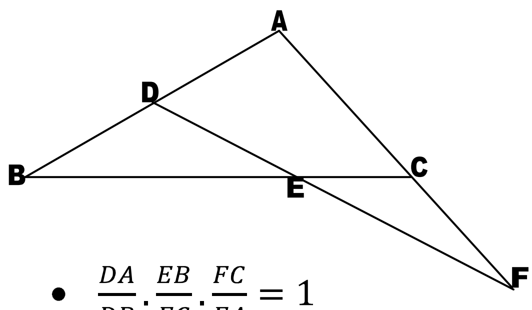
⇒ Triângulos Quaisquer

- $\alpha + \beta + \gamma = \pi$
- $\frac{a}{\operatorname{sen}\alpha} = \frac{b}{\operatorname{sen}\beta} = \frac{c}{\operatorname{sen}\gamma} = 2R$
- $a^2 = b^2 + c^2 - 2.b.c.\cos\alpha$
- $4m_A^2 = 2(b^2 + c^2) - a^2$
- $b_{iA} = \frac{2}{b+c}\sqrt{b.c.(p-a).p}$
- $b_{eA} = \frac{2}{|b-c|}\sqrt{b.c.(p-b).(p-c)}$
- $\frac{1}{r} = \frac{1}{r_a} + \frac{1}{r_b} + \frac{1}{r_c} = \frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c}$
- $\frac{1}{r_a} = \frac{1}{h_b} + \frac{1}{h_c} - \frac{1}{h_a}$
- $r_a.r_b + r_a.r_c + r_b.r_c = p^2$
- $r_a + r_b + r_c - r = 4R$
- $S = \frac{a.h_A}{2} = p.r = \frac{a.b.c}{4R} = \sqrt{p.(p-a).(p-b).(p-c)} = (p-a).r_A = \sqrt{r.r_A.r_B.r_C} = \sqrt{\frac{1}{2}R.h_A.h_B.h_C} = \frac{r_A.r_B.r_C}{p} = \frac{1}{2}a.b.\operatorname{sen}\gamma$

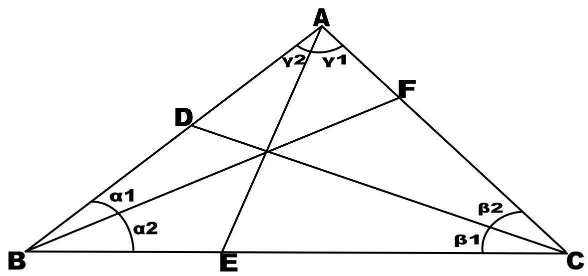
⇒ Triângulos Retângulos

- $a^2 = b^2 + c^2$
- $a = m + m$
- $h_A^2 = m.n$
- $b^2 = a.n$
- $c^2 = a.m$
- $b.c = a.h_A$
- $\frac{1}{h_A^2} = \frac{1}{b^2} + \frac{1}{c^2}$

⇒ Menelaus e Ceva

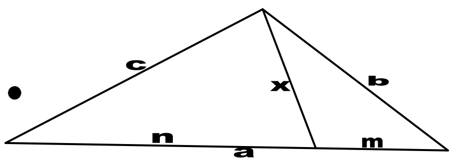


- $\frac{DA}{DB}.\frac{EB}{EC}.\frac{FC}{FA} = 1$



- $\frac{\operatorname{sen}\alpha_1.\operatorname{sen}\beta_1.\operatorname{sen}\gamma_1}{\operatorname{sen}\alpha_2.\operatorname{sen}\beta_2.\operatorname{sen}\gamma_2} = 1$

⇒ Stewart

- 

$n.b^2 + m.c^2 = m.n.a + x^2.a$

⇒ Quadriláteros

1. Quaisquer
   - $a^2 + b^2 + c^2 + d^2 = p^2 + q^2 + 4\overline{MN}^2$
   - $S = \frac{1}{4}\sqrt{4p^2q^2 - (a^2 - b^2 + c^2 - d^2)}$

2. $p \perp q$
   - Paralelogramo
     - $a = c \;\; e \;\; b = d$
     - $p^2 + q^2 = 2(a^2 + b^2) = 2(b^2 + c^2)$
     - $S = a.h$
   - Losango
     - $a = b = c = d$
     - $p^2 + q^2 = 4a^2$
     - $S = \frac{1}{2}.p.q = 2.a.r = a.h$

