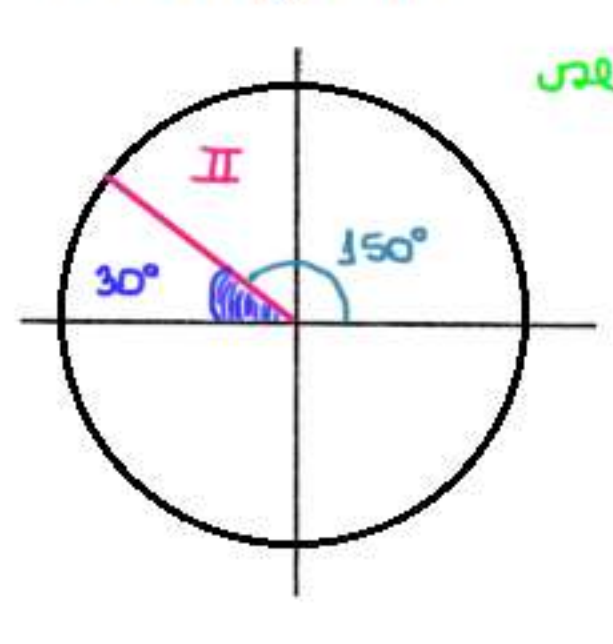
PROFESSOR FERRETTO

Teoria:

- $\text{cotg}\,x = \frac{1}{tg\,x} = \frac{\cos x}{\text{sen}\,x}$
- $\sec x = \frac{1}{\cos x}$
- $\text{cossec}\,x = \frac{1}{\text{sen}\,x}$

Dê o valor de:

1. cotg 150°



segundo quadrante = negativo

$\text{cotg}\,150° = \frac{1}{tg\,150°} \rightarrow \text{cotg}\,150° = \frac{1}{-\sqrt{3}/3}$

$\text{cotg}\,150° = 1 \cdot \frac{3}{-\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \rightarrow \text{cotg}\,150° = \frac{3\sqrt{3}}{-3}$

$\boxed{\text{cotg}\,150° = -\sqrt{3}}$

2. cotg 225°

III quadrante
cotg é positiva

$\text{cotg}\,225° = \frac{1}{tg\,225°}$

$\text{cotg}\,225° = \frac{1}{1}$

$\boxed{\text{cotg}\,225° = 1}$

3. Dado $\text{sen}\,x = \frac{1}{2}$, quais são os possíveis valores de $\text{cotg}\,x$?

$\text{sen}^2 x + \cos^2 x = 1$

$\left(\frac{1}{2}\right)^2 + \cos^2 x = 1$

$\cos^2 x = 3/4$

$\cos x = \pm\sqrt{3/4}$

$\cos x = \pm\sqrt{3}/2$

$\text{cotg}\,x = \frac{\cos x}{\text{sen}\,x}$

$\text{cotg}\,x = \frac{\cos x}{1/2}$

$\text{cotg}\,x = 2 \cdot \cos x$

$\text{cotg}\,x = 2 \cdot \sqrt{3}/2 = \boxed{\sqrt{3}}$

$\text{cotg}\,x = 2 \cdot -\sqrt{3}/2 = \boxed{-\sqrt{3}}$

Dê o valor da secante e da cossecante de?

4. 120° → é equivalente à 60°, mas está no II quadrante

sec 120°

$\sec 120° = \frac{1}{\cos 120°}$

$\sec 120° = \frac{1}{-1/2}$

$\boxed{\sec 120° = -2}$

cossec 120°

$\text{cossec}\,120° = \frac{1}{\text{sen}\,120°}$

$\text{cossec}\,120° = \frac{1}{\sqrt{3}/2}$

$\text{cossec}\,120° = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\boxed{\text{cossec}\,120° = \frac{2\sqrt{3}}{3}}$

