Relembrando:

$\bullet\ z_1 \cdot z_2 = \rho_1 \cdot \rho_2 \cdot (\cos(\theta_1+\theta_2) + i\,\text{sen}(\theta_1+\theta_2))$

$\bullet\ \dfrac{z_1}{z_2} = \dfrac{\rho_1}{\rho_2} \cdot (\cos(\theta_1-\theta_2) + i\,\text{sen}(\theta_1-\theta_2))$

$\bullet\ z^n = \rho^n \cdot (\cos n\theta + i\,\text{sen}\, n\theta)$

Expresse na forma trigonométrica o produto $z_1 z_2$:

1.

$z_1 = 3\left(cos\frac{\pi}{5} + i\,sen\frac{\pi}{5}\right)$ $\quad z_1 = \rho = 3$ e $\theta = \frac{\pi}{5}$

$z_2 = 4\left(cos\frac{2\pi}{5} + i\,sen\frac{2\pi}{5}\right)$ $\quad z_2 = \rho = 4$ e $\theta = \frac{2\pi}{5}$

$z_1 \cdot z_2 = 3.4.\left[\cos\left(\frac{\pi}{5}+\frac{2\pi}{5}\right) + i.\,\text{sen}\left(\frac{\pi}{5}+\frac{2\pi}{5}\right)\right]$

$\boxed{z_1 \cdot z_2 = 12.(\cos 3\pi/5 + i.\,\text{sen}\, 3\pi/5)}$

2.

$z_1 = 6\left(cos\frac{3\pi}{10} + i\,sen\frac{3\pi}{10}\right)$ $\quad z_1: \rho = 6,\ \theta = \frac{3\pi}{10}$

$z_2 = 5\left(cos\frac{2\pi}{10} + i\,sen\frac{2\pi}{10}\right)$ $\quad z_2: \rho = 5;\ \theta = 2\pi/10$

$z_1 \cdot z_2 = 6.5\left[\cos\left(\frac{3\pi}{10}+\frac{2\pi}{10}\right) + i.\,\text{sen}\left(\frac{3\pi}{10}+\frac{2\pi}{10}\right)\right]$

$\boxed{z_1 \cdot z_2 = 30.(\cos \pi/2 + i.\,\text{sen}\, \pi/2)}$

3.

$z_1 = 5(cos\,30° + i\,sen\,30°)$ $\quad z_1: \rho = 5,\ \theta = 30°$

$z_2 = 2(cos\,60° + i\,sen\,60°)$ $\quad z_2: \rho = 2,\ \theta = 60°$

$z_1 \cdot z_2 = 2.5.[\cos(30°+60°) + i.\,\text{sen}(30°+60°)]$

$z_1 \cdot z_2 = 10(\cancelto{0}{\cos 90°} + i.\,\cancelto{1}{\text{sen}\, 90°})$

$z_1 \cdot z_2 = 10(0+1i)$

$\to \boxed{z_1 \cdot z_2 = 10i}$

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4. Calcule $z_1 z_2 z_3$, sendo dados:

$z_1 = 2\left(cos\frac{\pi}{5} + i\,sen\frac{\pi}{5}\right);$ $\quad z_1: \rho = 2,\ \theta = \pi/5$

$z_2 = \sqrt{3}\left(cos\frac{3\pi}{5} + i\,sen\frac{3\pi}{5}\right)$ $\quad z_2: \rho = \sqrt{3},\ \theta = 3\pi/5$

e $z_3 = 2\sqrt{3}\left(cos\frac{6\pi}{5} + i\,sen\frac{6\pi}{5}\right).$ $\quad z_3: \rho = 2\sqrt{3},\ \theta = 6\pi/5$

$z_1 \cdot z_2 \cdot z_3 = 2.\sqrt{3}.2\sqrt{3}\left[\cos\left(\frac{\pi}{5}+\frac{3\pi}{5}+\frac{6\pi}{5}\right) + i.\,\text{sen}\left(\frac{\pi}{5}+\frac{3\pi}{5}+\frac{6\pi}{5}\right)\right]$

$z_1 \cdot z_2 \cdot z_3 = 12.(\underbrace{\cos 10\pi/5}_{\to \cos 2\pi = 1} + i.\,\underbrace{\text{sen}\, 10\pi/5}_{\to \text{sen}\, 2\pi = 0})$

