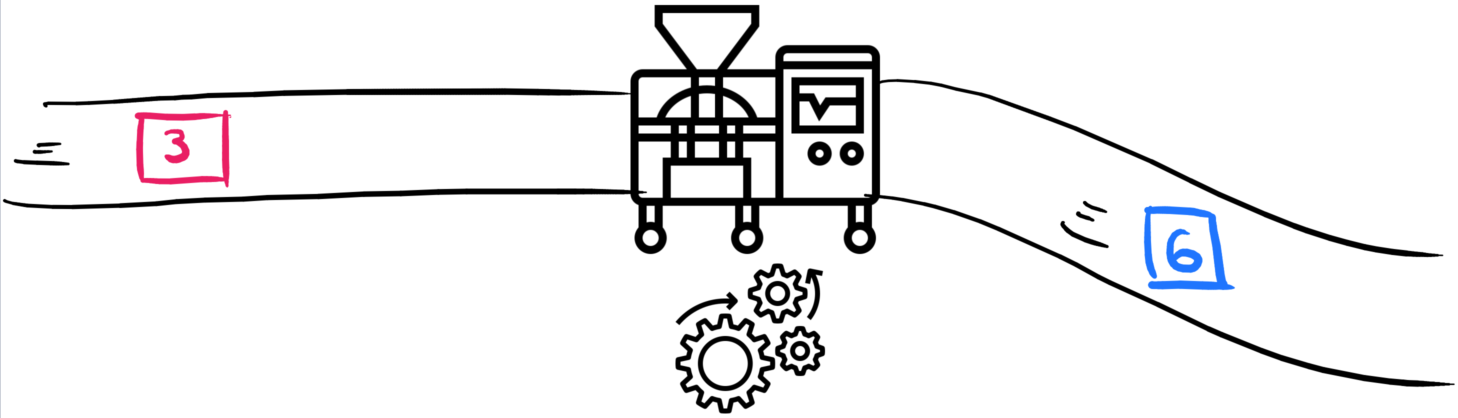
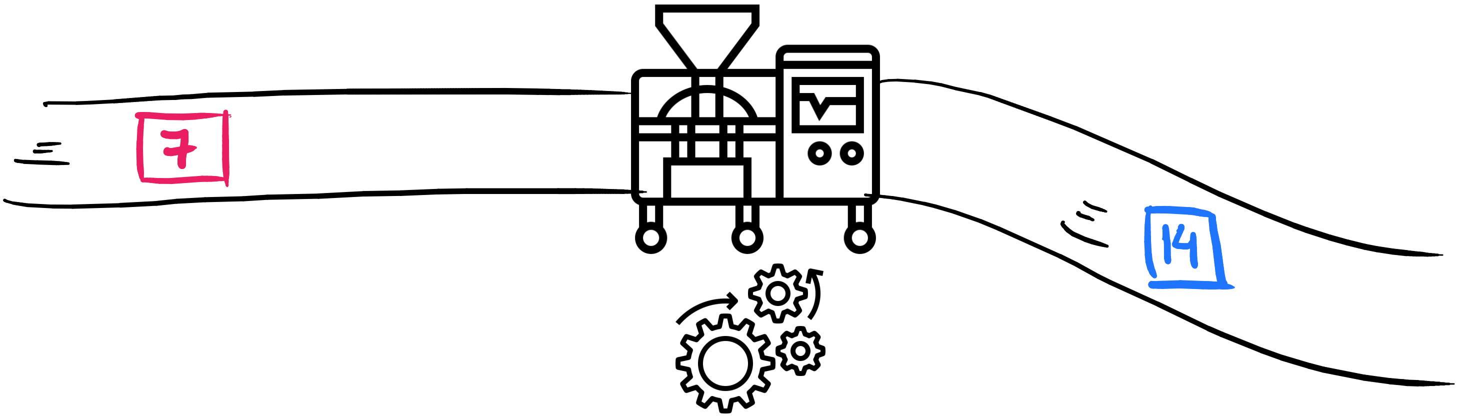
Módulo 08

· INTRODUÇÃO ÀS FUNÇÕES ·

8.1 . Introdução





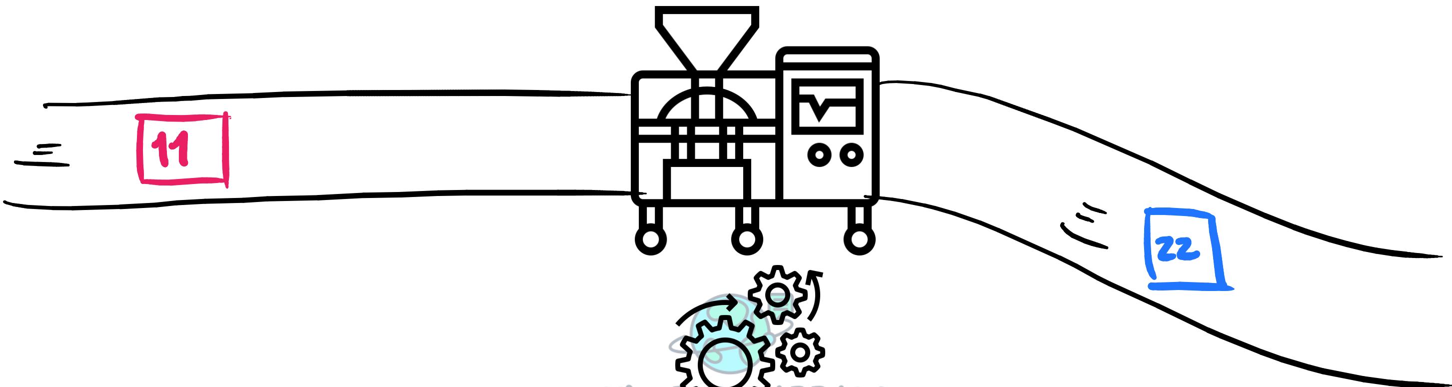
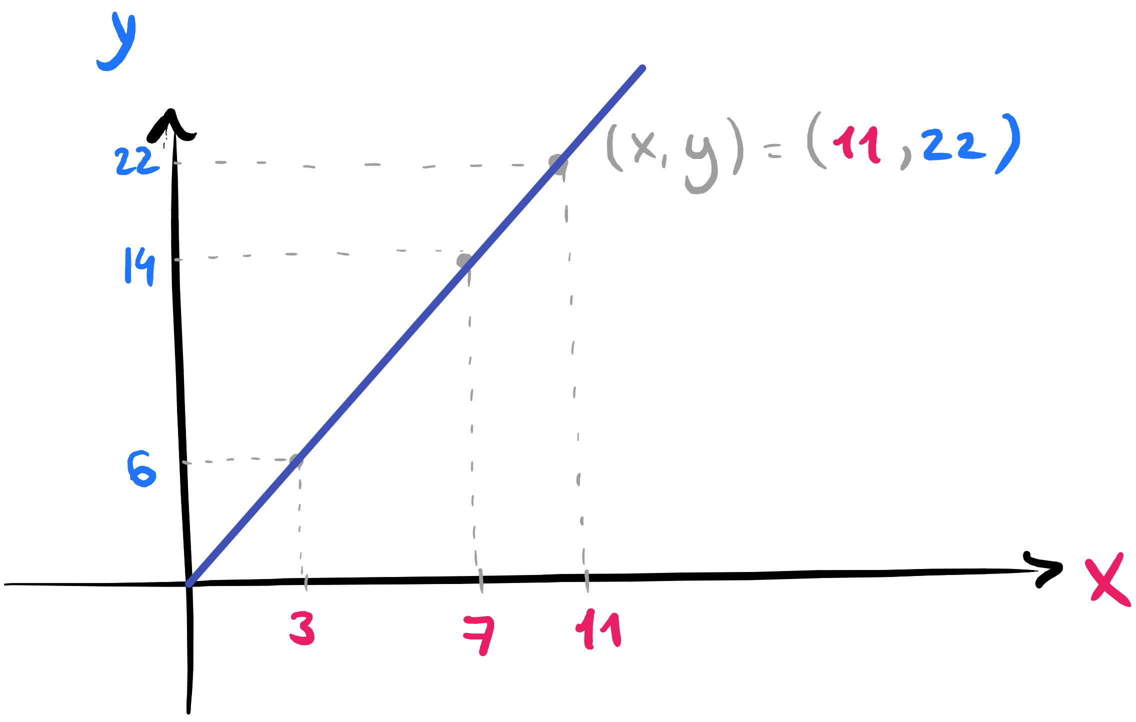


Tabela:

y	x
6	3
14	7
22	11

Gráfico:



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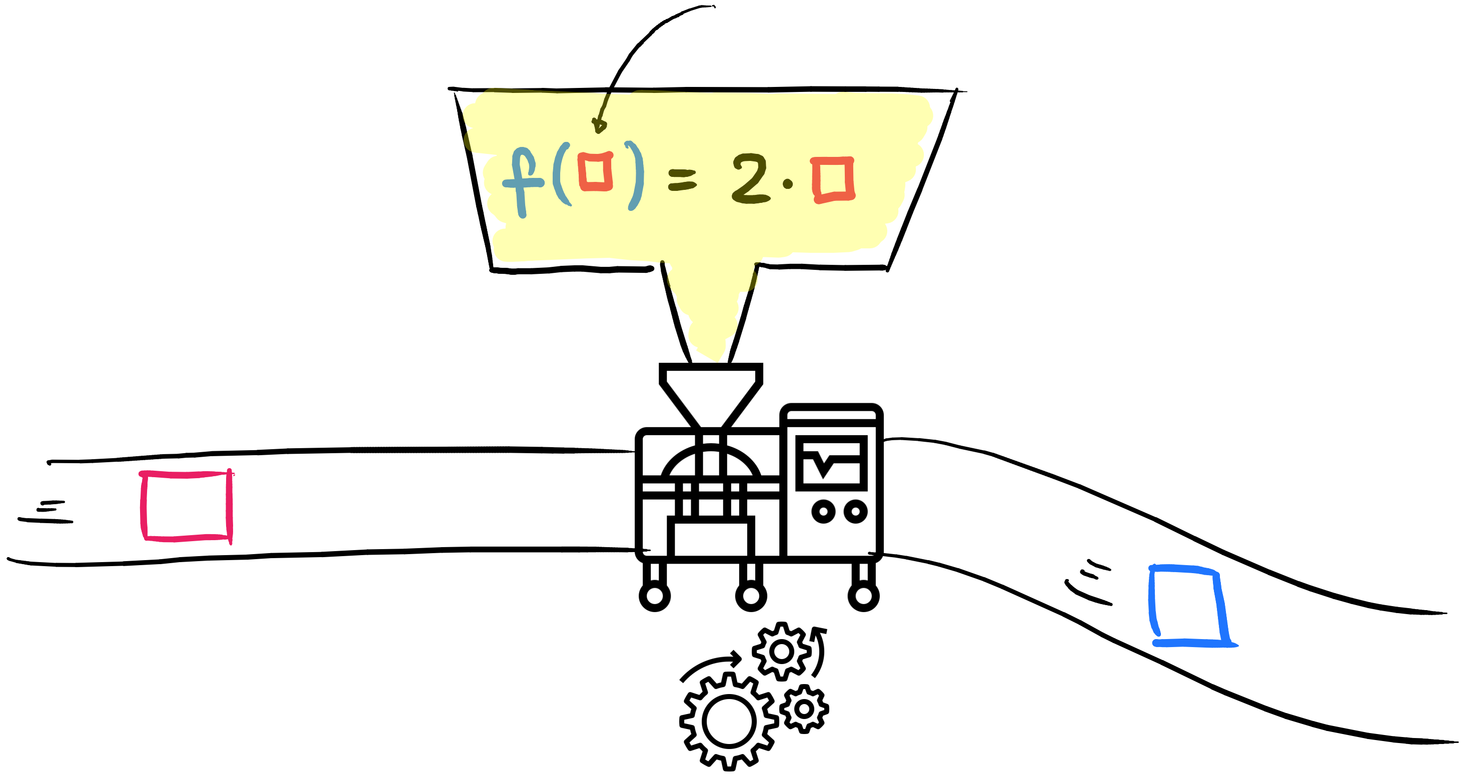
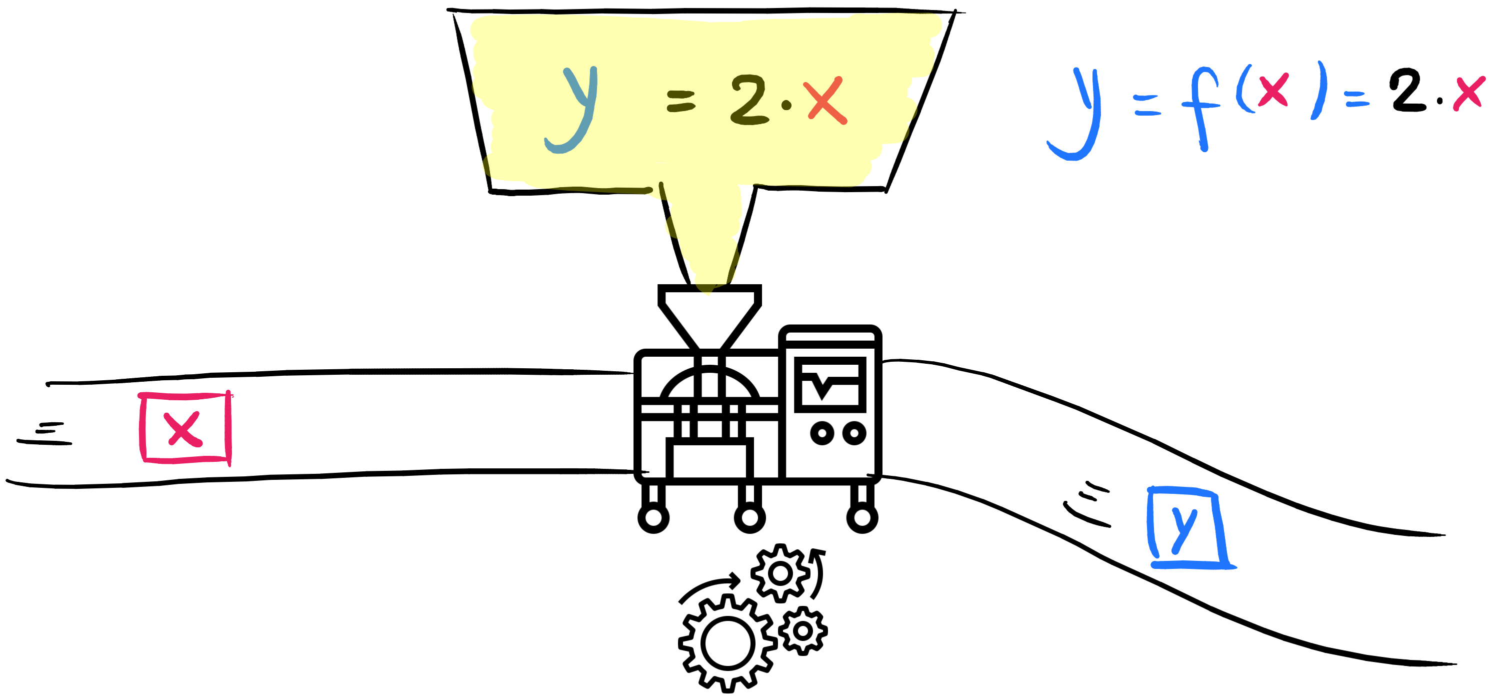
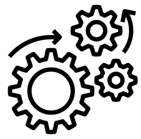

f(□) = 2·□

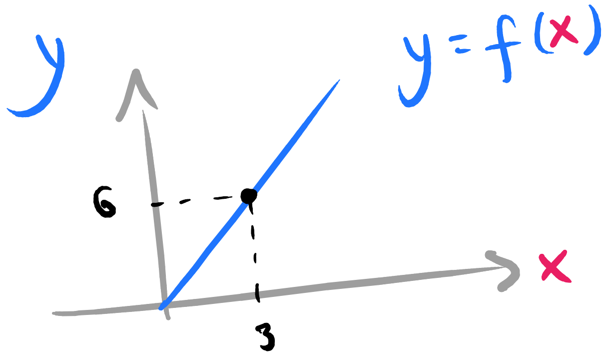

y = 2·x
x
y
y = f(x) = 2.x

4 formas

- MÁQUINA $= \boxed{x}$  $\rightarrow \boxed{y}$
- TABELA

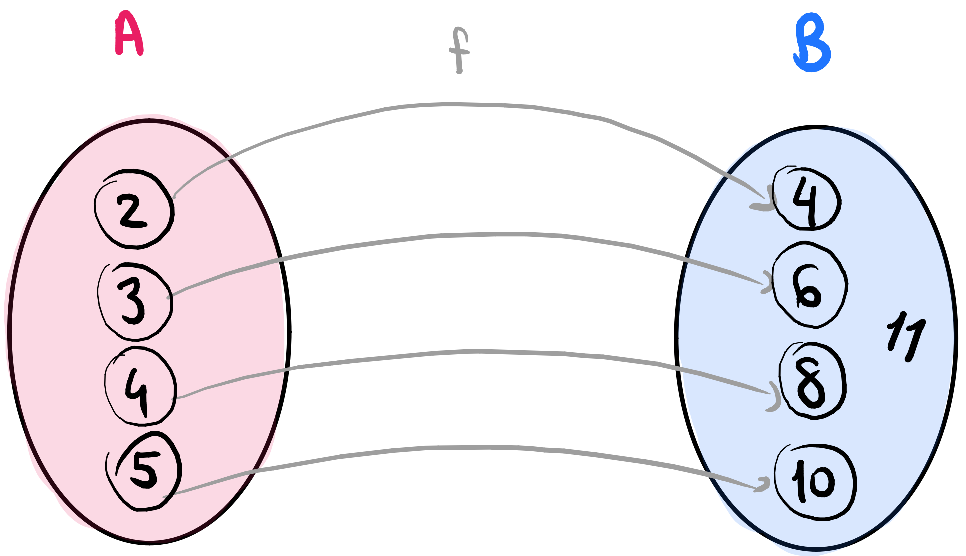
$y=f(x)$	x
⋮	⋮

- EQUAÇÃO $y = f(x) = 2 \cdot x$
- GRÁFICO



Uma função é uma <u>relação</u> entre um número x, de entrada, e um número y, de saída.