5. 300°

sec 300°

$300° = -60°$

$\sec 300° = \frac{1}{\cos 300°} = \frac{1}{\cos(-60°)}$

$\sec 300° = \frac{1}{1/2}$

$\sec 300° = 2$

cossec 300°

$\text{cossec}\,300° = \frac{1}{\text{sen}\,300°} = \frac{1}{\text{sen}(-60°)}$

$\text{cossec}\,300° = \frac{1}{-\sqrt{3}/2}$

$\text{cossec}\,300° = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\text{cossec}\,300° = -\frac{2\sqrt{3}}{3}$

6. Dado $\cos x = \frac{1}{4}$, $0 < x < \frac{\pi}{2}$, calcule $\text{tg}\,x$.

$\cos x = 1/4$

$\text{sen}^2 x + \cos^2 x = 1$

$\text{sen}^2 x + (1/4)^2 = 1$

$\text{sen}^2 x = 1 - 1/16$

$\text{sen}^2 x = 15/16$

$\text{sen}\,x = \pm\sqrt{15/16}$

$\text{sen}\,x = \pm\sqrt{15}/4$

entre 0 e $\pi/2$ = I quadrante
sen x é positivo

$tg\,x = \frac{\text{sen}\,x}{\cos x} \rightarrow tg\,x = \frac{\sqrt{15}/4}{1/4} \rightarrow \boxed{tg\,x = \sqrt{15}}$

7. Dada $\text{tg}\,x = 2$ e $0 < x < \frac{\pi}{2}$, calcule $\text{sen}\,x$.

$\frac{\text{sen}\,x}{\cos x} = 2 \rightarrow \cos x = \frac{\text{sen}\,x}{2}$

$\text{sen}^2 x + \cos^2 x = 1$

$\text{sen}^2 x + \left(\frac{\text{sen}\,x}{2}\right)^2 = 1$

$\text{sen}^2 x + \frac{\text{sen}^2 x}{4} = 1$

$\frac{5}{4}\,\text{sen}^2 x = 1$

$\text{sen}^2 x = \frac{4}{5}$

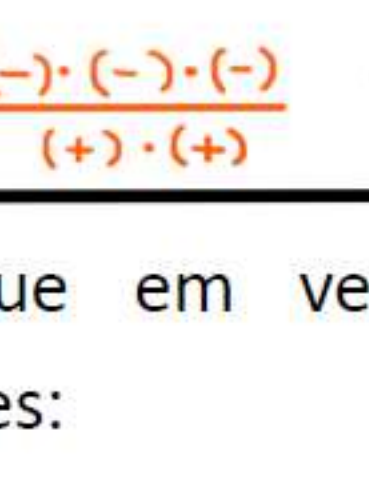
$\text{sen}\,x = \pm\sqrt{4/5}$ → é positivo, 1° quad. $0 < x < \pi/2$

$\text{sen}\,x = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

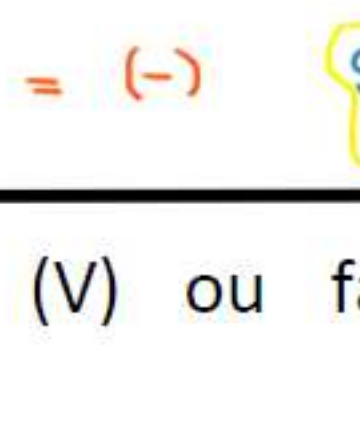
$\boxed{\text{sen}\,x = \frac{2\sqrt{5}}{5}}$

8. Sendo x um arco do 3° quadrante, qual é o sinal da expressão

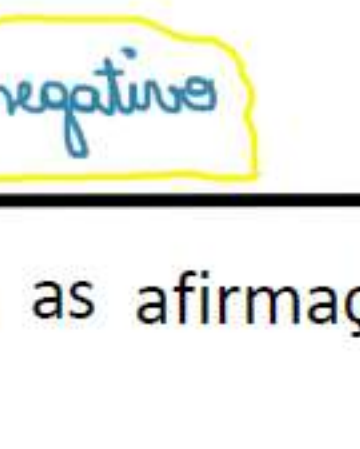
$$y = \frac{sen\,x \cdot \cos x \cdot \sec x}{tg\,x \cdot \sec(x - \pi)}?$$



