Matemática

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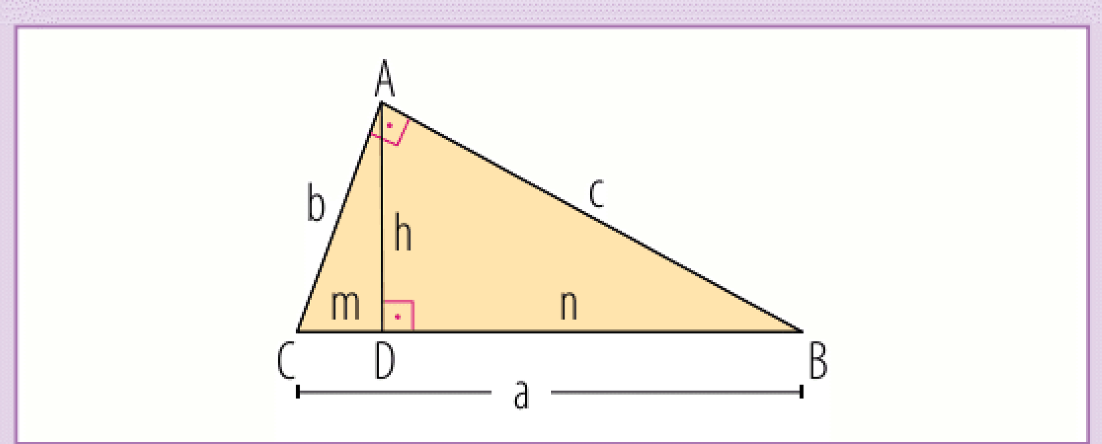
Trigonometria

Relações métricas no triângulo retângulo



$h^2 = m \cdot n$

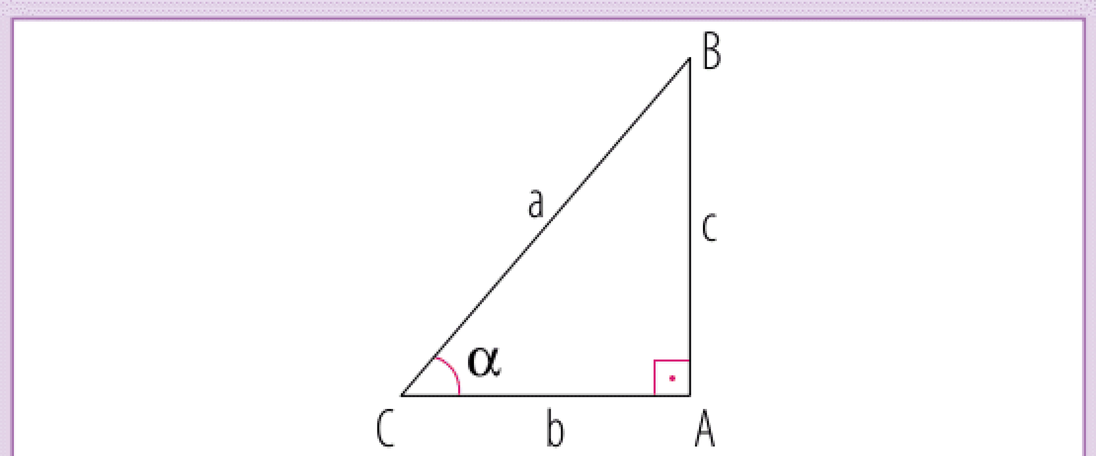
$b^2 = a \cdot m$

$c^2 = a \cdot n$

$b \cdot c = a \cdot h$

$a^2 = b^2 + c^2$ (Pitágoras)

Razões trigonométricas

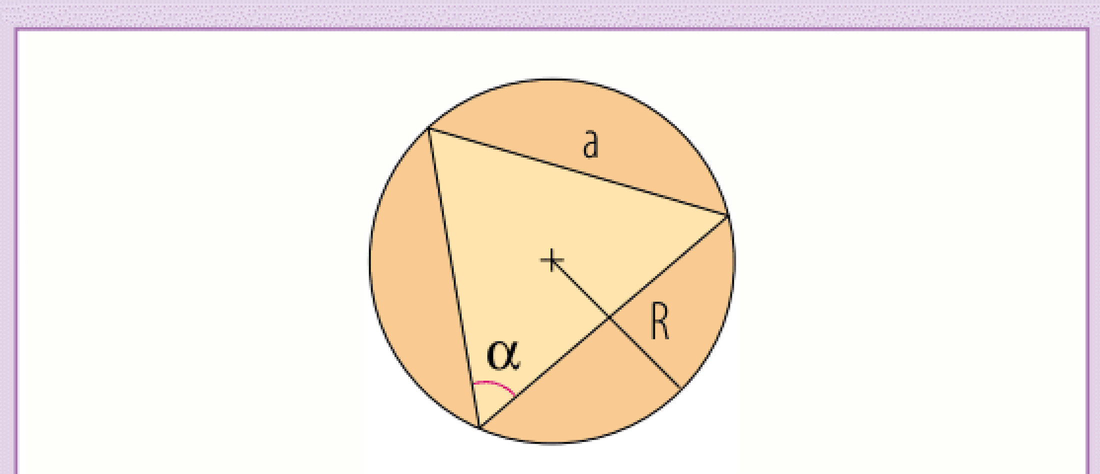


$\text{sen}\alpha = \frac{c}{a}$

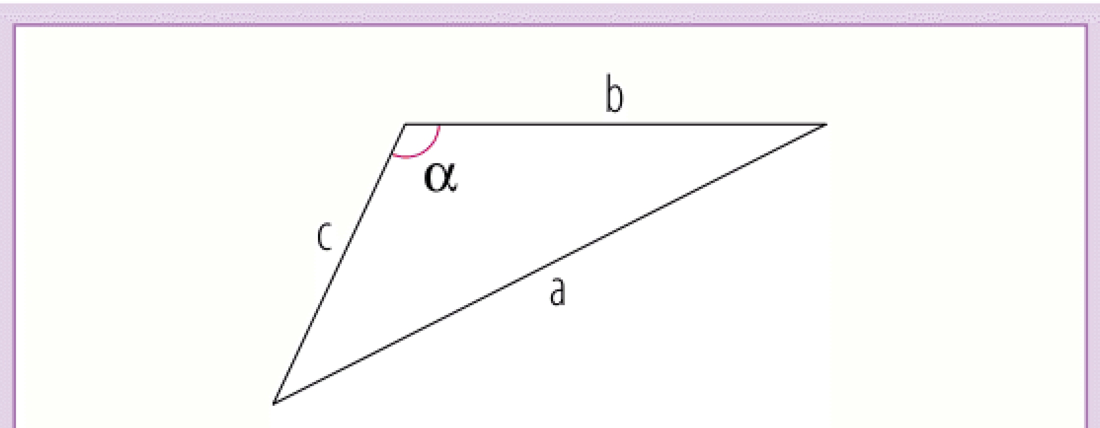
$\cos\alpha = \frac{b}{a}$

$\text{tg}\alpha = \frac{c}{b}$

Triângulo qualquer



$\frac{a}{\text{sen}\alpha} = 2R$



$a^2 = b^2 + c^2 - 2bc \cdot \cos\alpha$

Relações fundamentais

$\text{sen}^2x + \cos^2x = 1, \forall x \in \mathbb{R}$

$\text{tg}x = \frac{\text{sen}x}{\cos x}, \left(x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right)$

$\text{cotg}x = \frac{\cos x}{\text{sen}x}, \left(x \neq k\pi, k \in \mathbb{Z}\right)$

$\sec x = \frac{1}{\cos x}, \left(x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right)$

$\text{cossec}x = \frac{1}{\text{sen}x}, \left(x \neq k\pi, k \in \mathbb{Z}\right)$

Consequências $\left(x \neq \frac{k\pi}{2}\right)$

$\text{cotg}x = \frac{1}{\text{tg}x}$

$1 + \text{tg}^2x = \sec^2x$

$1 + \text{cotg}^2x = \text{cossec}^2x$

$\cos^2x = \frac{1}{1 + \text{tg}^2x}$

$\text{sen}^2x = \frac{\text{tg}^2x}{1 + \text{tg}^2x}$

Fórmulas de adição

$\cos(a+b) = \cos a \cdot \cos b - \text{sen}a \cdot \text{sen}b$

$\cos(a-b) = \cos a \cdot \cos b + \text{sen}a \cdot \text{sen}b$

$\text{sen}(a+b) = \text{sen}a \cdot \cos b + \text{sen}b \cdot \cos a$

$\text{sen}(a-b) = \text{sen}a \cdot \cos b - \text{sen}b \cdot \cos a$

$\text{tg}(a+b) = \frac{\text{tg}a + \text{tg}b}{1 - \text{tg}a \cdot \text{tg}b}$

$\text{tg}(a-b) = \frac{\text{tg}a - \text{tg}b}{1 + \text{tg}a \cdot \text{tg}b}$

Fórmulas da multiplicação

$\text{sen}2a = 2 \cdot \text{sen}a \cdot \cos a$

$$\cos 2a = \begin{cases} \cos^2a - \text{sen}^2a \\ \text{ou} \\ 2\cos^2a - 1 \\ \text{ou} \\ 1 - 2\text{sen}^2a \end{cases}$$