- Trapézio
  - $B_m = \frac{a+c}{2} \qquad \overline{MN} = \frac{a-c}{2}$
  - $p^2 + q^2 = b^2 + d^2 + 2a.c$
  - $S = \frac{1}{2}(a+c).h$
- Trapézio Isósceles
  - $p = q = \sqrt{b^2 + a.c}$
  - $h = \frac{1}{2}\sqrt{4b^2-(a-c)^2}$
  - $S = \frac{(a+c)}{2}.h$

3. Inscritível ($A + C = B + D = \pi$)
   - $Ptolomeu\text{:}\ p.q = a.c + b.d$
   - $R = \frac{1}{4S}\sqrt{(a.b+c.d)(a.c+b.d)(a.d+b.c)}$
   - $S = \sqrt{(p-a)(p-b)(p-c)(p-d)}$
   - $Hiparco\text{:}\ \frac{p}{q} = \frac{a.b+c.d}{a.d+b.c}$
4. Circunscritível
   - $a + c = b + d$
   - $S = (a+c).r = p.r$
5. Inscritível & Circunscritível
   - $r = \frac{\sqrt{a.b.c.d}}{a+c} = \frac{\sqrt{a.b.c.d}}{b+d}$
   - $S = \sqrt{a.b.c.d}$

⇒ Polígonos Regulares

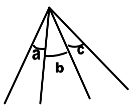
- $L_n = 2R\,\mathrm{sen}(\frac{\pi}{n})$
- $a_n = R\cos(\frac{\pi}{n})$
- $S_n = \frac{R^2}{2}\mathrm{sen}(\frac{2\pi}{n})$
- $\theta_{central} = p.(\frac{2\pi}{n})$
- $p \to espécie \begin{cases} p = 1 \to convexo \\ p > 1 \to estrelado \end{cases}$
- $p < \frac{n}{2}\ \ e\ \ (p,n) =$

⇒ Polígonos Estrelados

Geometria Espacial

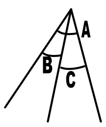
⇒ Teorema das três perpendiculares

⇒ Triedros



- $a > b > c > d$
- $a + b + c + \cdots < 360°$
- $a < b + c + \cdots$

⇒ Diedros

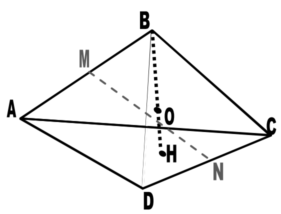


- $A > B > C$
- $A + B < C + \pi$
- $\pi < A + B + C < 3\pi$

⇒ Poliedros

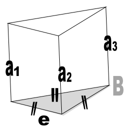
- $V + F = A + 2$
- $F = f_3 + f_4 + \cdots + f_n$
- $V = v_3 + v_4 + \cdots + v_n$
- $2A = 3f_3 + 4f_4 + \cdots + nf_n$
- $2A = 3v_3 + 4v_4 + \cdots + nv_n$

⇒ Tetraedro Regular



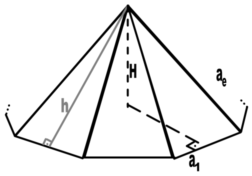
- $AD \perp BC$
- $\frac{OM}{1} = \frac{OA}{2} = \frac{AH}{4}$
- $H = \frac{a\sqrt{6}}{3}$
- $MN = \frac{a\sqrt{2}}{2}$
- $S = a^2\sqrt{3}$
- $V = \frac{a^3\sqrt{2}}{12}$

⇒ Tronco de Prisma Regular



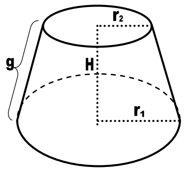
- $S_e = e.(a_1 + a_2 + \cdots + a_n)$
- $V = \frac{B.(a_1+a_2+\cdots+a_n)}{3}$

⇒ Pirâmide Regular



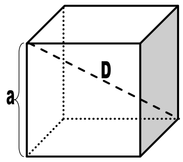
- $S_e = p.h$
- $S_t = p.(h + a_1)$
- $V = \frac{B.H}{3}$
- $V_T^{1^a} = \frac{h}{3}(S + \sqrt{S\Delta} + \Delta)$
- $V_T^{2^a} = \frac{h}{3}(S - \sqrt{S\Delta} + \Delta)$