8.1.1. Domínio e Imagem



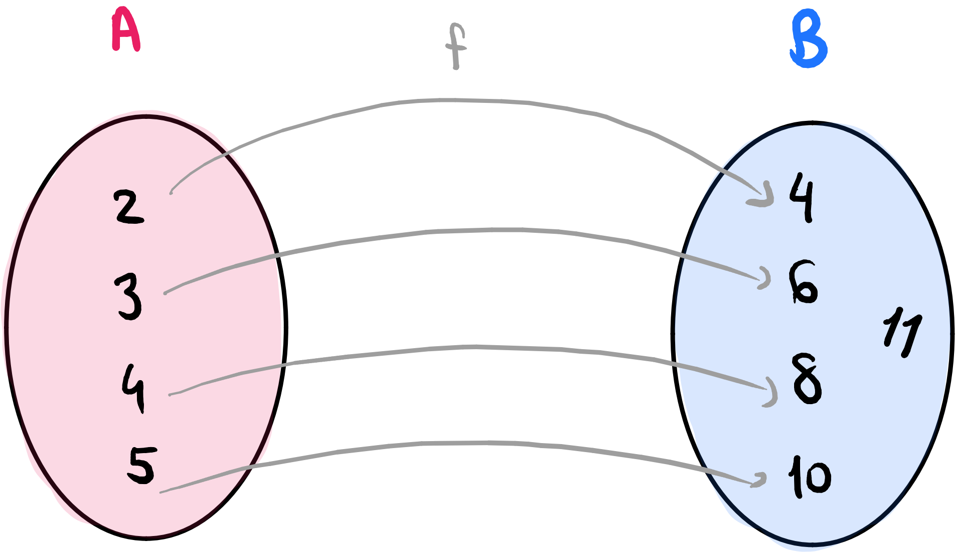
$$\boxed{f: A \to B}$$

DOMÍNIO

CONTRADOMÍNIO

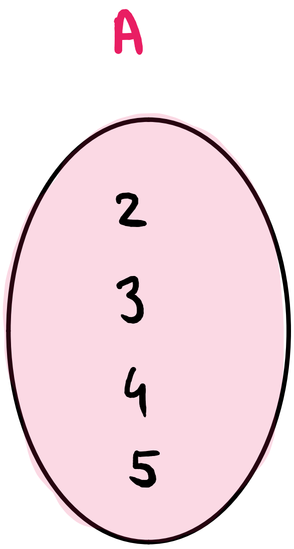
A imagem é $\{4, 6, 8, 10\}$

$$\boxed{f(x) = 2 \cdot x}$$



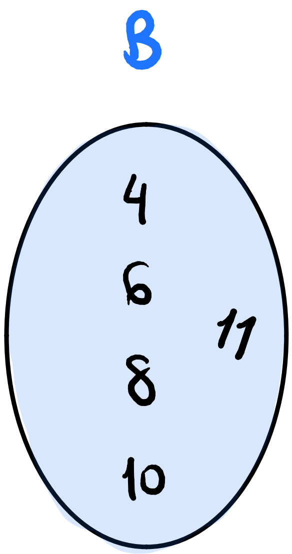
$$f: A \to B$$

DOMÍNIO



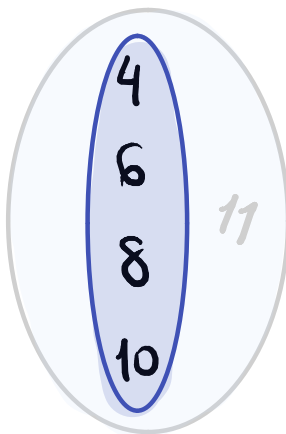
Valores de x

CONTRADOMÍNIO



Possíveis valores de y

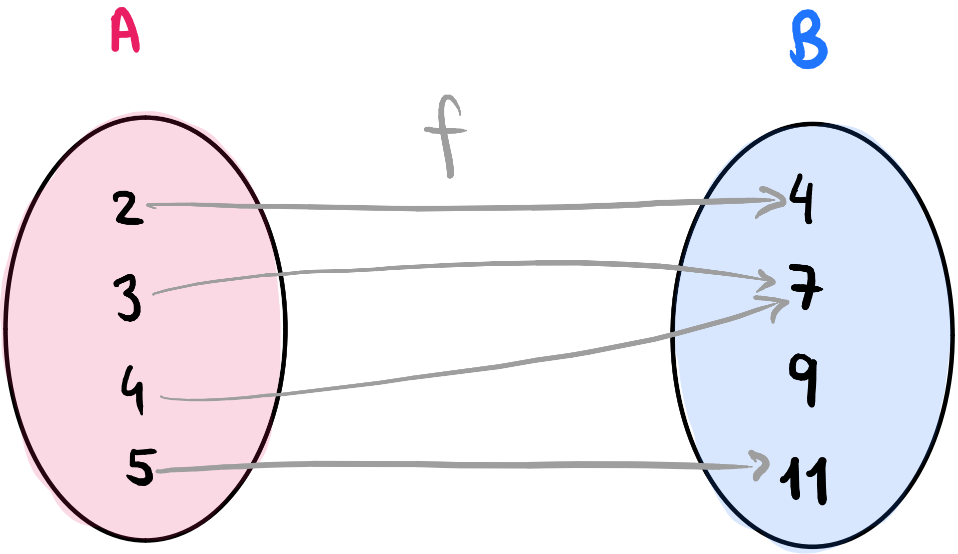
IMAGEM



Valores que y assume

Exemplo

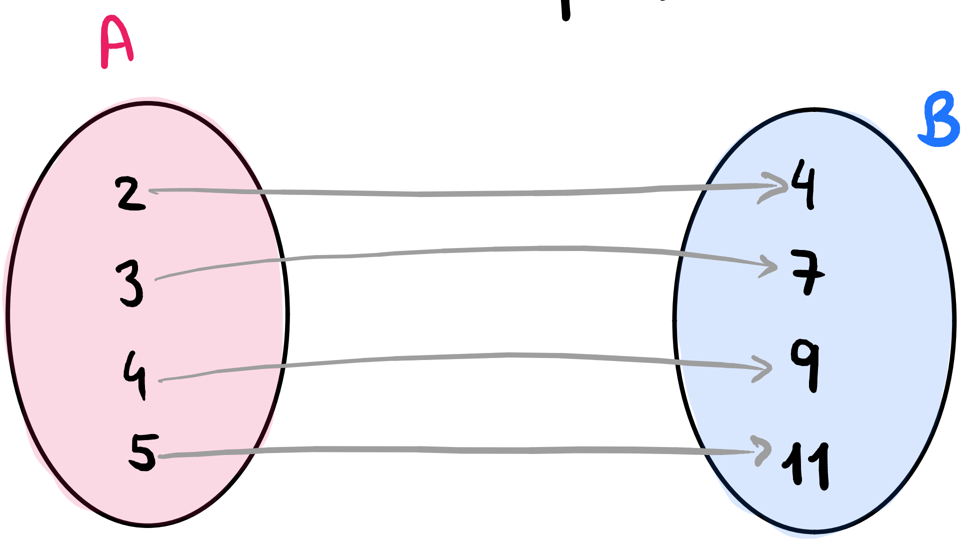
(i)



- $D = \{2, 3, 4, 5\}$
- $CD = \{4, 7, 9, 11\}$
- $I = \{4, 7, 11\}$

- $f(2) = 4$
- $f(3) = 7$
- $f(4) = 7$
- $f(5) = 11$

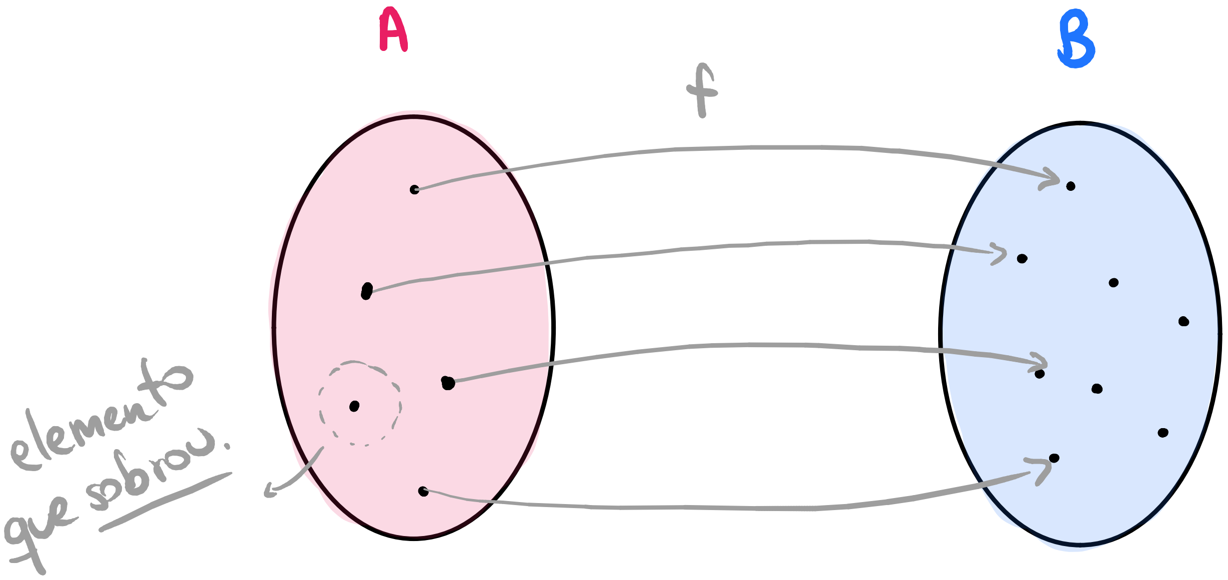
(ii)