$\text{tg}2a = \frac{2\text{tg}a}{1 - \text{tg}^2a}$

$\text{sen}3a = 3\text{sen}a - 4\text{sen}^3a$

$\cos 3a = 4\cos^3a - 3\cos a$

$\text{tg}3a = \frac{3\text{tg}a - \text{tg}^3a}{1 - 3\text{tg}^2a}$

Fórmulas de divisão

$\text{sen}\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$

$\cos\frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$

$\text{tg}\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}}$

Fórmulas de transformação em produto

$\cos p + \cos q = 2 \cdot \cos\frac{p+q}{2} \cdot \cos\frac{p-q}{2}$

$\cos p - \cos q = -2 \cdot \text{sen}\frac{p+q}{2} \cdot \text{sen}\frac{p-q}{2}$

$\text{sen}p + \text{sen}q = 2 \cdot \text{sen}\frac{p+q}{2} \cdot \cos\frac{p-q}{2}$

$\text{sen}p - \text{sen}q = 2 \cdot \text{sen}\frac{p-q}{2} \cdot \cos\frac{p+q}{2}$

$\text{tg}p + \text{tg}q = \frac{\text{sen}(p+q)}{\cos p \cdot \cos q}$

$\text{tg}p - \text{tg}q = \frac{\text{sen}(p-q)}{\cos p \cdot \cos q}$

Equações trigonométricas fundamentais

$\text{sen}\alpha = \text{sen}\beta \Rightarrow \alpha = \beta + 2k\pi$ ou $\alpha = (\pi - \beta) + 2k\pi$

$\cos\alpha = \cos\beta \Rightarrow \alpha = \pm\beta + 2k\pi$

$\text{tg}\alpha = \text{tg}\beta \Rightarrow \alpha = \beta + k\pi$

Funções circulares inversas

$y = \text{arc sen}x \Leftrightarrow \text{sen}y = x$ e $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$y = \text{arc cos}x \Leftrightarrow \cos y = x$ e $0 \leq y \leq \pi$

$y = \text{arc tg}x \Leftrightarrow \text{tg}y = x$ e $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Polígonos convexos

Sendo n = número de lados
d = número de diagonais
S_i = soma dos ângulos internos
S_e = soma dos ângulos externos

Temos:

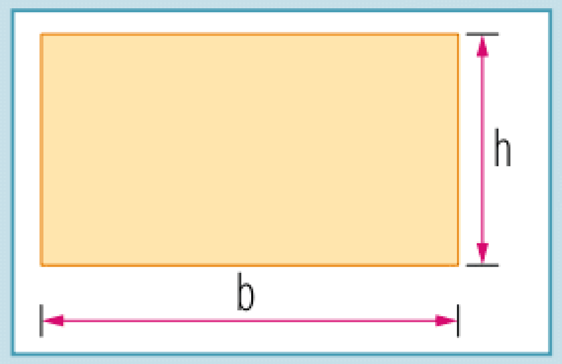
$d = \frac{n(n-3)}{2}$

$S_i = (n-2) \cdot 180^\circ$

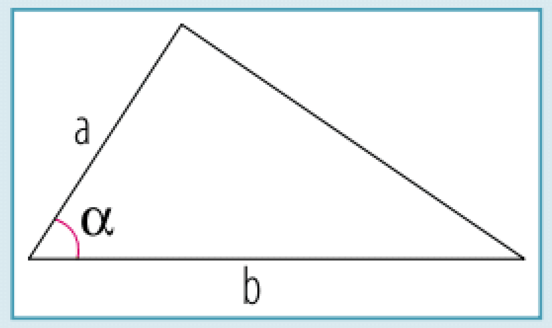
$S_e = 360^\circ$

Áreas

Retângulo

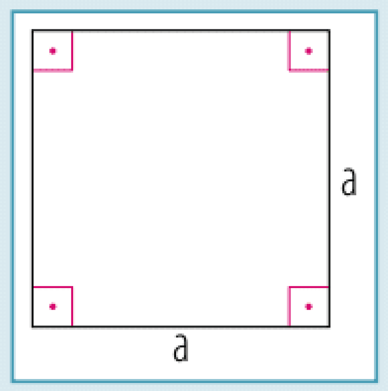


área = $b \cdot h$

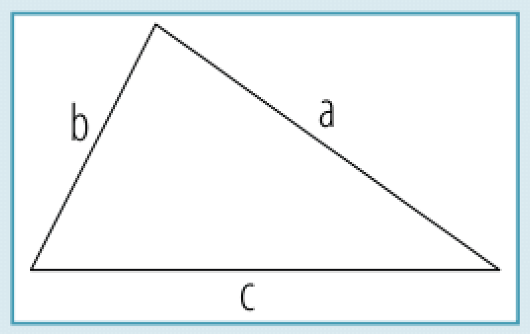


área = $\frac{1}{2} ab \cdot \operatorname{sen}\alpha$

Quadrado



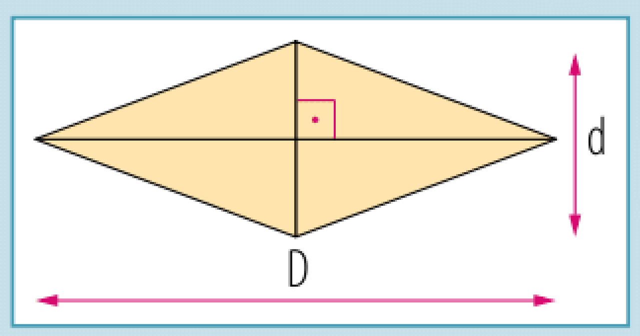
área = a^2



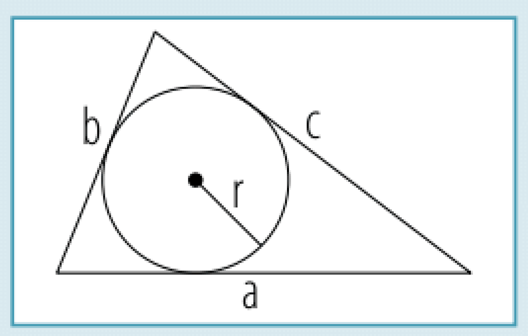
área = $\sqrt{p(p-a)\cdot(p-b)\cdot(p-c)}$