⇒ Tronco de Cone Reto



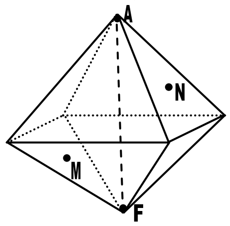
- $S_l = \pi.(r_1 + r_2).g$
- $S_t = \pi.r_1.(g + r_1) + \pi.r_2.(g + r_2)$
- $V = \frac{\pi.h}{3}(r_1^2 + r_1.r_2 + r_2^2)$

⇒ Cubo



- $D = a\sqrt{3}$
- $S_t = 6a^2 = 2D^2$
- $V = a^3$

⇒ Octaedro Regular



- $MN = \frac{a\sqrt{6}}{3}$
- $AF = a\sqrt{2}$
- $S = 2a^2\sqrt{3}$
- $V = \frac{a^3\sqrt{2}}{3}$

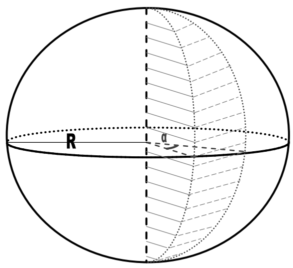
⇒ Cilindro

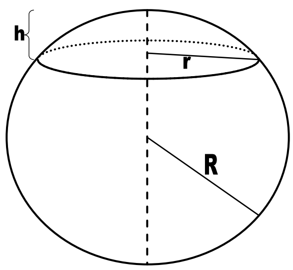
- $S_e = 2.\pi.r.h$
- $S_t = 2.\pi.r.(r + h)$
- $V = \pi.r^2.h$

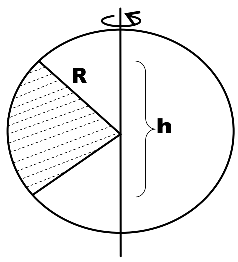
⇒ Cone Reto

- $S_e = \pi.r.g$
- $S_t = \pi.r.(g + r)$
- $V = \frac{1}{3}.\pi.r^2.h$
- $R_c = \frac{g^2}{2.h}$
- $r_i = \frac{r.h}{r+g}$
- $R^* = \frac{r.g}{h}$

⇒ Esfera







- $S_{fuso} = 2.\alpha.R^2$
- $V_{cunha} = \frac{2}{3}.\alpha.R^3$

- $V = \frac{\pi.h}{6}(h^2 + 3r^2)$
- $V = \frac{\pi.h^2}{3}(3R - h)$
- $S = 2.\pi.R.h$

- $V = \frac{2}{3}.\pi.R^2.h$

Geometria Analítica

- $versor: \vec{V} = \frac{\vec{e}}{|\vec{e}|}$
- $\vec{I} = \frac{a.\vec{A}+b.\vec{B}+c.\vec{C}}{a+b+c}$
- $\vec{P} = \frac{a.\vec{A}+b.\vec{B}}{a+b}$
- $\vec{G} = \frac{\vec{A}+\vec{B}+\vec{C}}{3}$
- $\vec{H} = \frac{\tan\alpha.\vec{A}+\tan\beta.\vec{B}+\tan\gamma.\vec{C}}{\tan\alpha+\tan\beta+\tan\gamma}$

⇒ Produto Escalar

- $\vec{u}.\vec{v} = |\vec{u}||\vec{v}|\cos\theta$
- $Cauchy - Schwartz: -|\vec{u}||\vec{v}| \leq \vec{u}.\vec{v} \leq |\vec{u}||\vec{v}|$
- $\vec{u}.\vec{v} = (x_1x_2 + y_1y_2 + z_1z_2)$
- $\vec{u} \perp \vec{v} \Leftrightarrow \vec{u}.\vec{v} = 0$
- $\vec{U}\vec{v} = \frac{\vec{u}.\vec{v}}{|\vec{v}|}.\frac{\vec{v}}{|\vec{v}|}$

⇒ Produto Vetorial

- $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}|\,\mathrm{sen}\,\theta$
- Regra da Mão Esquerda
- $S_{\#} = \frac{1}{2}\left|\overrightarrow{AC} \times \overrightarrow{BD}\right|$
- $S_{\triangle} = \frac{1}{2}|\vec{u} \times \vec{v}|$
- $S_{\square} = |\vec{u} \times \vec{v}|$
- $\vec{u} \parallel \vec{v} \Leftrightarrow \vec{u} \times \vec{v} = 0$
- $\vec{u} \times \vec{v} = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{bmatrix}$