$\boxed{z_1 \cdot z_2 \cdot z_3 = 12}$

Dado $z = \sqrt{2}\left(cos\frac{\pi}{4} + i\,sen\frac{\pi}{4}\right)$, calcule as potências:

5. $z^2 =$

$z: \rho = \sqrt{2},\ \theta = \frac{\pi}{4}$

$z^2 = (\sqrt{2})^2 \cdot (\cos 2.\pi/4 + i.\,\text{sen}\, 2.\pi/4)$

$z^2 = 2(\cancelto{0}{\cos \pi/2} + i\,\cancelto{1}{\text{sen}\,\pi/2})$

$z^2 = 2.0 + 2.i.1$

$\boxed{z^2 = 2i}$

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6. $z^5 =$

$\rho = \sqrt{2},\ \theta = \frac{\pi}{4}$

$z^5 = (\sqrt{2})^5.(\cos 5.\pi/4 + i.\,\text{sen}\, 5.\pi/4)$

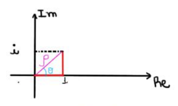
$z^5 = 4.\sqrt{2}\cdot(\cos 5\pi/4 + i.\,\text{sen}\, 5\pi/4)$ $\to$ $5\pi/4 = 225°$; $\cos 255° = -\sqrt{2}/2$; $\text{sen}\, 255 = -\sqrt{2}/2$

$z^5 = 4\sqrt{2}.(-\sqrt{2}/2 - i\sqrt{2}/2)$

$\boxed{z^5 = -4 - i4}$

Calcule:

7. $(1+i)^8 =$



$\rho = \sqrt{1^2+1^2} \to \rho = \sqrt{2}$

$\theta = tg^{-1}\left(\frac{1}{1}\right) \to \theta = 45°$

$z = \sqrt{2}.(\cos 45° + i.\,\text{sen}\, 45°)$

$z^8 = (\sqrt{2})^8.(\cos 8\cdot 45° + i.\,\text{sen}\, 8\cdot 45°)$

$z^8 = 16.(\cancelto{=1}{\cos 360°} + i.\,\cancelto{=0}{\text{sen}\, 360°})$

$z^8 = 16\cdot(1 + i.0)$

$\boxed{z^8 = 16}$

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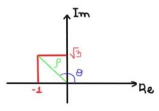
8. $\left(-1+\sqrt{3}\,i\right)^3 =$



$\rho^2 = (-1)^2 + \sqrt{3}^2 \to \rho = \sqrt{4} \to \rho = 2$

$\theta = tg^{-1}\left(\frac{\sqrt{3}}{-1}\right) \to \theta = 120°$

$z = 2.(\cos 120° + i.\,\text{sen}\, 120°)$

$z^3 = 2^3.(\cos 3.120° + i.\,\text{sen}\, 3.120°)$

$z^3 = 8.(\cancelto{=1}{\cos 360°} + i.\,\cancelto{=0}{\text{sen}\, 360°})$

$z^3 = 8.(1 + i.0)$

$\boxed{z^3 = 8}$

Dados:

$Z = 6\left(cos\frac{5\pi}{6} + i\,sen\frac{5\pi}{6}\right); W = 3\left(cos\frac{\pi}{4} + i\,sen\,\frac{\pi}{4}\right)$

$Z: \rho = 6,\ \theta = 5\pi/6$ $\qquad W: \rho = 3,\ \theta = \pi/4$

Calcule:

9. $\frac{Z}{W}$

$\frac{Z}{W} = \frac{6}{3}\left[\cos\left(\frac{5\pi}{6}-\frac{\pi}{4}\right) + i.\,\text{sen}\left(\frac{5\pi}{6}-\frac{\pi}{4}\right)\right]$

$\boxed{\frac{Z}{W} = 2.(\cos 7\pi/12 + i.\,\text{sen}\, 7\pi/12)}$

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10. $\frac{W}{Z}$

$\frac{W}{Z} = \frac{3}{6}\left[\cos\left(\frac{\pi}{4}-\frac{5\pi}{6}\right) + i.\,\text{sen}\left(\frac{\pi}{4}-\frac{5\pi}{6}\right)\right]$

$\frac{W}{Z} = \frac{1}{2}.(\cos -7\pi/12 + i.\,\text{sen} -7\pi/12)$ $\to 2\pi - \frac{7\pi}{12} = \frac{17\pi}{12}$

$\boxed{\frac{W}{Z} = \frac{1}{2}.(\cos 17\pi/12 + i.\,\text{sen}\, 17\pi/12)}$