$p = \frac{a+b+c}{2}$

Losango



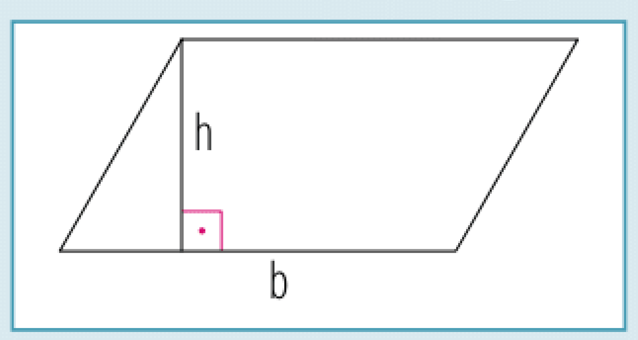
área = $\frac{D \cdot d}{2}$



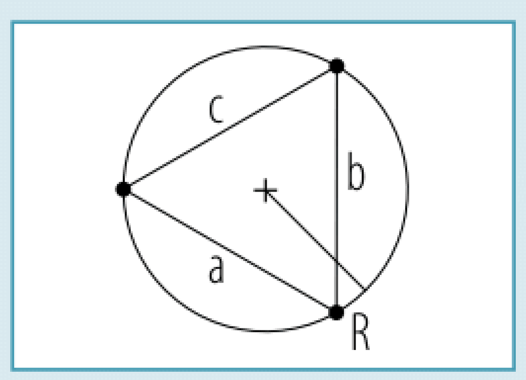
área = pr

$p = \frac{a+b+c}{2}$

Paralelogramo

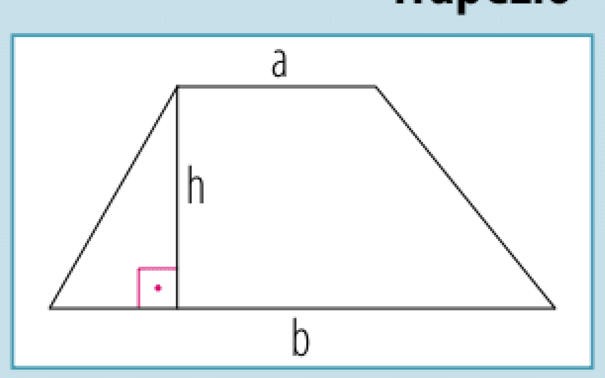


área = bh



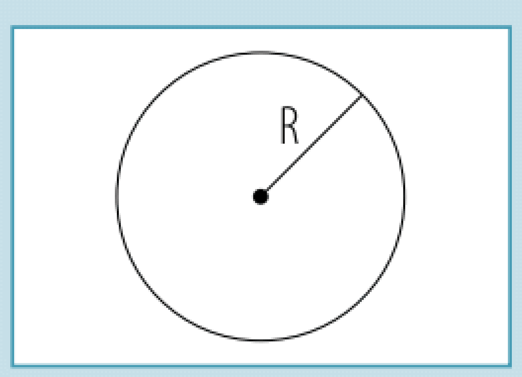
área = $\frac{abc}{4R}$

Trapézio



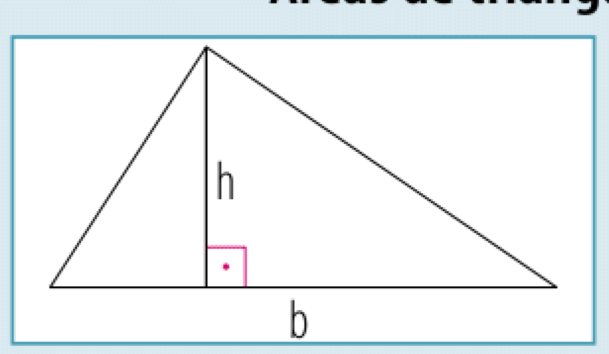
área = $\frac{(a+b)\cdot h}{2}$

Círculo



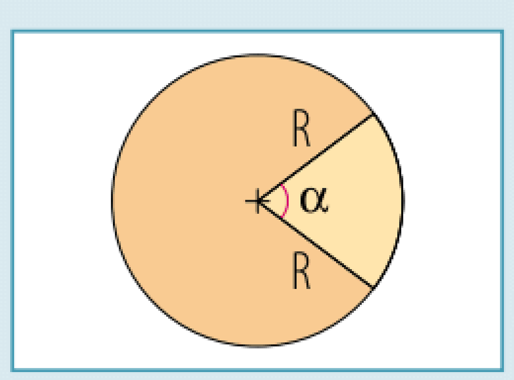
área = πR^2

Áreas de triângulos



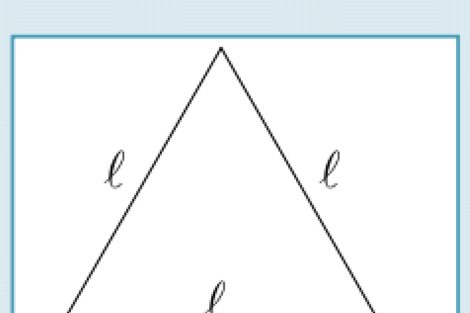
área = $\frac{bh}{2}$

Setor circular

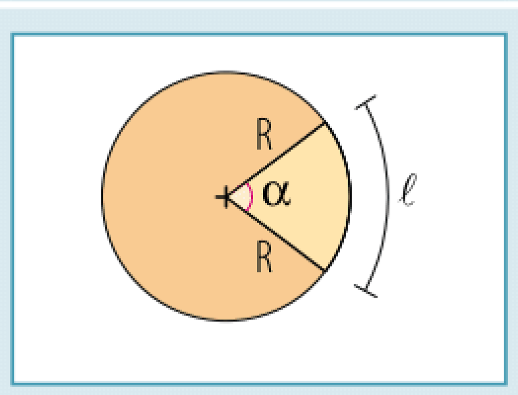


área = $\frac{\alpha \cdot \pi \cdot R^2}{360^\circ}$

área = $\frac{\alpha \cdot R^2}{2}$, α em radianos



área = $\frac{\ell^2 \cdot \sqrt{3}}{4}$

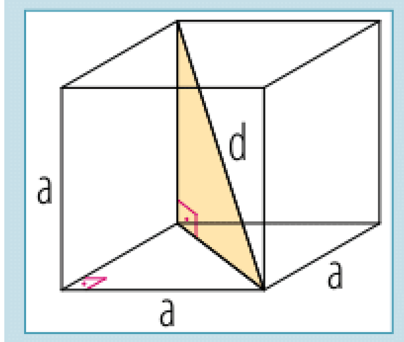


$A = \frac{\ell \cdot R}{2}$

Geometria espacial

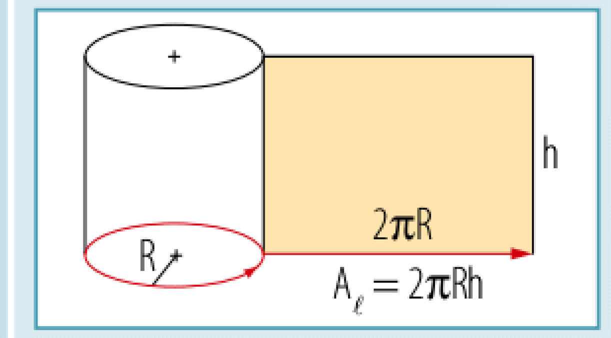
Sólidos

Cubo



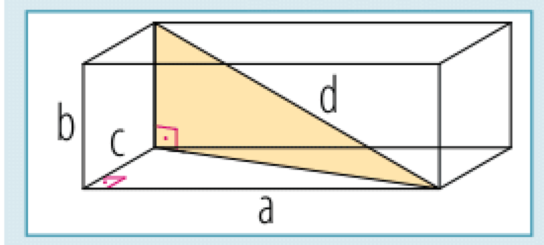
área = $6a^2$
Volume = a^3
Diagonal = $a\sqrt{3}$

Cilindro



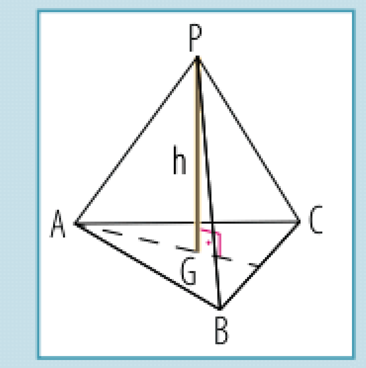
Volume = B · h
B = área da base

Paralelepípedo reto-retângulo



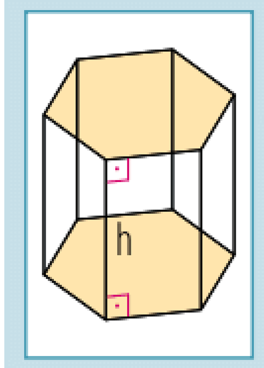
área = 2 (ab + ac + bc)
Volume = a · b · c
Diagonal = $\sqrt{a^2+b^2+c^2}$

Pirâmide



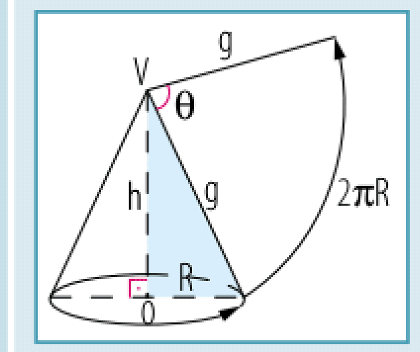
Volume = $\frac{1}{3} A_B \cdot h$, A_B = área da base

Prismas



Volume = B · h
B = área da base

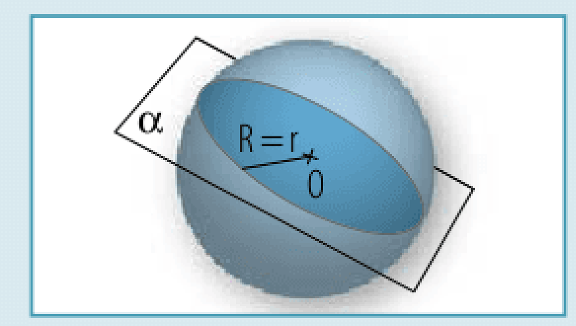
Cone



$A_\ell = \pi Rg$

$\theta = \frac{360^\circ \cdot R}{g}$

Esfera



$A = 4\pi R^2$

$V = \frac{4}{3}\pi R^3$

Geometria analítica

Equação

Equação geral da reta: ax + by + c = 0
Equação reduzida da reta: y = mx + n

Feixe de retas por $(x_0; y_0)$

$y - y_0 = m(x - x_0)$

Posições relativas entre retas

$r // s \Leftrightarrow m_r = m_s$

$r \perp s \Leftrightarrow m_r \cdot m_s = -1$

Distância entre dois pontos A e B:

$d_{A;B} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

Distância do ponto P à reta r:

$d_{P;r} = \frac{|ax_P + by_P + c|}{\sqrt{a^2+b^2}}$

Baricentro do triângulo ABC

$G = \left(\frac{x_A + x_B + x_C}{3}; \frac{y_A + y_B + y_C}{3}\right)$

Condição de alinhamento para três pontos

$$\begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix} = 0$$

Área do triângulo ABC

$$A = \frac{1}{2}\left\|\begin{matrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{matrix}\right\|$$

Equações da circunferência

Equação geral da circunferência:
$x^2 + y^2 - 2ax - 2by + a^2 + b^2 - r^2 = 0$

Equação reduzida da circunferência de centro (x_0, y_0):
$(x - x_0)^2 + (y - y_0)^2 = r^2$

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Potenciação e radiciação

Potenciação

$$a^n = \underbrace{a \cdot a \cdot a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ fatores}}$$

$$a^0 = 1$$

Para bases positivas

1. $a^m \cdot a^n = a^{m+n}$
2. $\dfrac{a^m}{a^n} = a^{m-n}, a \neq 0$
3. $\left(a^m\right)^n = a^{m \cdot n}$
4. $(a \cdot b)^n = a^n \cdot b^n$
5. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n},\ b \neq 0$
6. $a^{-n} = \dfrac{1}{a^n},\ a \neq 0$

Radiciação

$$\sqrt[n]{b} = a \Rightarrow a^n = b$$

1. $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
2. $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}},\ b \uparrow 0$
3. $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$
4. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[n \cdot m]{a}$
5. $\sqrt[n]{a^m} = a^{\frac{m}{n}}$
6. $\sqrt[n \cdot p]{a^{m \cdot p}} = \sqrt[n]{a^m}$

Produtos notáveis e fatoração

$$(a+b)\cdot(a-b) = a^2 - b^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2\cdot(ab+ac+bc)$$