$\text{sen}\,x = \text{cossec}\,x$



$\cos x = \sec x$



$tg\,x = \text{cotg}\,x$

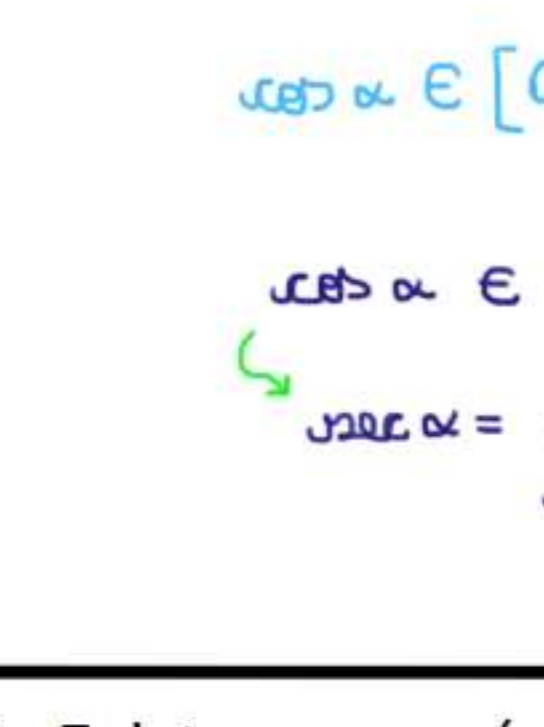
$\frac{(-\text{sen}\,x) \cdot (-\cos x) \cdot (-\sec x)}{(+tg\,x) \cdot (+\sec(x - \pi))}$

↳ x = 3° quad.
→ $x - \pi$ = 1° quad

$\frac{(-)\cdot(-)\cdot(-)}{(+)\cdot(+)} = \frac{(-)\cdot(+)}{(+)} = (-)$ Sinal negativo

Classifique em verdadeiras (V) ou falsas (F) as afirmações seguintes:

9. Existe um número real $\alpha \in [0, 2\pi]$ tal que $sec\,\alpha = 1/2$.



→ máx valor para cos α

$\sec\alpha = \frac{1}{\cos\alpha}$ $\quad \frac{1}{2} = \frac{1}{\cos\alpha}$

$\cos\alpha = 2$

Falso! cos α não é igual à 2, o valor máximo para cos α é = 1.

10. Se $\alpha \in \left[0, \frac{\pi}{2}\right]$, então $sec\,\alpha \geq 1$.

$\cos\alpha \in [0, \pi/2]$

$\cos\alpha \in [1, 0]$

$\sec\alpha = \frac{1}{\cos\alpha}$

$\sec\alpha = \left[\frac{1}{1}, \frac{1}{0}\right]$

$\sec\alpha = [1, \infty[$

Verdadeira

11. Existe um número real $\alpha \in [0, 2\pi]$ tal que

$cotg\,\alpha = 3$ e $cossec\,\alpha = 3$.