⇒ Fórmula "Mágica" de Áreas

- ♦ $\frac{1}{2}\sqrt{\begin{bmatrix} x_1 & \cdots & x_1 \\ y_1 & \cdots & y_1 \end{bmatrix}^2 + \begin{bmatrix} x_1 & \cdots & x_1 \\ z_1 & \cdots & z_1 \end{bmatrix}^2 + \begin{bmatrix} y_1 & \cdots & y_1 \\ z_1 & \cdots & z_1 \end{bmatrix}^2}$

⇒ Transformação Afim

- ♦ $\frac{S'}{A'} = \frac{S}{A}$

⇒ Produto Misto

- $V_{paralelogramo} = |[\vec{u}\vec{v}\vec{w}]|$
- $V_{tetraedro} = \frac{|[\vec{u}\vec{v}\vec{w}]|}{6}$
- $Coplanares: \left[\overrightarrow{AB}.\overrightarrow{AC}.\overrightarrow{AD}\right] = 0$
- $[\vec{u}\vec{v}\vec{w}] = \vec{u}(v \times \vec{w}) = |\vec{u}||\vec{v}||\vec{w}|\,\mathrm{sen}\,\theta\cos\varphi$
- $[\vec{u}\vec{v}\vec{w}] = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$

⇒ Plano

- $\alpha \begin{cases} \vec{N} = (a,b,c) \\ P_0 = (x_0,y_0,z_0) \end{cases}$
- $\alpha: \ ax + by + cz + d = 0$
- $\alpha: \ \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

⇒ Superfície Esférica

- $\mathcal{E}: \ (x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 = R^2$
- $\mathcal{E}: \ x^2 + y^2 + z^2 + Ax + By + Cz + D = 0$

⇒ Plano Radical

- $(A_1 - A_2)x + (B_1 - B_2)y + (C_1 - C_2)z + (D_1 - D_2) = 0$

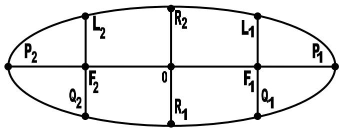
⇒ Reta

- $r: ax + by + c = 0$
- $d = \frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$
- $\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

⇒ Elipse

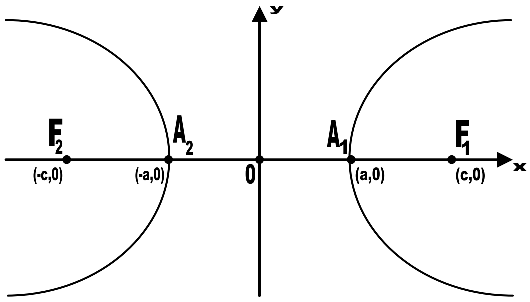
- $\mathcal{E}$: $\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$
- $a^2=b^2+c^2$
- $excentricidade$: $e=\frac{c}{a}<1(elipse)$
- $S=\pi.a.b$

- $\overline{R_1R_2}=2b$
- $\overline{F_1F_2}=2c$
- $\overline{P_1P_2}=2a$
- $Parâmetro$: $\overline{L_1Q_1}=\frac{2b^2}{a}$

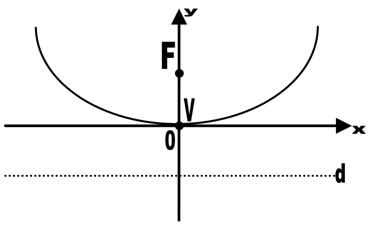


⇒ Hipérbole

- $\mathcal{H}$: $\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}=1$
- $c^2=a^2+b^2$
- $excentricidade$: $e=\frac{c}{a}>1(hipérbole)$
- $\overline{F_1F_2}=2c$
- $\overline{A_1A_2}=2a$
- $Assíntotas$: $y=\pm\left(\frac{b}{a}\right)x$



⇒ Parábola



- $\mathcal{P}$: $y=\frac{x^2}{2p}$
- $F=(x_V,y_V+\frac{p}{2})$
- $diretriz\ (d)$: $y=-\frac{p}{2}$