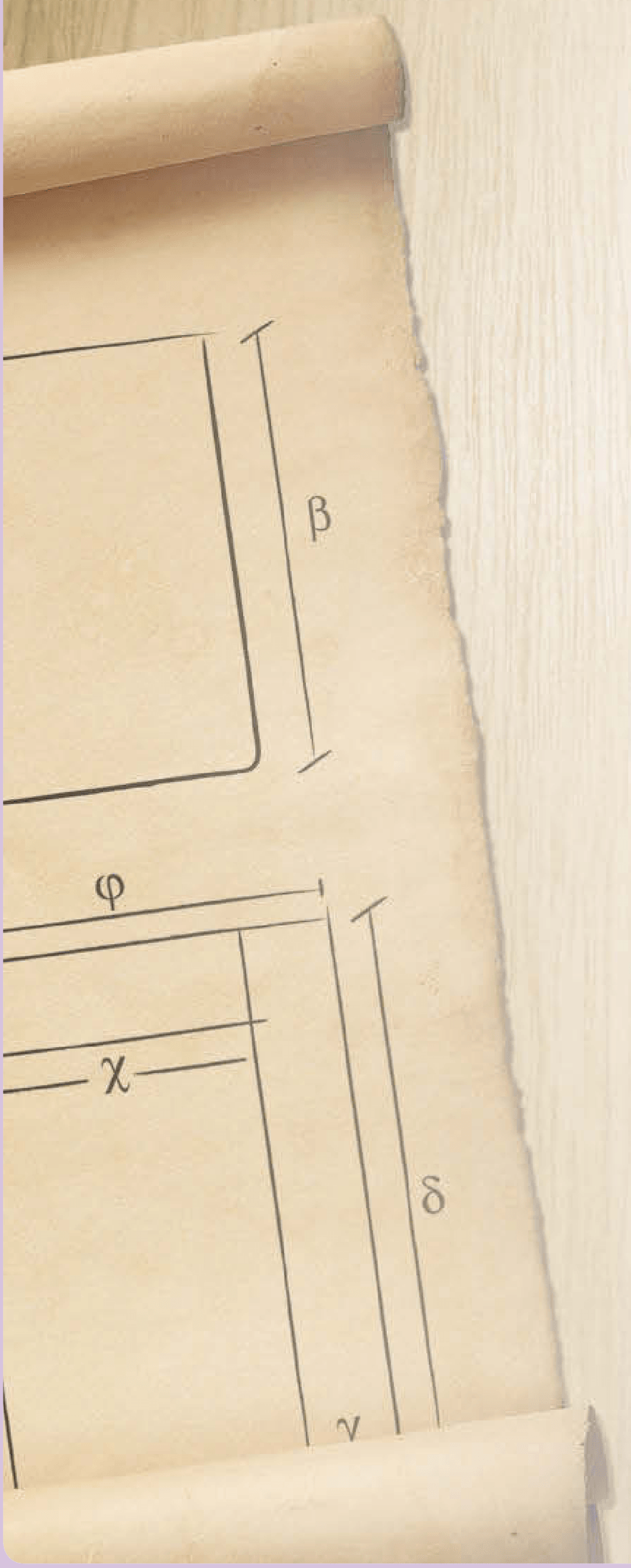
$$ab + ac = a\cdot(b+c)$$

$$ab + ac + db + dc = a\cdot(b+c) + d\cdot(b+c) = (b+c)\cdot(a+d)$$

$ax^2 + bx + c = a\cdot(x-a_1)\cdot(x-a_2)$, onde a_1 e a_2 são raízes de $ax^2 + bx + c = 0$

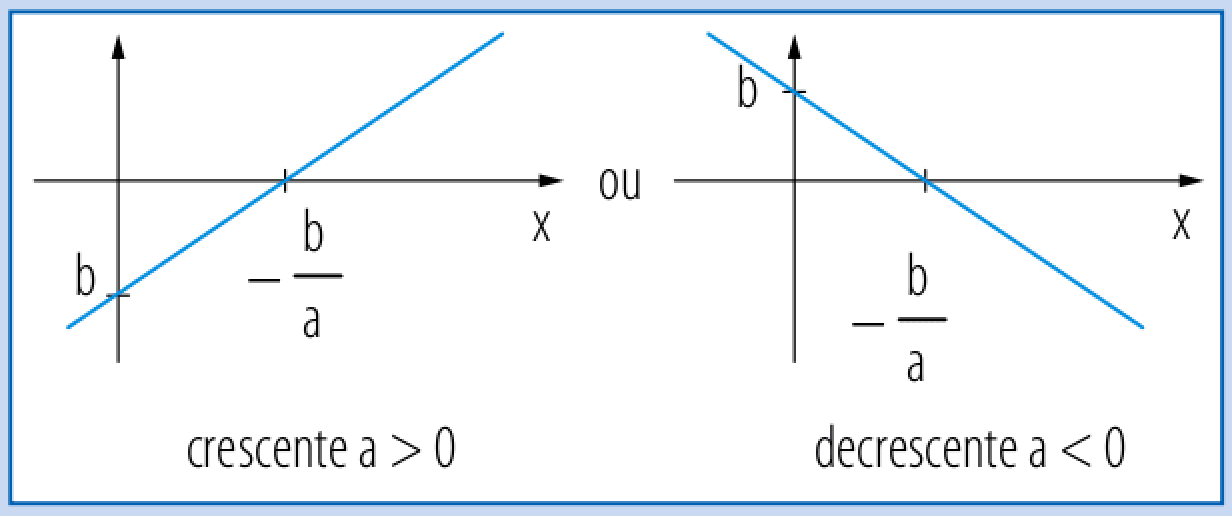
$$a^3 + b^3 = (a+b)\cdot(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)\cdot(a^2 + ab + b^2)$$



Função do 1° grau

$f(x) = ax + b$	$a \neq 0$



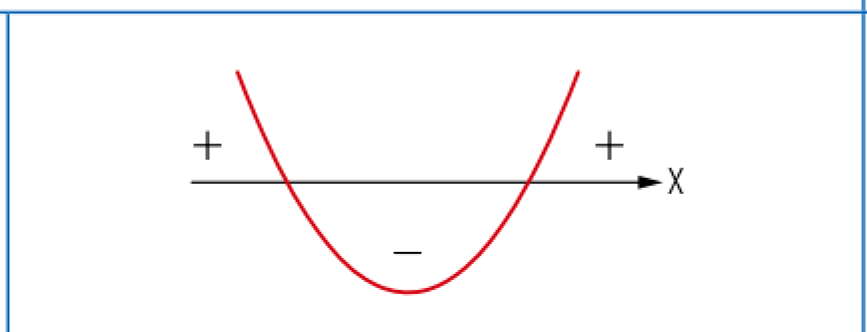
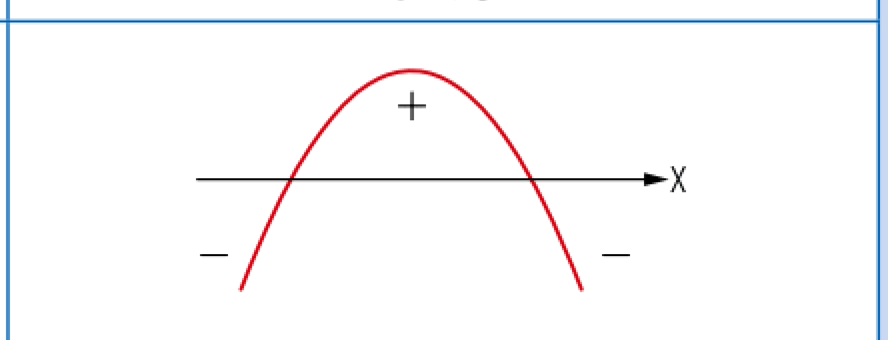
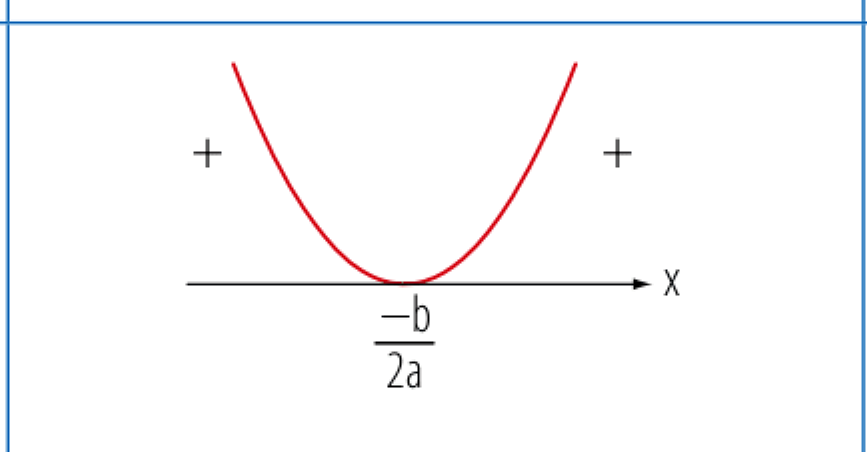
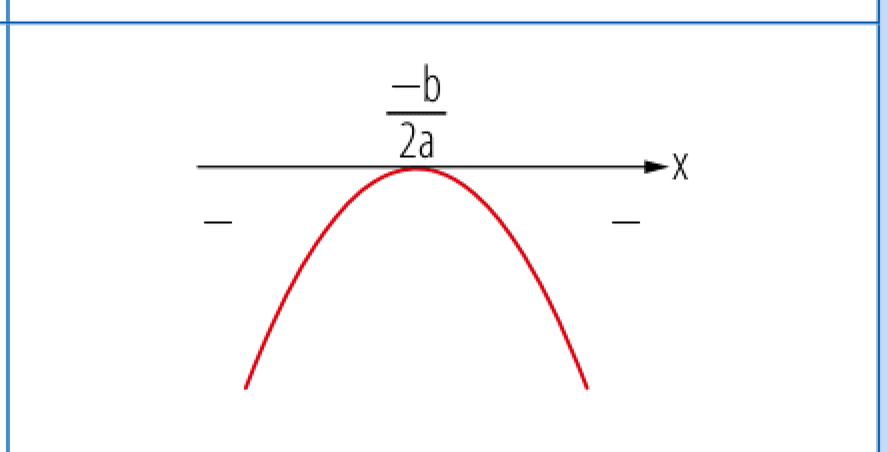
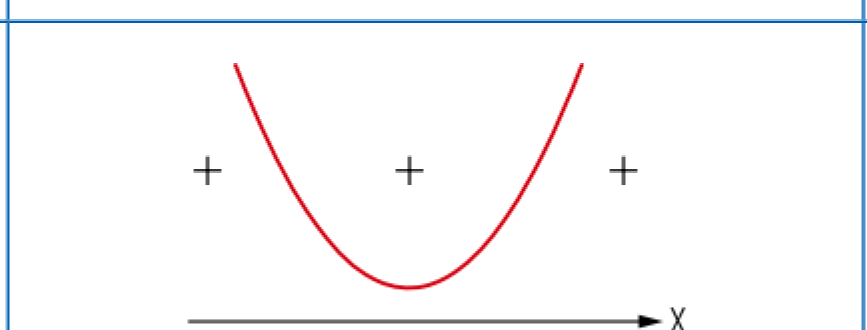
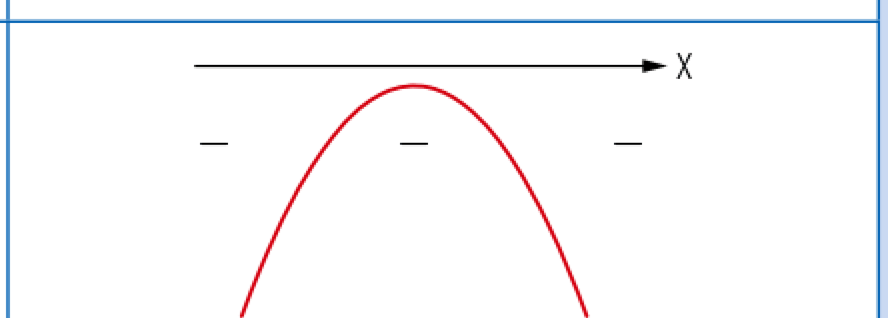
Função do 2° grau