- $D = \{2, 3, 4, 5\}$
- $CD = \{4, 7, 9, 11\}$
- $I = \{4, 7, 9, 11\}$

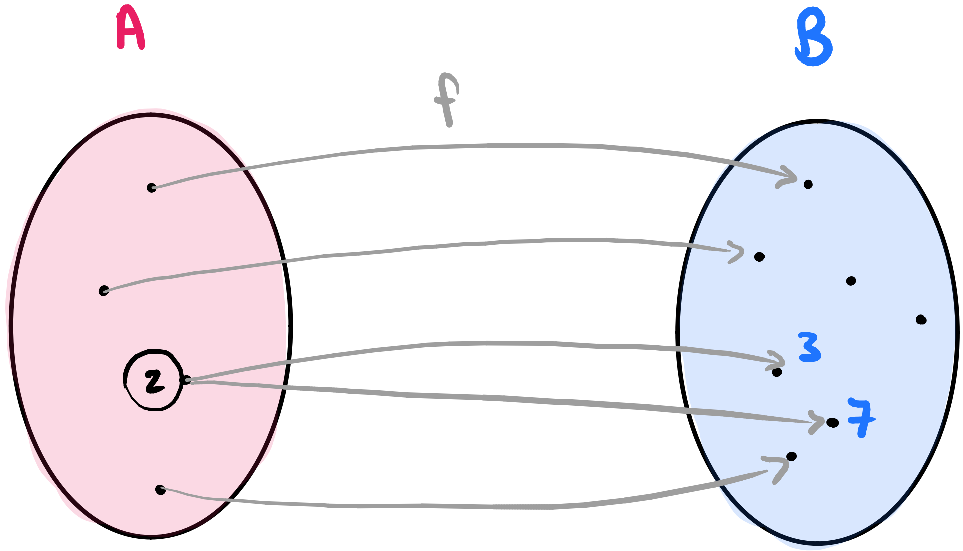
8.1.2. Definição

(i) Todo elemento do domínio deve participar da relação.



f não é uma função.

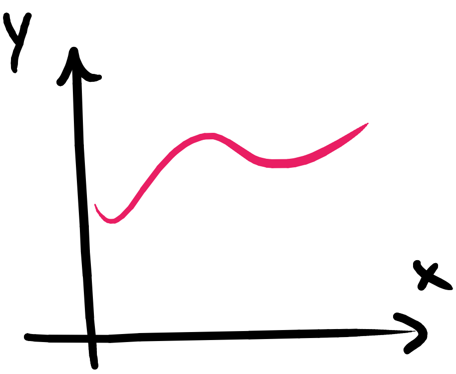
(ii) Os elementos do domínio estão associados a um único valor no contra-domínio.

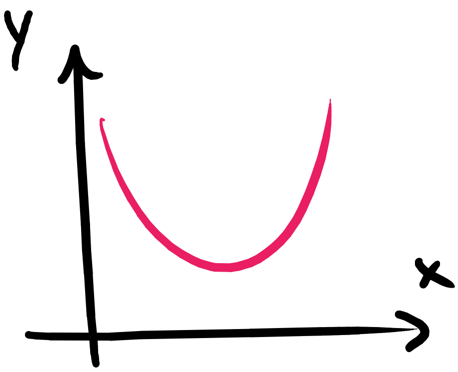


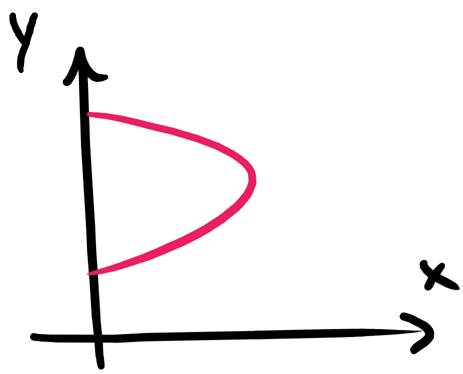
f não é uma função.

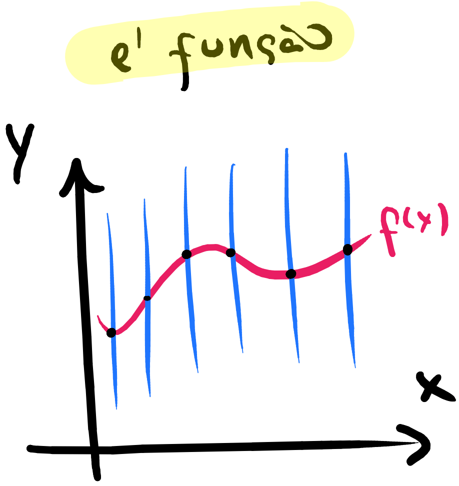
Há ambiguidade: $f(2) = 3$ ou $f(2) = 7$?

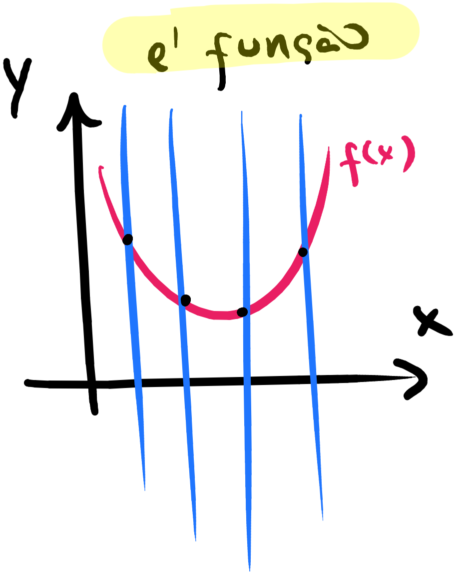
Teste: retas verticais

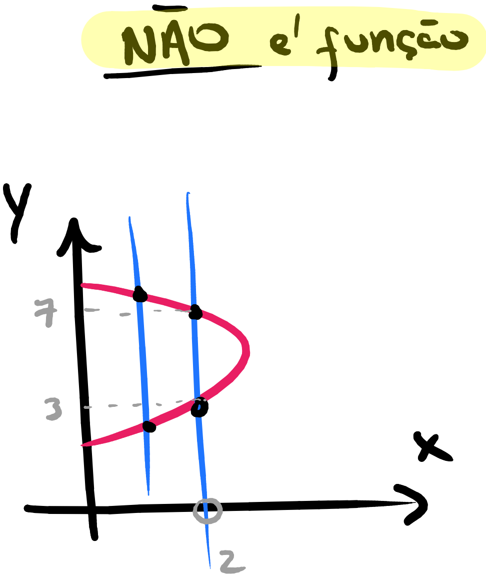












Se uma reta vertical interceptar o gráfico em mais de um ponto então <u>não</u> se trata de uma função.

f é aplicação de A em B se e somente se

$$\boxed{\forall x \in A \quad \exists| \ y \in B \quad | \quad (x, y) \in f}$$

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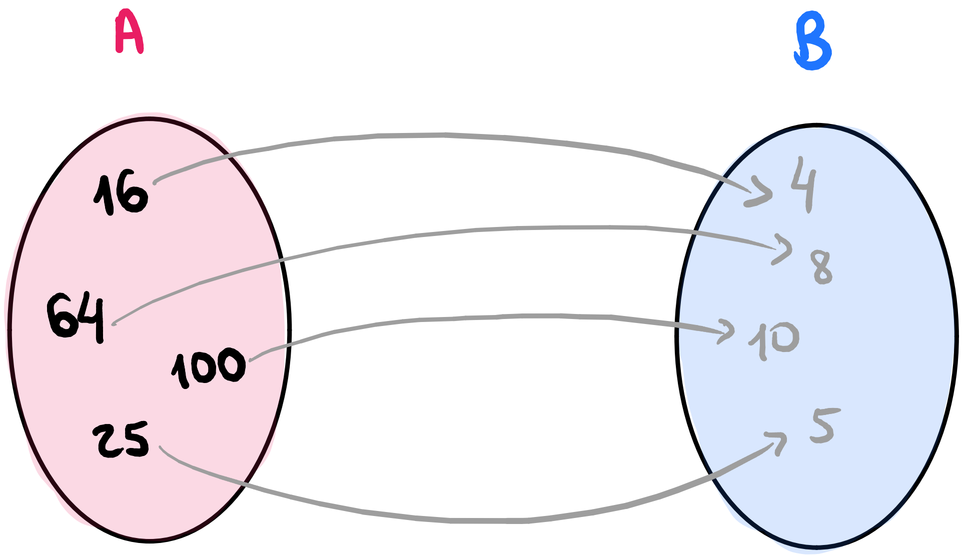
Exemplo