Trigonometria

- $\mathrm{f}(x)=A+B\,\mathrm{sen}(Cx+D)$
  - $período$: $\frac{2\pi}{|c|}$
  - $imagem$: $[A-B,A+B]$
- $\mathrm{sen}^2 x+\cos^2 x=1$
- $\tan^2 x+1=\sec^2 x$
- $\mathrm{ctg}^2 x+1=\csc^2 x$
- $\mathrm{sen}\,2x=\frac{2\tan x}{1+\tan^2 x}$
- $\cos 2x=\frac{1-\tan^2 x}{1+\tan^2 x}$
- $\mathrm{sen}(a\pm b)=\mathrm{sen}\,a\cos b\pm\mathrm{sen}\,b\cos a$
- $\cos(a\pm b)=\cos a\cos b\mp\mathrm{sen}\,a\,\mathrm{sen}\,b$
- $\tan(a\pm b)=\frac{\tan a\pm\tan b}{1\mp\tan a\tan b}$
- $\mathrm{sen}\,a\pm\mathrm{sen}\,b=2\,\mathrm{sen}\left(\frac{(a\pm b)}{2}\right)\cos\left(\frac{a\mp b}{2}\right)$
- $\cos a+\cos b=2\cos\left(\frac{(a+b)}{2}\right)\cos\left(\frac{(a-b)}{2}\right)$
- $\cos a-\cos b=-2\,\mathrm{sen}\left(\frac{(a+b)}{2}\right)\mathrm{sen}\left(\frac{(a-b)}{2}\right)$
- $\mathrm{sen}\,3x=3\,\mathrm{sen}\,x-4\,\mathrm{sen}^3 x$
- $\cos 3x=4\cos^3 x-3\cos x$

Ângulo	Seno	Cosseno	Tangente
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
15°	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$
75°	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$
22,5°	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{\sqrt{2+\sqrt{2}}}{2}$	...
67,5°	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	...
18°	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	...
72°	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{5}-1}{4}$	...
36°	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{\sqrt{5}+1}{4}$	...
64°	$\frac{\sqrt{5}+1}{4}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	...

- Equações
  - $\operatorname{sen} x = \operatorname{sen} \alpha$
    $\Rightarrow x = (-1)^k \alpha + k\pi, k \in \mathbb{Z}.$
  - $\cos x = \cos \alpha$
    $\Rightarrow x = \pm\alpha + 2k\pi, k \in \mathbb{Z}.$
  - $\operatorname{sen} x = \operatorname{sen} \alpha$
    $\Rightarrow x = \alpha + k\pi, k \in \mathbb{Z}.$

- $a \operatorname{sen} x + b \cos x = c$
  - $dividir\ por\ \sqrt{a^2+b^2}$

- $\mathrm{g}(x) = m + n.\mathrm{f}(ax+b) = c$
  - $P = \frac{p'}{|a|},$
    - $p'\!: período\ de\ f(x)$
    - $P\!: período\ de\ g(x)$

Teorema de Rouche-Capelli

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \Rightarrow plano\ \alpha_1 \\ a_2x + b_2y + c_2z = d_2 \Rightarrow plano\ \alpha_2 \\ a_3x + b_3y + c_3z = d_3 \Rightarrow plano\ \alpha_3 \end{cases}$$

- $\alpha_1 \cap \alpha_2 \cap \alpha_3 = \begin{cases} um\ ponto \Rightarrow Sistema\ Possível\ e\ Determinado \\ uma\ reta \Rightarrow Sistema\ Possível\ e\ Indeterminado \\ nada \Rightarrow Sistema\ Impossível \end{cases}$
- $\Delta = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$
- $M = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{bmatrix}$
- $Se\!: posto\ de\ \Delta = posto\ de\ M = número\ de\ incógnitas$
  $\Rightarrow Sistema\ Possível\ e\ Determinado$
- $Se\!: posto\ de\ \Delta = posto\ de\ M < número\ de\ incógnitas$
  $\Rightarrow Sistema\ Possível\ e\ Indeterminado$
- $Se\!: posto\ de\ \Delta < posto\ de\ M$
  $\Rightarrow Sistema\ Possível\ e\ Determinado$

- $Exemplo_1$:

$$\begin{cases} x + 2y - 3z = 1 \\ 2x + y + z = 2 \\ 3x - y - z = 3 \end{cases} \qquad M_i = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & 1 \\ 3 & -1 & -1 \end{bmatrix},\ \det(M_i) \neq 0;$$

$$\Delta p = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & 1 \\ 3 & -1 & -1 \end{bmatrix},\ p = n = 3. \quad \boldsymbol{S.P.D.}$$