$f(x) = ax^2 + bx + c$	$a \neq 0$

Raízes: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminante: $\Delta = b^2 - 4ac$

- $\Delta > 0$: duas raízes reais diferentes
- $\Delta = 0$: duas raízes reais iguais
- $\Delta < 0$: $\nexists$ raízes reais

	a > 0	**a < 0**
Δ > 0		
Δ = 0		
Δ < 0	 $y > 0 : \forall x \in \mathbb{R}$	 $y < 0 : \forall x \in \mathbb{R}$

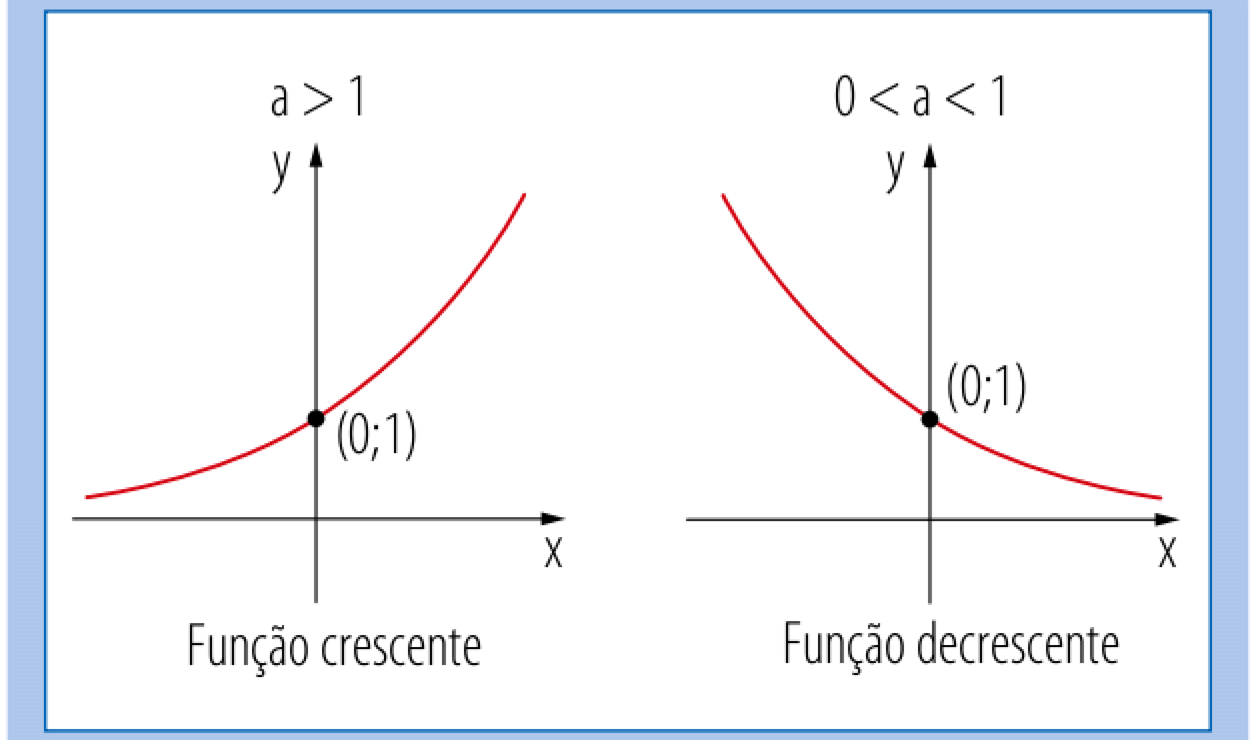
Vértice: $x_v = \dfrac{-b}{2a}$ e $y_v = \dfrac{-\Delta}{4a}$ **Domínio:** $\mathbb{R}$

Imagem: $\left\{ y \in \mathbb{R} \,\middle|\, y \geq \dfrac{-\Delta}{4a} \text{ se } a > 0 \text{ ou } y \leq \dfrac{-\Delta}{4a} \text{ se } a < 0 \right\}$

Função exponencial

$f(x) = a^x$: $a > 0$ e $a \neq 1$

$Im_f = \mathbb{R}_+^*$ $D_f = \mathbb{R}$



- $a^{f(x)} = a^{g(x)} \Leftrightarrow f(x) = g(x)$
- Se $a > 1$; $a^{f(x)} > a^{g(x)} \Leftrightarrow f(x) > g(x)$
- Se $0 < a < 1$; $a^{f(x)} > a^{g(x)} \Leftrightarrow f(x) < g(x)$

Função logarítmica

$\log_b a = x \Leftrightarrow a = b^x$ com $a > 0, 0 < b \neq 1$

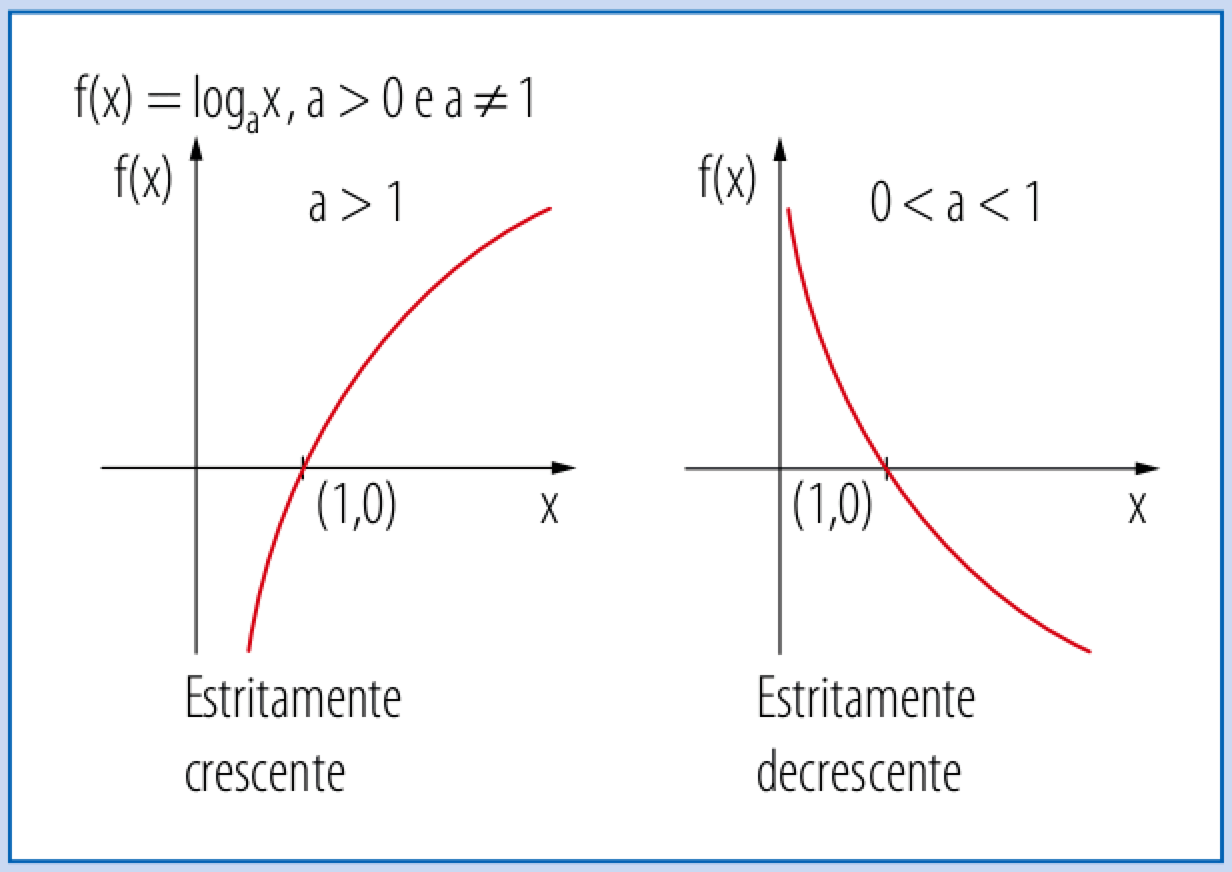
$Im_f = \mathbb{R}$ $D_f = \mathbb{R}_+^*$