$$f(x) = \sqrt{x}$$

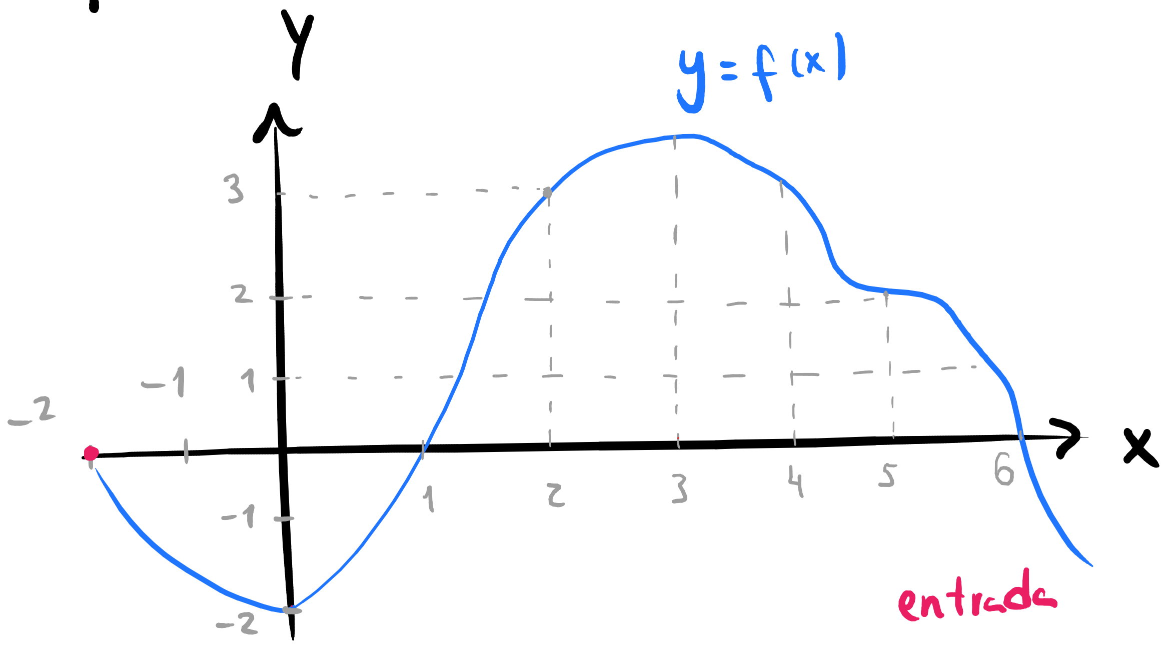


$$f : A \longrightarrow B$$

$$f : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$$

$$\boxed{f(x) = \sqrt{x}}$$

Exemplo



entrada

$$y = f(x)$$

saída

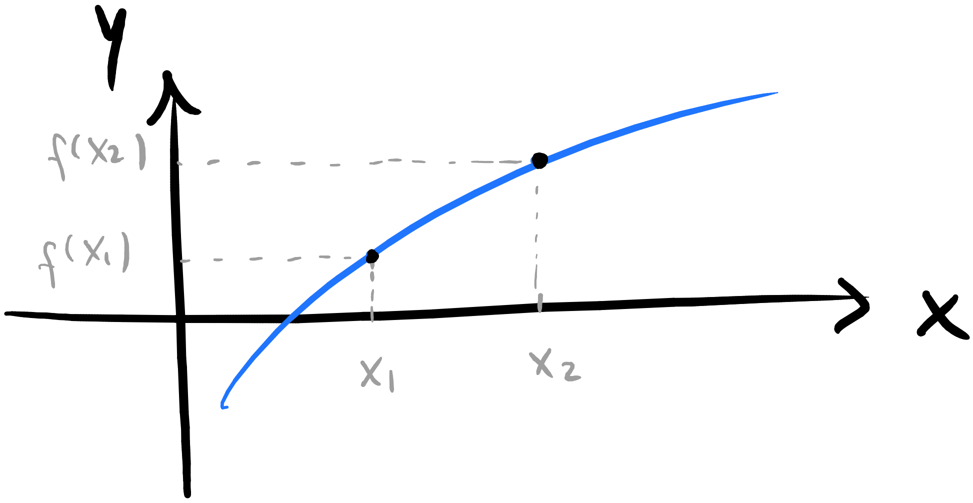
- $f(1) = 0$
- $f(2) = 3$
- $f(5) = 2$
- $f(6) = 0$
- $f(0) = -2$
- $f(-2) = 0$

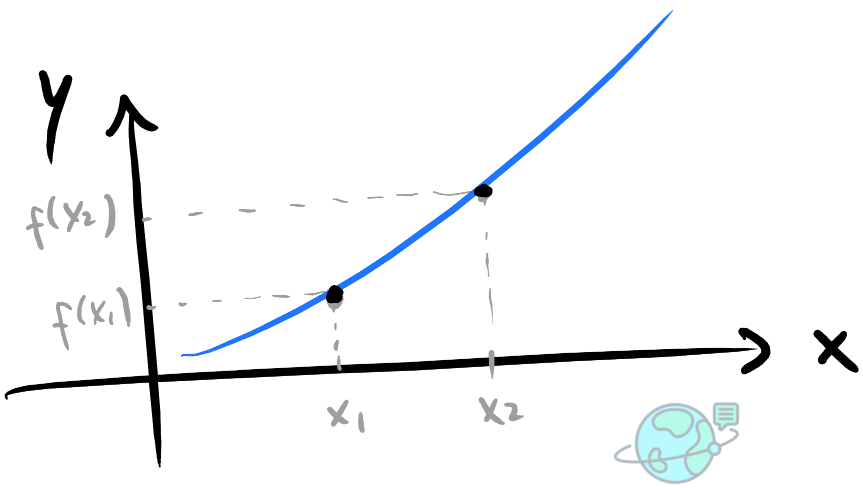
8.1.3 Crescente | Decrescente

a) **Crescente**

$f: A \to B$ é função crescente se, para todo x_1 e x_2 em A, com $x_2 > x_1$, temos:

$$f(x_2) > f(x_1)$$

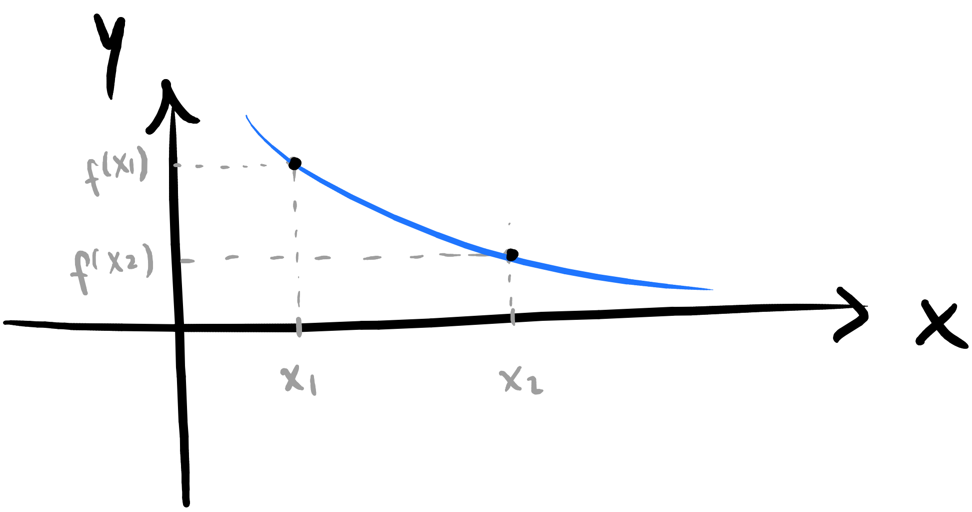


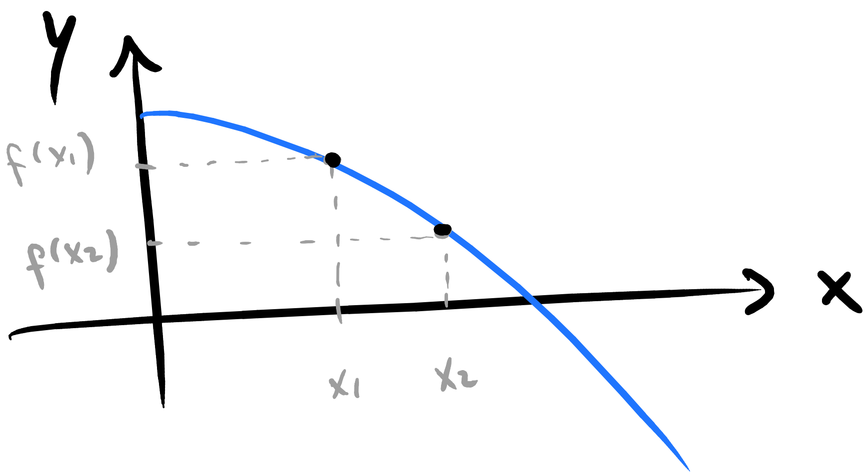


b) Decrescente

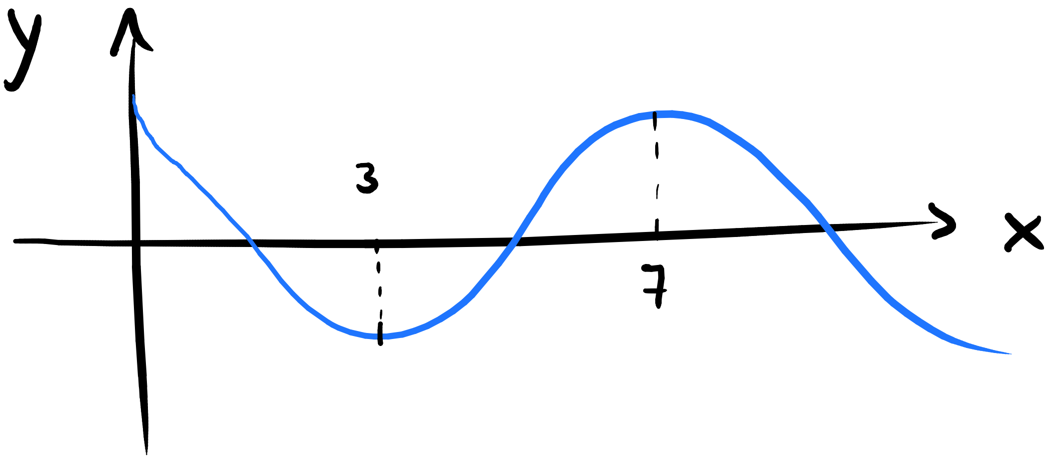
$f: A \to B$ é função decrescente se, para todo x_1 e x_2 em A, com $x_2 > x_1$, temos:

$$f(x_2) < f(x_1)$$





Exemplo



$0 < x < 3$: DECRESCENTE

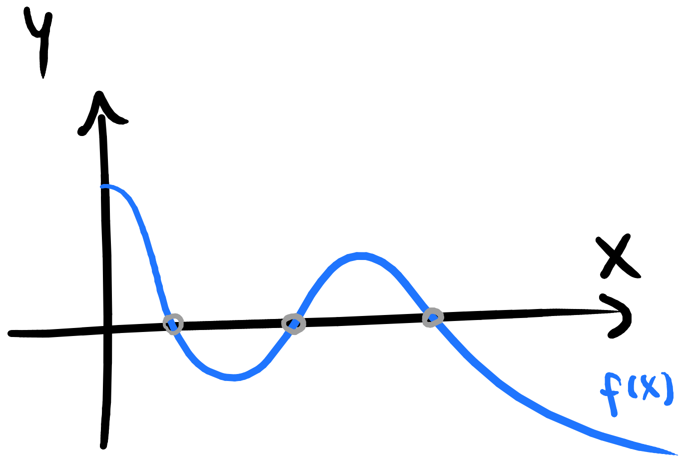
$3 < x < 7$: CRESCENTE

$x > 7$: DECRESCENTE

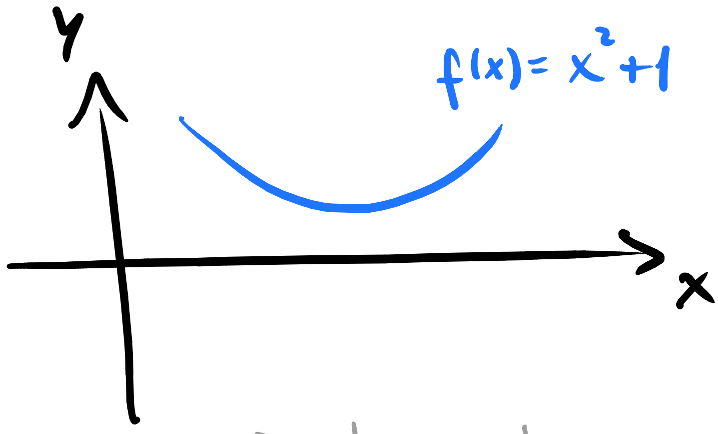
8.1.4. Raízes

- Raiz ou zero de uma função é todo número $\underline{x}$ cuja imagem é nula:

$$f(x) = 0$$



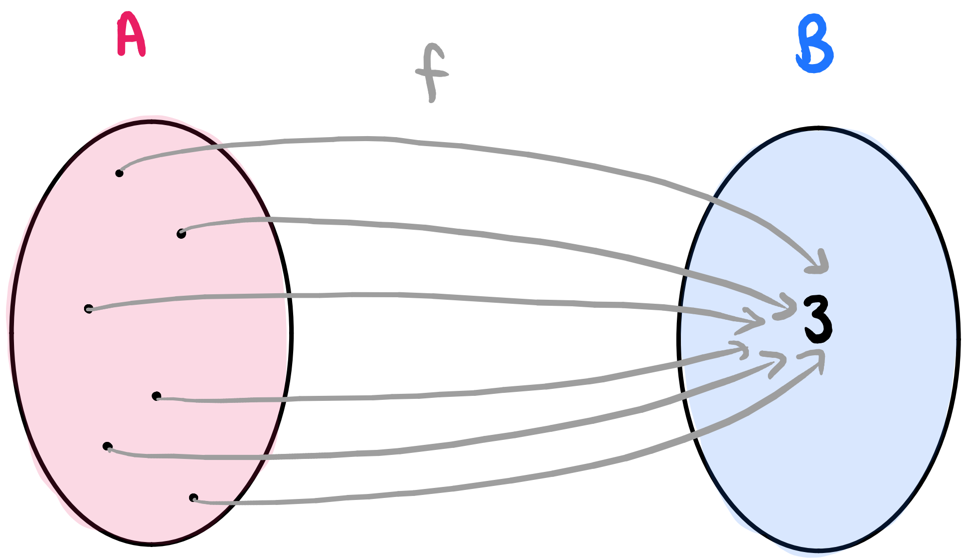
3 raízes



não tem raízes

↳ graficamente, trata-se do ponto onde o gráfico corta o eixo x.

8.2. Função Constante



$$\boxed{f(x) = 3}$$

$$\begin{cases} f(4) = 3 \\ f(17) = 3 \\ f(-8) = 3 \end{cases}$$

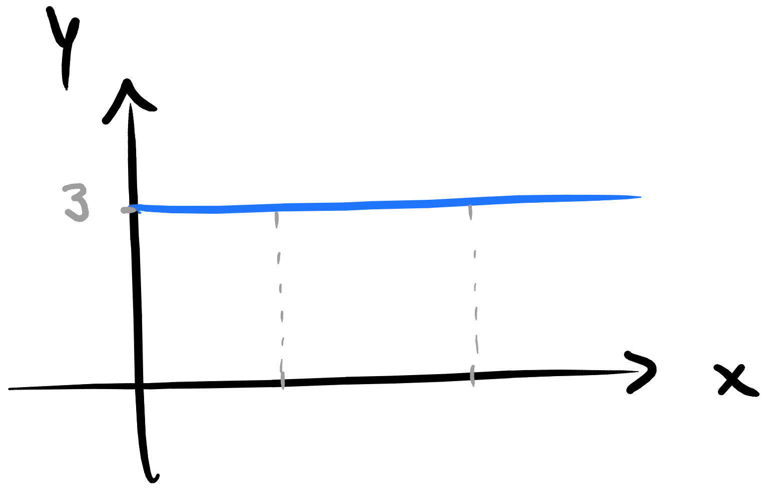
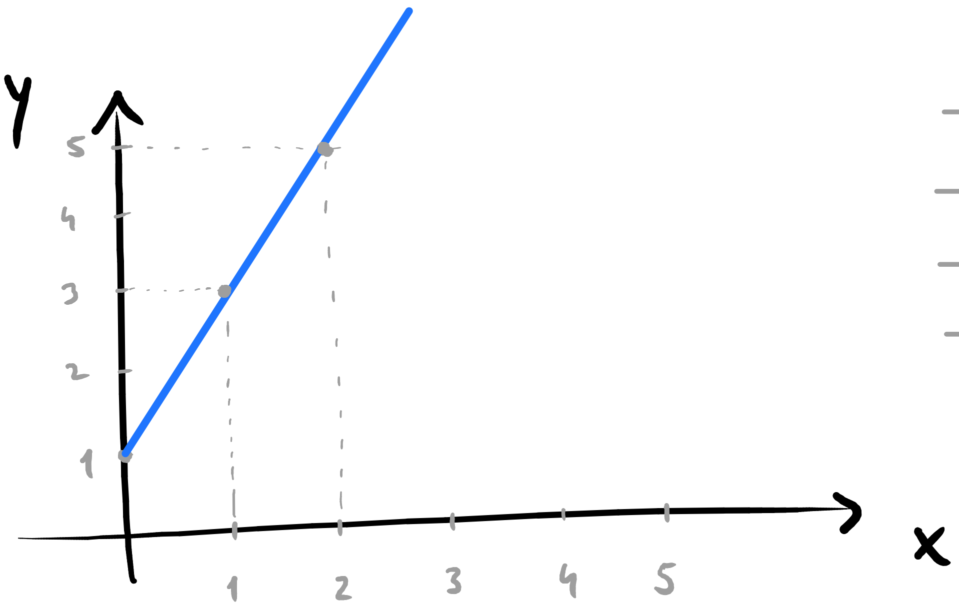


Gráfico: reta horizontal

8.3 . Função do 1º grau

$$\boxed{f(x) = a \cdot x + b} \quad , \quad a \neq 0$$

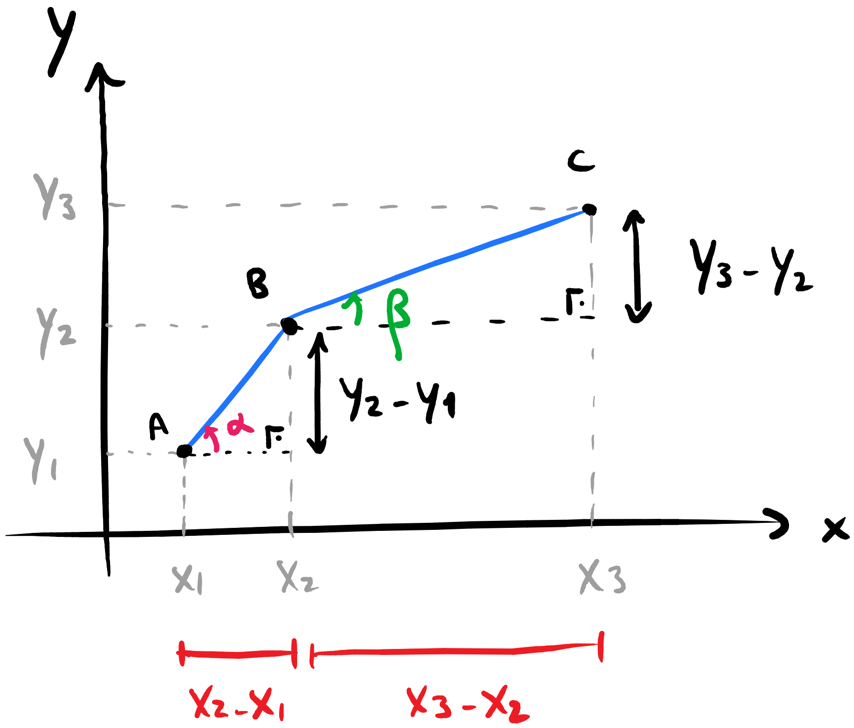
Gráfico: linha reta



$f(x) = 2x + 1$

x	$f(x)$
0	1
1	3
2	5
5	1
⋮	⋮

Prova:



$$y = f(x) = ax + b$$

A: $\quad y_1 = a x_1 + b$

B: $\quad y_2 = a x_2 + b$

C: $\quad y_3 = a x_3 + b$

$(-)$

$(-)$

$$y_2 - y_1 = a(x_2 - x_1) \quad \therefore \quad a = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y_3 - y_2 = a(x_3 - x_2) \quad \therefore \quad a = \frac{y_3 - y_2}{x_3 - x_2}$$

$$\frac{y_3 - y_2}{x_3 - x_2} = \frac{y_2 \;\; y_1}{x_2 - x_1}$$

$$\tan\beta = \tan\alpha$$

$$\boxed{\alpha = \beta}$$

↳ o gráfico é uma <u>reta</u>!