- $Exemplo_2$:

$$\begin{cases} 2x + 3y + 2z = 5 \\ x - 2y - z = 3 \\ 3x + y + z = 8 \end{cases} \qquad M_i = \begin{bmatrix} 2 & 3 & 2 \\ 1 & -2 & -1 \\ 3 & 1 & 1 \end{bmatrix}, \det(M_i) = 0;$$

$$m_i = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}, \det(m_i) \neq 0; \quad \Delta p = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix};$$

$$C_r = \begin{bmatrix} 2 & 3 & 5 \\ 1 & -2 & 3 \\ 3 & 1 & 8 \end{bmatrix},\ p = 2,\ n = 3. \quad \boldsymbol{S.P.I.}$$

Complexos

- $Z = r(\cos\theta + i \operatorname{sen}\theta) = re^{i\theta} = x + yi \begin{cases} x = r\cos\theta \\ y = r \operatorname{sen}\theta \end{cases}$
- $Z^n = r^n \operatorname{cis} n\theta$
- $Z^n = r \operatorname{cis}\theta \Longrightarrow Z = \sqrt[n]{r} \operatorname{cis}\left(\frac{\theta + 2k\pi}{n}\right), k \in \{0,1,\dots,n-1\}$
  - $1 + \operatorname{cis} 2\alpha = 2\cos\alpha \operatorname{cis}\alpha$
  - $1 - \operatorname{cis} 2\alpha = \frac{2}{i} \operatorname{sen}\alpha \operatorname{cis}\alpha$

$$\sum_{k=1}^{n} \operatorname{sen} k\alpha = \frac{\operatorname{sen}\left(\frac{n}{2}\alpha\right)\operatorname{sen}\left(\frac{n+1}{2}\alpha\right)}{\operatorname{sen}\left(\frac{\alpha}{2}\right)} \qquad \sum_{k=1}^{n} \cos k\alpha = \frac{\operatorname{sen}\left(\frac{n}{2}\alpha\right)\cos\left(\frac{n+1}{2}\alpha\right)}{\operatorname{sen}\left(\frac{\alpha}{2}\right)}$$

Jansey

- Se $f''(a) < 0 \Longrightarrow$  $\quad \frac{f(a_1) + \cdots + f(a_n)}{n} \leq f\left(\frac{a_1 + \cdots + a_n}{n}\right)$
- Se $f''(a) > 0 \Longrightarrow$  $\quad \frac{f(a_1) + \cdots + f(a_n)}{n} \geq f\left(\frac{a_1 + \cdots + a_n}{n}\right)$

Funções

1) Reflexiva: aRa
2) Simétrica: aRb $\Longrightarrow$ bRa
3) Transitiva: aRb e bRc $\Longrightarrow$ aRc

- Equivalência $\rightarrow 1,2\ e\ 3$
- Ordem $\rightarrow 1, \overline{2}\ e\ 3$

Congruência

- $a \equiv b \ (mod\ m) \Rightarrow a = km + b$
- $a \pm c \equiv b \pm c \ (mod\ m)$
- $ac \equiv bc \ (mod\ m) \Rightarrow ac \equiv bc \ (mod\ mc)$
- $a^n \equiv b^n \ (mod\ m)$
- $ac \equiv bc \ (mod\ m) \Rightarrow a \equiv b \ \left(mod\ \frac{c}{(c,m)}\right)$

- *Teorema de Fermat*: $a^{p-1} \equiv 1 \ (mod\ p),\ p\ primo.$
- *Teorema de Euler*: $a^{\phi(m)} \equiv 1 \ (mod\ m),\ (a,m) = 1.$
- *Teorema de Wilson´s*: $(p-1)! + 1 \equiv 0 \ (mod\ p).$

- $n = p_1^{\alpha_1}.p_2^{\alpha_2}\dots p_m^{\alpha_m}$
- $\phi(n) = \left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\dots\left(1-\frac{1}{p_m}\right)$

- $Cubos\ Perfeitos \Rightarrow \equiv 0, \pm 1\ (mod 7)$
- $Quadrados \Rightarrow \equiv 0{,}1\ (mod\ 3\ ou\ 4)\ e \equiv 0{,}1{,}4\ (mod\ 8)$