$\log_c(a \cdot b) = \log_c a + \log_c b$; $a > 0, b > 0, 0 < c \neq 1$

$\log_c\left(\dfrac{a}{b}\right) = \log_c a - \log_c b$; $a > 0, b > 0, 0 < c \neq 1$

$\log_c a^m = m \cdot \log_c a$; $a > 0, 0 < c \neq 1$ e $m \in \mathbb{R}$

$\log_{c^m} a = \dfrac{1}{m} \cdot \log_c a$; $a > 0, 0 < c \neq 1$ e $m \in \mathbb{R}^*$



PA e PG

PA

$(a_1, a_2, a_3, a_4, ..., a_{n-1}, a_n)\ n \in \mathbb{N}$

$a_n = a_{n-1} + r$

- PG $(..., a, b, c, ...)$
  $a + c = 2b$
- PG $(a_1, a_2, a_3, ..., a_{n-1}, a_n)$
  $a_1 + a_n = a_2 + a_{n-1} = ...$

Termo geral: $a_n = a_1 + (n-1)r$

Soma dos n termos: $S_n = \dfrac{(a_1 + a_n)n}{2}$

PG

$(a_1, a_2, a_3, a_4, ..., a_{n-1}, a_n)\ n \in \mathbb{N}$

$a_n = a_{n-1} \cdot q$

- PG $(..., a, b, c, ...)$
  $a \cdot c = b^2$
- PG $(a_1, a_2, a_3, ..., a_{n-1}, a_n)$
  $a_1 \cdot a_n = a_2 \cdot a_{n-1} = ...$

Termo geral: $a_n = a_1 \cdot q^{n-1}$

Soma dos n termos: $S_n = \dfrac{a_1(1-q^n)}{1-q}$

$S_\infty = \dfrac{a_1}{1-q},\ -1 < q < 1$

Análise combinatória

Fatorial

$0! = 1;\ 1! = 1$

$n! = n \cdot (n-1)!\ |\ n \in \mathbb{N}$ e $n > 1$

Permutação

$P_n = n!$

$P_n^{\alpha,\beta,\gamma} = \dfrac{n!}{\alpha!\beta!\gamma!}$

Arranjo

$A_{n,p} = \dfrac{n!}{(n-p)!}$

Arranjo com repetição: $(AR)_{n,p} = n^p$

Combinação simples

$C_{n;p} = \dfrac{n!}{p!(n-p)!} = \dbinom{n}{p}$

Números complexos

$i^2 = -1$

$i^0 = 1;\ i^1 = i;\ i^2 = -1;\ i^3 = -i;\ i^4 = 1;\ ...$

$z = a + bi\ |\ a \in \mathbb{R}$ e $b \in \mathbb{R}$

Conjugado de z

$z = a - bi \Rightarrow \bar{z} = a - bi$

Módulo de z

$|z| = \sqrt{a^2 + b^2}$

Forma trigonométrica de z

$= |z|(\cos\theta + i \cdot \text{sen}\theta)$

Operações na forma trigonométrica

Sejam:

$z = |z|(\cos\theta + i \cdot \text{sen}\theta)$

$z_1 = |z_1|(\cos\theta_1 + i \cdot \text{sen}\theta_1)$

$z_2 = |z_2|(\cos\theta_2 + i \cdot \text{sen}\theta_2)$

$z_1 \cdot z_2 = |z_1| \cdot |z_2| \cdot [\cos(\theta_1 + \theta_2) + i \cdot \text{sen}(\theta_1 + \theta_2)]$

$\dfrac{z_1}{z_2} = \dfrac{|z_1|}{|z_2|}[\cos(\theta_1 - \theta_2) + i \cdot \text{sen}(\theta_1 - \theta_2)]$

$z^n = |z|^n \cdot [\cos(n\theta) + i \cdot \text{sen}(n\theta)]$

$\sqrt[n]{z} = \sqrt[n]{|z|} \cdot \left[\cos\left(\dfrac{\theta}{n} + \dfrac{2k\pi}{n}\right) + i \cdot \text{sen}\left(\dfrac{\theta}{n} + \dfrac{2k\pi}{n}\right)\right]$

e $0 \le k \le n;\ k \in \mathbb{N}$

Matemática financeira básica

Porcentagem

$x\% = \dfrac{x}{100}$

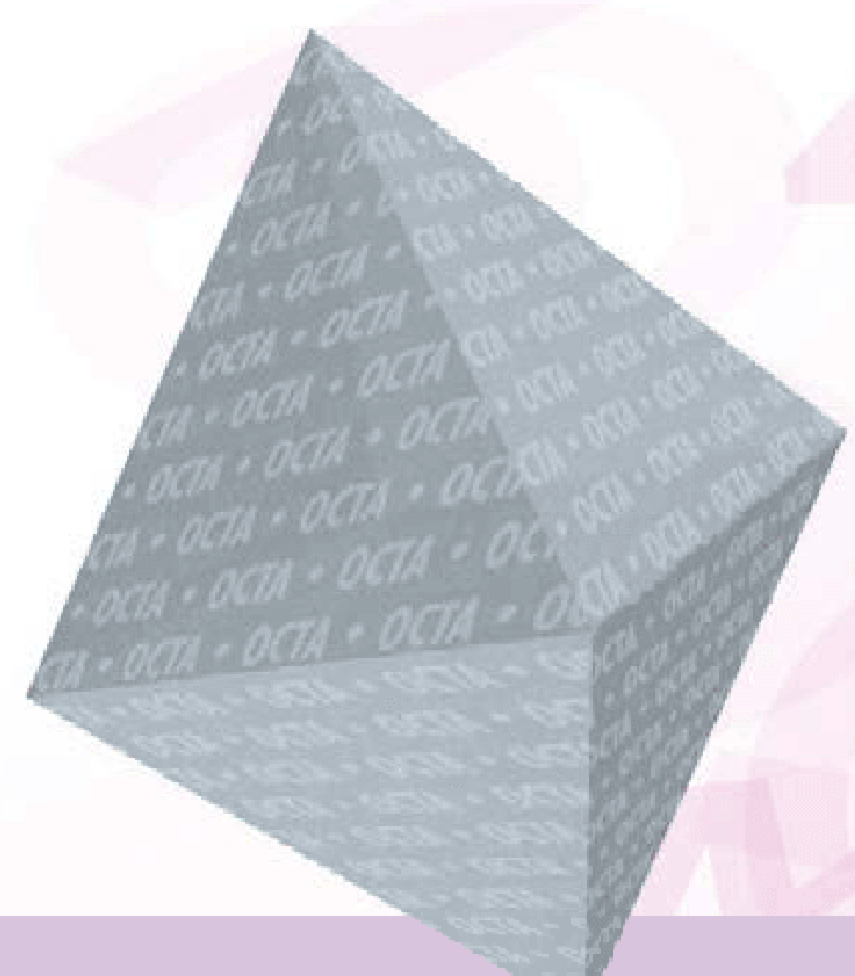
$x\%$ do total $= \dfrac{x}{100} \cdot \text{total}$

Juros simples

$J = C \cdot i \cdot t$

Juros compostos

$C_n = C_0(1 + i)^t$



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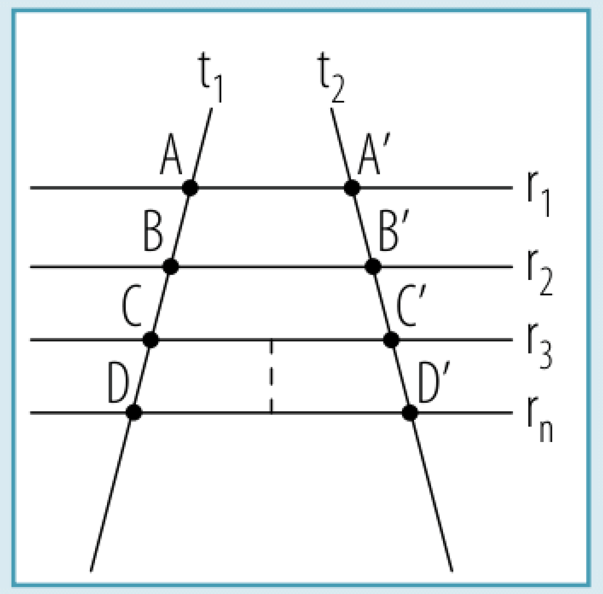
POLIEDRO
SISTEMA DE ENSINO