$\text{cossec}\,\alpha = 3$

$\frac{1}{\text{sen}\,\alpha} = 3$

$\text{sen}\,\alpha = 1/3$

$\text{sen}^2\alpha + \cos^2\alpha = 1$

$\cos^2\alpha = 1 - (1/3)^2$

$\cos\alpha = \pm\sqrt{8/9}$

$\cos\alpha = \pm 2\sqrt{2}/3$

$\text{cotg}\,\alpha = \frac{\cos\alpha}{\text{sen}\,\alpha}$

$\text{cotg}\,\alpha = \frac{2\sqrt{2}/3}{1/3}$

$\text{cotg}\,\alpha = \pm 2\sqrt{2}$

FALSA

12. $cotg\,\frac{7\pi}{8} \cdot \sec\frac{7\pi}{8} > 0$

$\frac{\cos 7\pi/8}{\text{sen}\,7\pi/8} \cdot \frac{1}{\cos 7\pi/8} > 0$

$\frac{1}{\text{sen}\,7\pi/8} > 0$

↳ $7\pi/8 = 157{,}5°$
↓
2° quadrante
↓
$\text{cossec}(7\pi/8) > 0$

$\text{cossec}\,7\pi/8 > 0$

$\text{sen}\,x = \text{cossec}\,x$

VERDADEIRA

13. Sabendo que $\cos x = 0{,}25$, determine o valor da expressão:

$$\frac{\sec x \cdot cossec\,x - sec^2 x}{cotg\,x - 1}$$

$\frac{\sec x \cdot \text{cossec}\,x - \sec^2 x}{\text{cotg}\,x - 1} = Y$

$\frac{\frac{1}{\cos x} \cdot \frac{1}{\text{sen}\,x} - \frac{1}{\cos^2 x}}{\frac{\cos x}{\text{sen}\,x} - 1}$ → mmc = $\cos^2 x \cdot \text{sen}\,x$; mmc = sen x

$\frac{\frac{\cos x - \text{sen}\,x}{\cos^2 x \cdot \text{sen}\,x}}{\frac{\cos x - \text{sen}\,x}{\text{sen}\,x}}$

$\frac{\cos x - \text{sen}\,x}{\cos^2 x \cdot \text{sen}\,x} \cdot \frac{\text{sen}\,x}{\cos x - \text{sen}\,x} \rightarrow \frac{1}{\cos^2 x}$

$\frac{1}{0{,}25^2} \rightarrow \boxed{Y = 16}$

14. Calcule m de modo que se tenha $tg\,x = m - 2$ e $cotg\,x = \frac{m}{3}$. ↳ m = ?

$\frac{\text{sen}\,x}{\cos x} = m - 2$ e $\frac{\cos x}{\text{sen}\,x} = m/3$

$\frac{\cos x}{\text{sen}\,x} = 1/(m-2) \rightarrow \frac{m}{3} = \frac{1}{m-2}$

$3 = m^2 - 2m \rightarrow m^2 - 2m - 3 = 0$

$-1 + 3 = -b/a = 2$

$-1 \cdot 3 = c/a = -3$

$\boxed{m_1 = 3 \text{ e } m_2 = -1}$

15. Sabendo que $tg\,x = \frac{7}{24}$ e $\pi < x < \frac{3\pi}{2}$, obtenha o valor da expressão

$$y = \frac{tg\,x \cdot \cos x}{(1 + \cos x) \cdot (1 - \cos x)}$$

$Y = \frac{\frac{\text{sen}\,x}{\cos x} \cdot \cos x}{1^2 - \cos x + \cos x - \cos^2 x}$

$Y = \frac{\text{sen}\,x}{1 - \cos^2 x} \rightarrow \text{sen}^2 x + \cos^2 x = 1$; $\text{sen}^2 x = 1 - \cos^2 x$

$Y = \frac{\text{sen}\,x}{\text{sen}^2 x}$

$Y = 1/\text{sen}\,x$

$tg\,x = 7/24$

$\frac{\text{sen}\,x}{\cos x} = 7/24 \rightarrow \cos x = \frac{24 \cdot \text{sen}\,x}{7}$

$\text{sen}^2 x + \cos^2 x = 1$

$\text{sen}^2 x + \left(\frac{24 \cdot \text{sen}\,x}{7}\right)^2 = 1$

$\text{sen}^2 x + \frac{576\,\text{sen}^2 x}{49} = 1$

$\frac{625}{49}\,\text{sen}^2 x = 1$

$\text{sen}\,x = \pm\sqrt{49/625}$ ↳ 3° quadrante → π

$\text{sen}\,x = -7/25 \rightarrow \boxed{\frac{1}{\text{sen}\,x} = -\frac{25}{7}}$