- $Euler$: $a^{\frac{\phi(p)}{2}} \equiv \left(\frac{a}{p}\right)\ (mod\ p), p\ primo.$

$$\left(\frac{a}{p}\right) = \begin{cases} 1, se\ \exists\ x\ tal\ que\ x^2 \equiv a\ (mod\ p) \\ 0, se\ p|a \\ -1, caso\ contrário \end{cases}$$

Matrizes

- $(A \pm B)^t = A^t \pm B^t$
- $(AB)^t = B^t A^t$
- $A = A^t: simétrica$
- $A = -A^t: antissimétrica$
- $\mathrm{tr}(A \pm B) = \mathrm{tr}\, A \pm \mathrm{tr}\, B$
- $\mathrm{tr}(AB) = \mathrm{tr}(BA)$

- $\det A^t = \det A$
- $\det(AB) = \det A . \det B$
- $\det A^{-1} = (\det A)^{-1}$
- $A^{-1} = \frac{1}{\det A}[\mathrm{cof}\, A]^t$
- $\det(kA) = k^n \det A$
- $\exists A^{-1} \Leftrightarrow \det A \neq 0$

- Auto-Valor e Auto-Vetor
  - $\underset{(n\times x)}{A} \cdot \underset{(n\times 1)}{V} = \lambda.V$ $\quad$ $\lambda: auto - valor$; $V: auto - vetor$
- Semelhança
  - $A = PBP^{-1}$
  - $A^n = PB^nP^{-1}$
  - $\det A = \det B$
  - $\mathrm{tr}\, A = \mathrm{tr}\, B$

- $(A - \lambda I)V = 0 \quad : sistema\ homogêneo$
- $\det(A - \lambda I) = 0 :\ \lambda$ é $auto - valor\ de\ A$

- $\lambda_1 + \lambda_2 + \dots + \lambda_n = \mathrm{tr}\, A$
- $\lambda_1.\lambda_2 \dots \lambda_n = \det A$

- Radical Duplo
  $\sqrt{A \pm \sqrt{B}} = \sqrt{\frac{A+C}{2}} \pm \sqrt{\frac{A-C}{2}}, C^2 = A^2 + B$
- Sophie-German
  $A^4 + 4B^4 = [(A+B)^2 + B^2][(A-B)^2 + B^2]$
- Desigualdades
  - $MQ \geq MA \geq MG \geq MH$
  - $e^x \geq x + 1$
  - $Médias\ Potenciais$: $\alpha \geq \beta \Rightarrow \left(\frac{a_1^\alpha + \dots + a_n^\alpha}{n}\right)^{\frac{1}{\alpha}} \geq \left(\frac{a_1^\beta + \dots + a_n^\beta}{n}\right)^{\frac{1}{\beta}}$
  - Chebyschev
    $Se\ \begin{matrix} \alpha_1 \leq \alpha_2 \leq \dots \leq \alpha_n \\ \beta_1 \leq \beta_2 \leq \dots \leq \beta_n \end{matrix}, então\ \left(\frac{\sum a_k}{n}\right)\left(\frac{\sum b_k}{n}\right) \leq \left(\frac{\sum a_k b_k}{n}\right)$

- Relações de Morgan
  - $(A \cup B)^c = A^c \cap B^c$
  - $(A \cap B)^c = A^c \cup B^c$

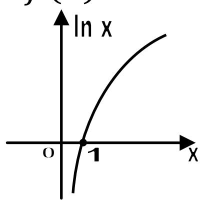
Cálculo

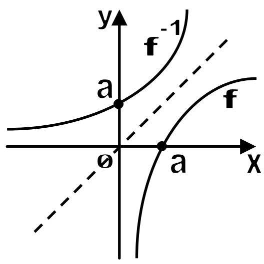
- Fórmula de Taylor
  - $f(x) = \frac{f(x_0)}{0!} + \frac{[f'(x_0)](x-x_0)}{1!} + \frac{[f''(x_0)](x-x_0)^2}{2!} + \cdots \frac{[f^{\overbrace{\prime\cdots\prime}^{n}}(x_0)](x-x_0)^n}{n!}$