Geometria plana

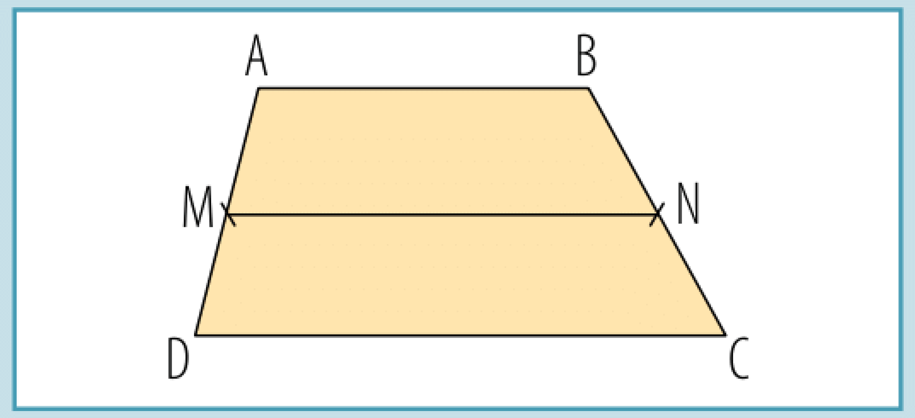
Teorema de Tales



$r_1 // r_2 // r_3 // \ldots // r_n$

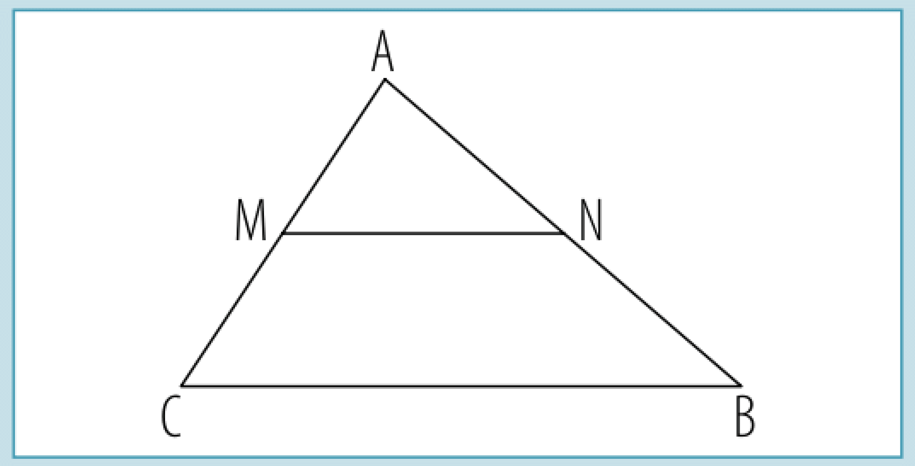
$$\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CD}{C'D'} = \frac{AC}{A'C'} = \frac{AD}{A'D'}$$

Base média do trapézio



$\overline{MN} // \overline{AB} // \overline{CD}$, temos: $MN = \frac{AB + CD}{2}$

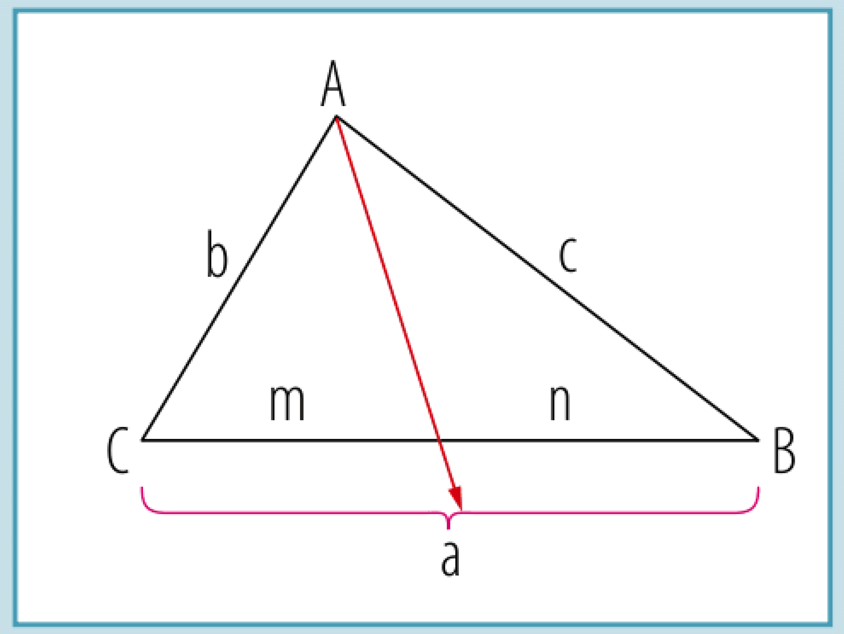
Base média do triângulo



$AM = MC; \; AN = NB$

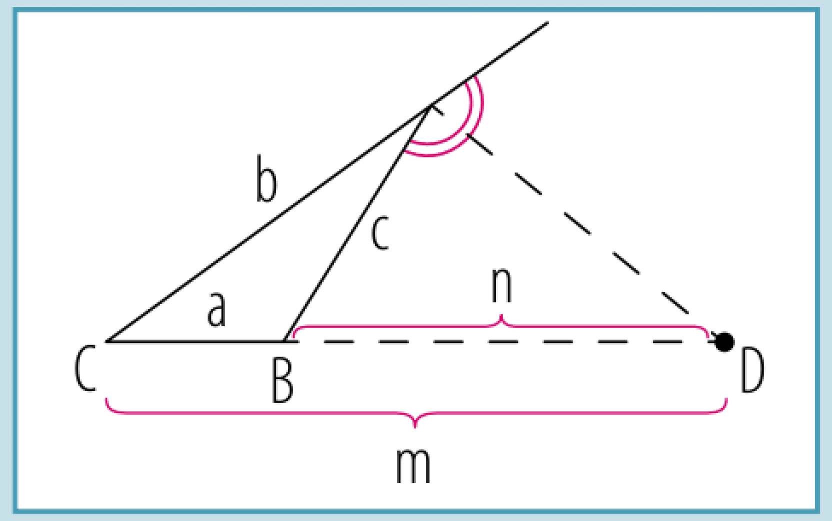
$\overline{MN}$ é a base média; $MN = \frac{BC}{2}$