- Série de Taylor
  - $e^x = 1 + \frac{x}{1!} + \frac{x^2}{1!} + \cdots + \frac{x^n}{n!} + \cdots$
  - $e^{ix} = \underbrace{\left(\frac{1}{0!} - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots\right)}_{cos\, x} + i\underbrace{\left(\frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)}_{sen\, x}$

- Comprimento
  - $c = \int \sqrt{1 + [f'(x)]^2} dx$
- Área
  - $S = \int f(x) dx$
- Volume
  - $V = \int [f(x)]^2 dx$

- $f(x) = \ln x$





1- $f \to par$
$\Rightarrow gof \to par$
2- $f \to ímpar\ e\ g \to par$
$\Rightarrow gof \to ímpar$

I. $f\ e\ g \to est.cres. \Rightarrow gof \to est.cres.$
II. $f\ e\ g \to est.decr. \Rightarrow gof \to est.decr.$
III. $f \to e.d.\ \&\ g \to e.c. \Rightarrow gof \to e.d.$
IV. $f \to e.c.\ \&\ g \to e.d. \Rightarrow gof \to e.d.$

$*\ f: A \to B\ e\ g: B \to C$
i. $f\ e\ g \to I \Rightarrow gof \to I(A \to C)$
ii. $f\ e\ g \to S \Rightarrow gof \to S(A \to C)$
iii. $f\ e\ g \to Bi \Rightarrow gof \to Bi(A \to C)$

Polinômio de Leibniz

- $\left(x_1 + x_2 + \cdots + x_p\right)^2 = \sum \frac{n!}{\alpha_1!\alpha_2!\ldots\alpha_p!} . x_1^{\alpha_1} . x_2^{\alpha_2} \ldots x_p^{\alpha_p}$

$\alpha_1 + \alpha_2 + \cdots + \alpha_p = n,\ \alpha_i \in \mathbb{N}.$

Número de Funções Sobrejetoras

- $f: E \to F$
  $\#|E| = n$
  $\#|F| = p$
  $n \geq p$

$$T = \sum_{k=0}^{p-1} \binom{p}{k} (p-k)^n (-1)^k$$

Teoria dos Números

$Dados: \quad a, b \in \mathbb{z}, \quad seja\ I = \{ax + by\ /\ x, y \in \mathbb{z}\}$

1- $Bézout: \quad menor\ elemento\ positivo\ de\ I = (a, b)$

2- $Corolário: \quad (a, b) = 1 \quad e \quad a|bc \quad \Rightarrow \quad a|c$

$\quad p\ \text{é}\ primo \quad e \quad p|ab \quad \Rightarrow \quad p|a \quad ou \quad p|b$

$\phi(p^{\alpha}) = p^{\alpha-1}(p-1)$

$\phi(m.n) = \phi(m).\phi(n), se\ (m, n) = 1$

3- $Euclides: \quad (a, b) = (a, ax + b),\ \forall x \in \mathbb{z}$

Combinatória - SOMAS

1- $\binom{p}{p} + \binom{p+1}{p} + \cdots + \binom{p+n}{p} = \binom{p+n+1}{p+1}$

2- $\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n+p}{p} = \binom{n+p+1}{p}$

3- $\binom{n}{0} + 2\binom{n}{1} + \cdots + (n+1)\binom{n}{n} = (n+2).2^{n-1}$

4- $\binom{n}{0} + \frac{\binom{n}{1}}{2} + \cdots + \frac{\binom{n}{n}}{n+1} = \frac{n.2^{n+1}}{(n+1)(n+2)} . \frac{2^{n+1}-1}{(n+1)}$

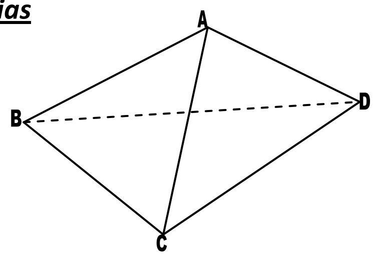
5- $\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n$

6- $\binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n}$

7- $\binom{m}{0}\binom{n}{k} + \binom{m}{1}\binom{n}{k-1} + \cdots + \binom{m}{k}\binom{n}{0} = \binom{m+n}{k}$

Matriz de Adjacências

Exemplo:



$$M = \begin{array}{c} \begin{matrix} A & B & C & D \end{matrix} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{matrix} A \\ B \\ C \\ D \end{matrix}$$

- Teorema: M^n dá o número de caminhos entre os vértices correspondentes em n passos.