Teorema da bissetriz interna



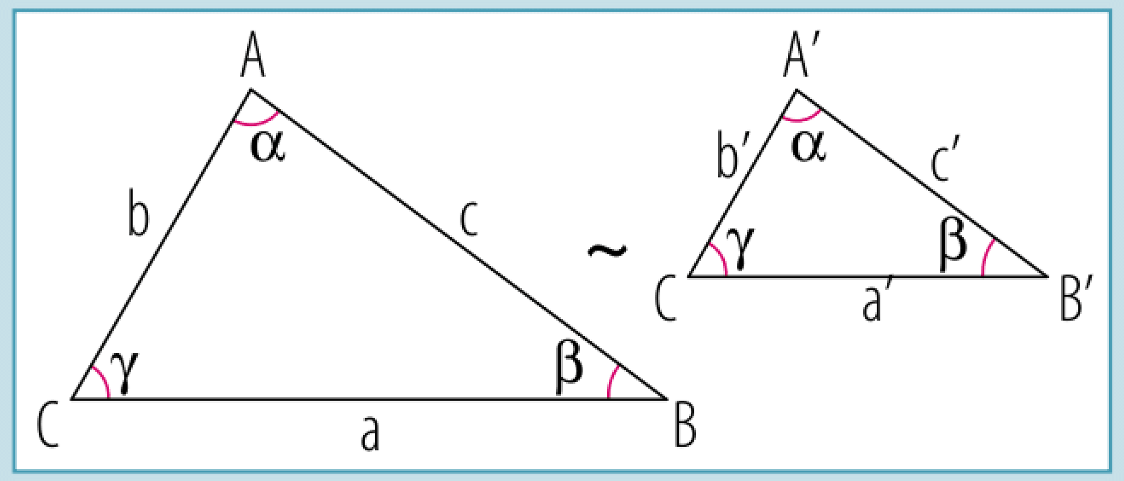
$$\frac{b}{m} = \frac{c}{n}$$

Teorema da bissetriz externa



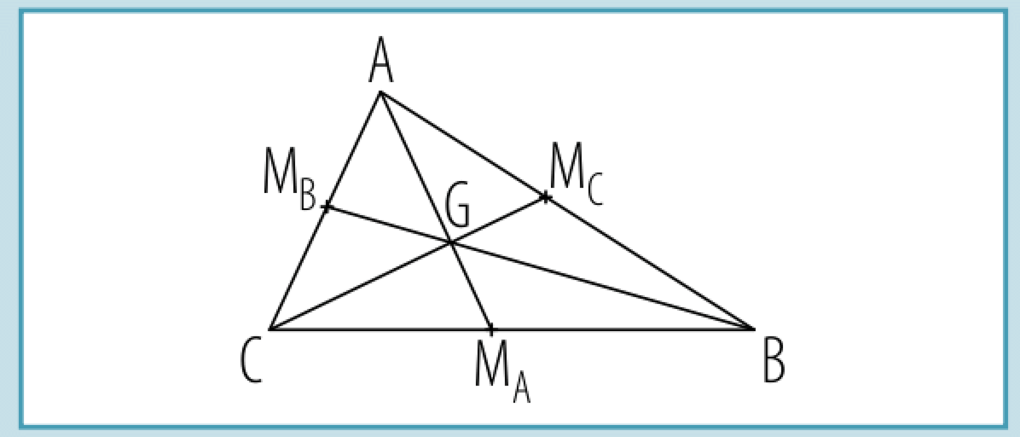
$$\frac{b}{m} = \frac{c}{n}$$

Semelhança de triângulos



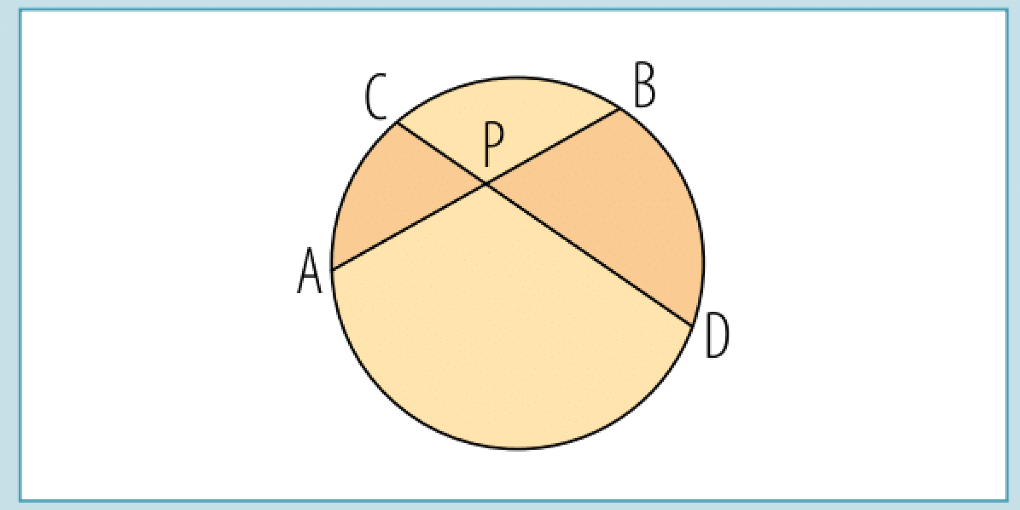
$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'} = k; \; \frac{\text{Área } \triangle ABC}{\text{Área } \triangle A'B'C'} = k^2$$

Baricentro do triângulo

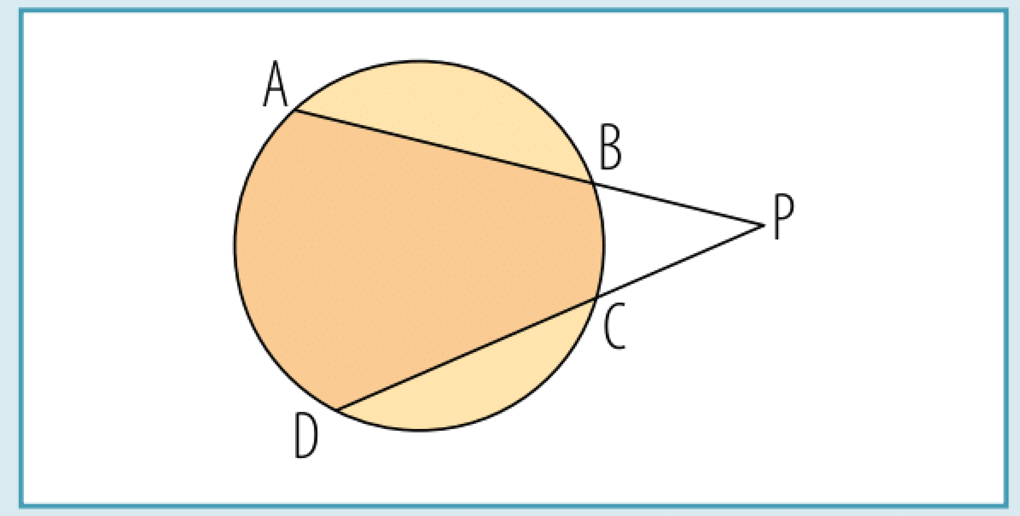


$$\frac{AG}{GM_A} = \frac{BG}{GM_B} = \frac{CG}{GM_C} = 2$$

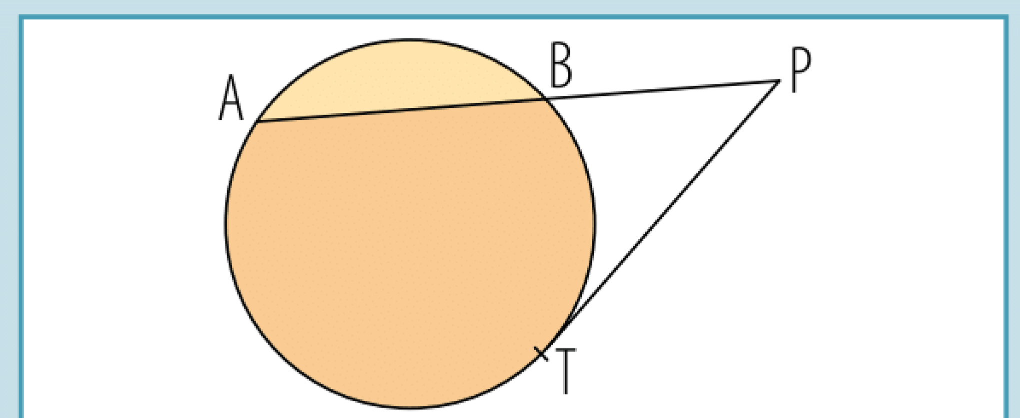
Relações métricas no círculo



$PA \cdot PB = PC \cdot PD$

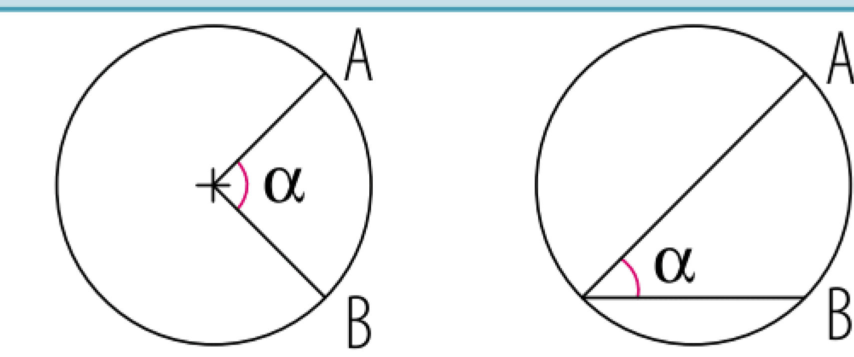


$PA \cdot PB = PC \cdot PD$

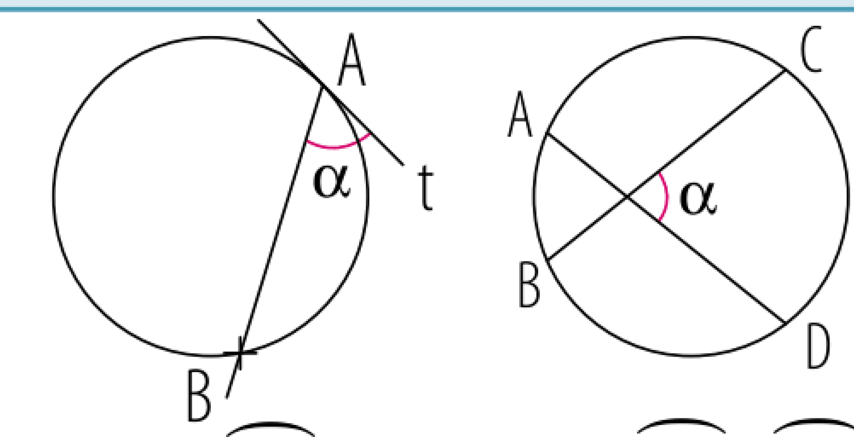


$(PT)^2 = PA \cdot PB$

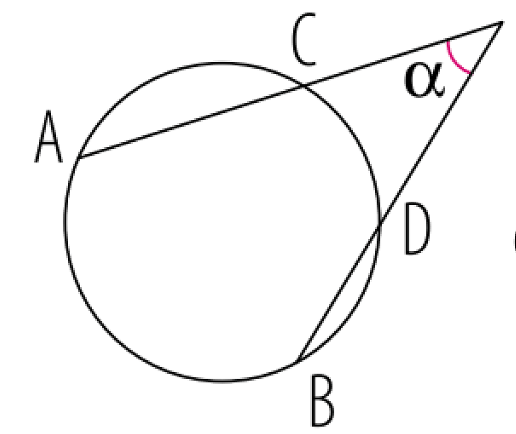
Arcos e ângulos



$\alpha = \overset{\frown}{AB}$ $\qquad$ $\alpha = \frac{\overset{\frown}{AB}}{2}$

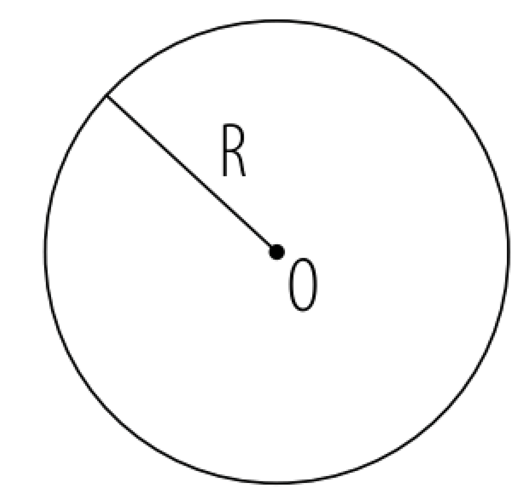


$\alpha = \frac{\overset{\frown}{AB}}{2}$ $\qquad$ $\alpha = \frac{\overset{\frown}{AB} + \overset{\frown}{CD}}{2}$



$\alpha = \frac{\overset{\frown}{AB} - \overset{\frown}{CD}}{2}$

Comprimento da circunferência



$C = 2\pi R$