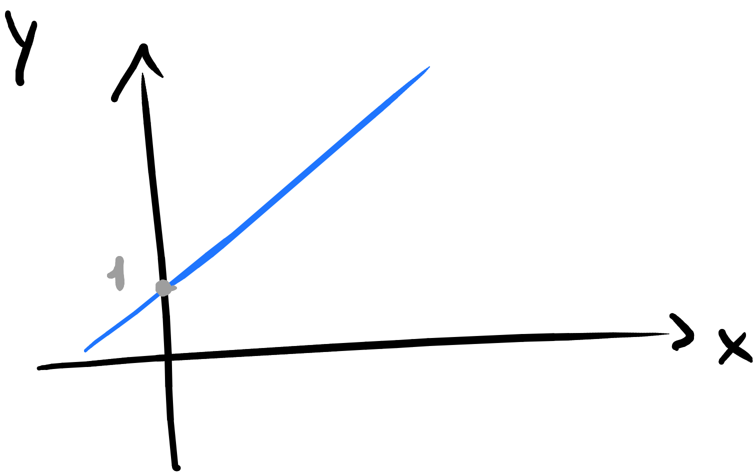
8.3.1 Coeficiente Linear

$f(x) = a \cdot x + b$

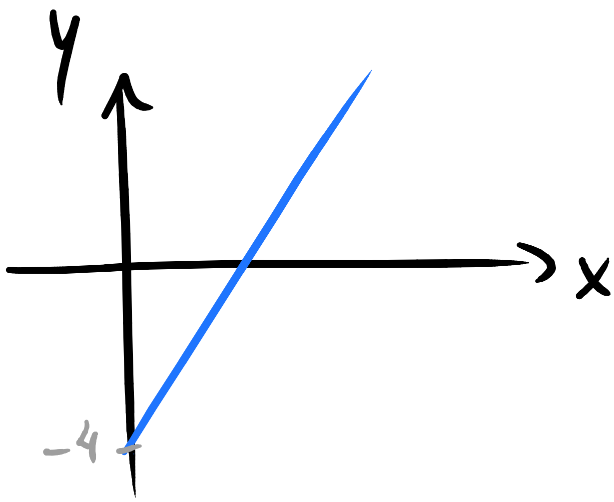
↳ "b": representa onde a reta corta o eixo vertical (valor de y quando x e' zero)

$$\boxed{f(x) = a \cdot x + b}$$

$$f(0) = \overset{0}{\cancel{a \cdot 0}} + b \quad \therefore \quad f(0) = b$$

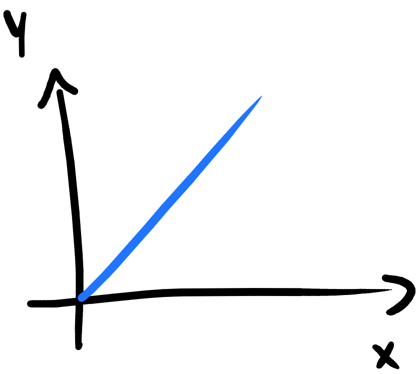


$f(x) = 3x + 1$

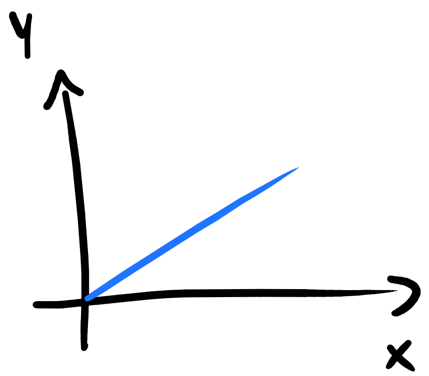


$f(x) = 5x - 4$

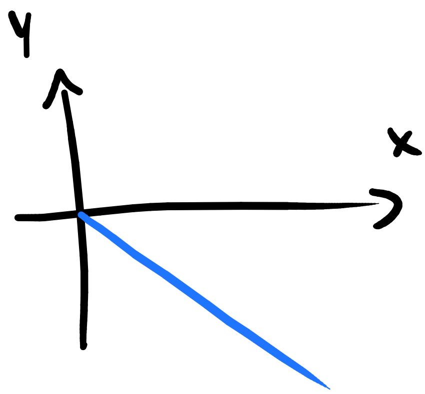
Se $b = 0$: a reta passa pela origem:



$f(x) = 4x$



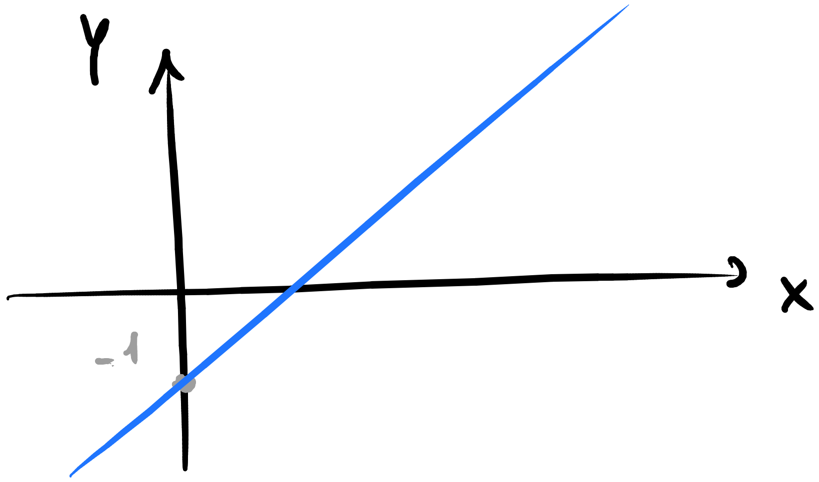
$f(x) = \frac{1}{2}x$



$f(x) = -2x$

Exemplo

$f(x) = 2x - 1$



8.3.2 Coeficiente Angular

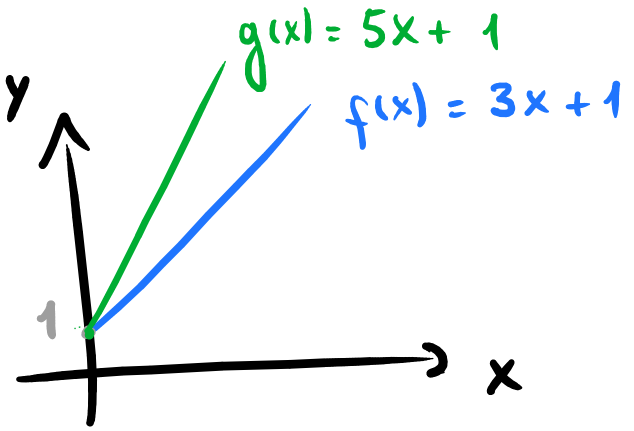
$f(x) = a \cdot x + b$

"a"

$$f(x) = \boxed{a}\, x + b$$

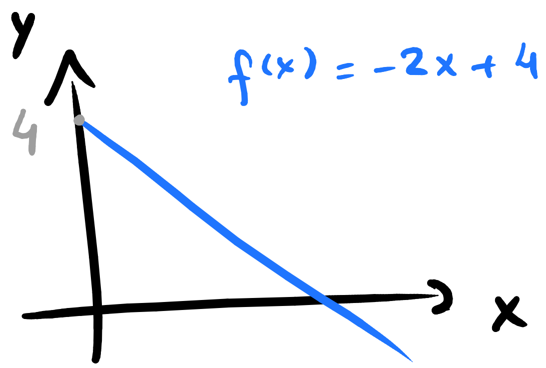
$a > 0$

FUNÇÃO CRESCENTE



$a < 0$

FUNÇÃO DECRESCENTE



Obs.: coeficiente angular mede a inclinação

Prova: função crescente

$f: A \to B$ é função crescente se, para todo x_1 e x_2 em A, com $x_2 > x_1$, temos:

$$f(x_2) > f(x_1)$$

Logo:

$$f(x_1) = ax_1 + b$$
$$f(x_2) = ax_2 + b$$

Se $x_2 > x_1$ precisamos ter $f(x_2) > f(x_1)$:

$$ax_2 + b > ax_1 + b$$

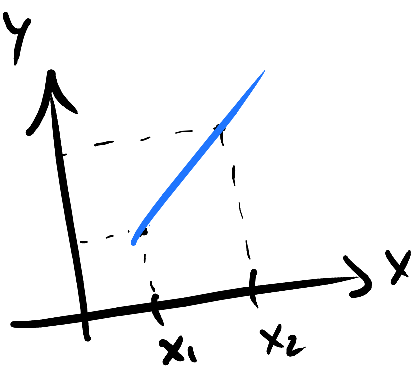
$-(b)$

$$ax_2 > ax_1$$

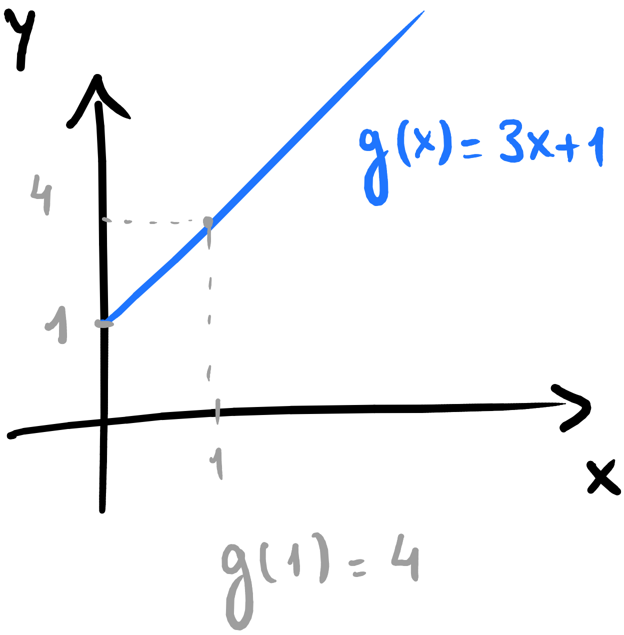
$$ax_2 - ax_1 > 0$$

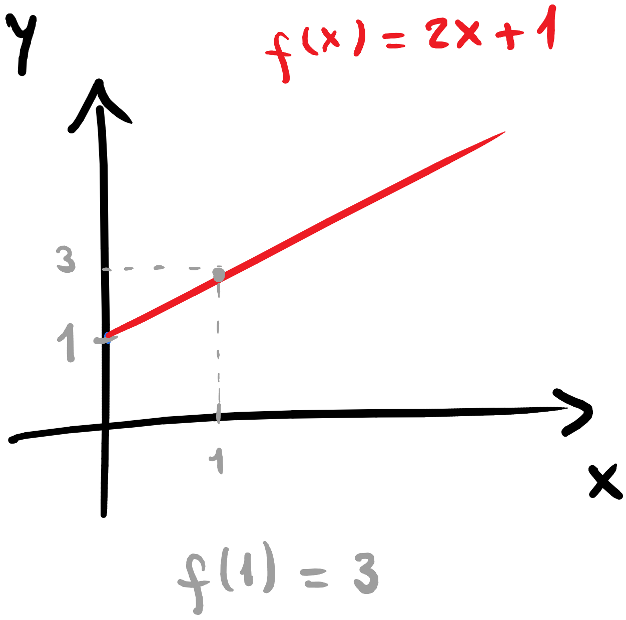
$$a(\underbrace{x_2 - x_1}_{>0}) > 0$$

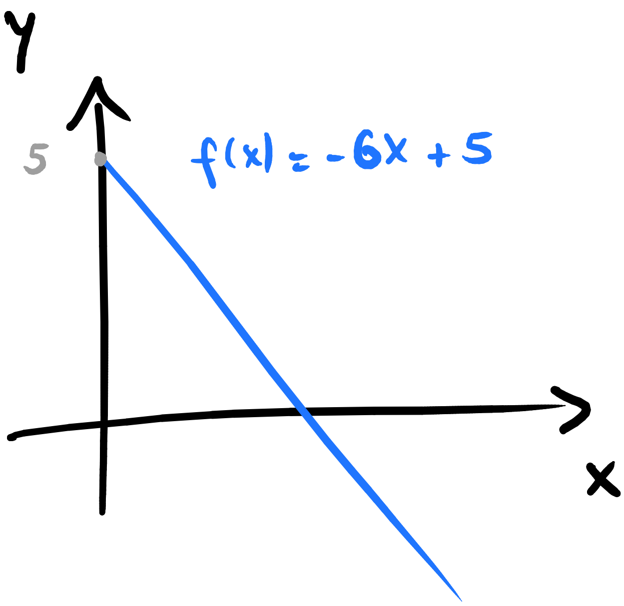
$$\boxed{a > 0}$$

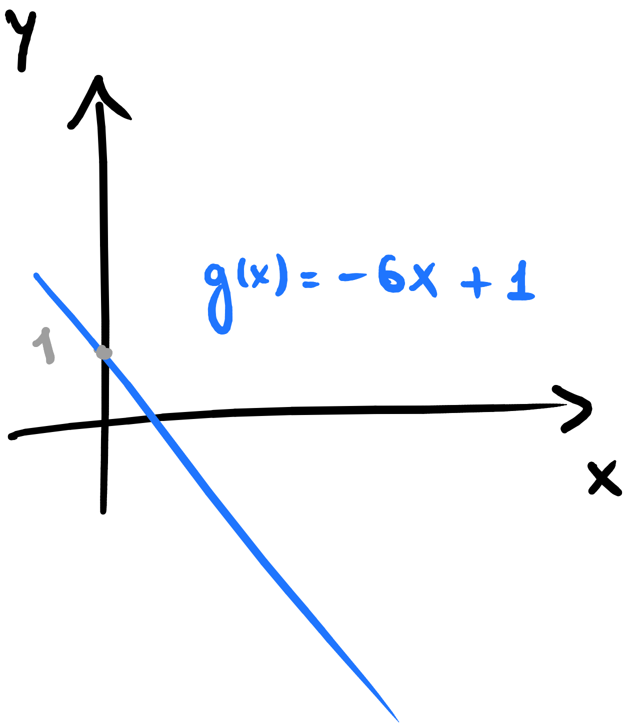


Exemplos









8.3.3 Raiz e sinal

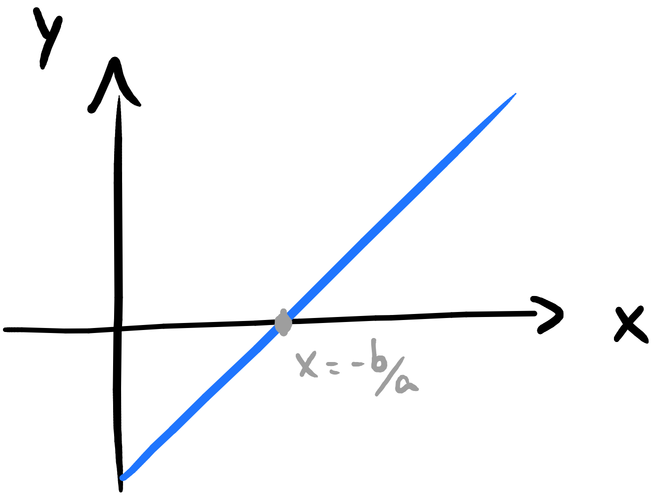
$$f(x) = a \cdot x + b$$

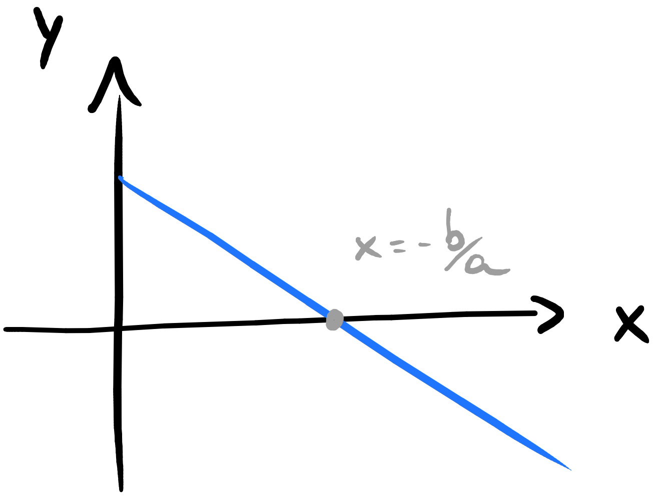
Raiz: $f(x) = 0$

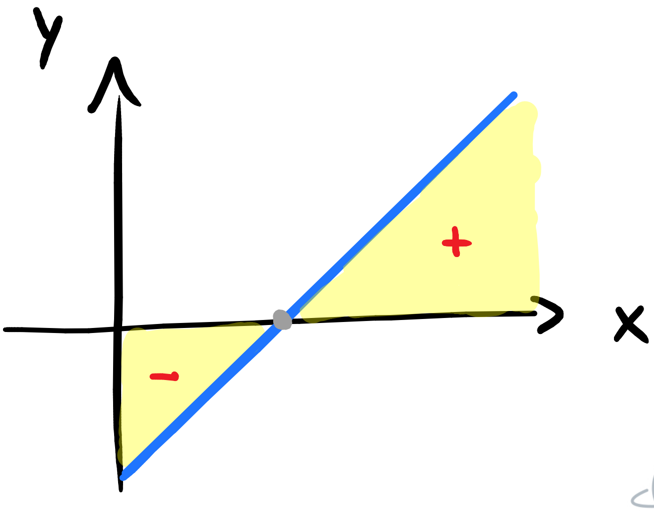
$$ax + b = 0 \quad \Big) -b$$

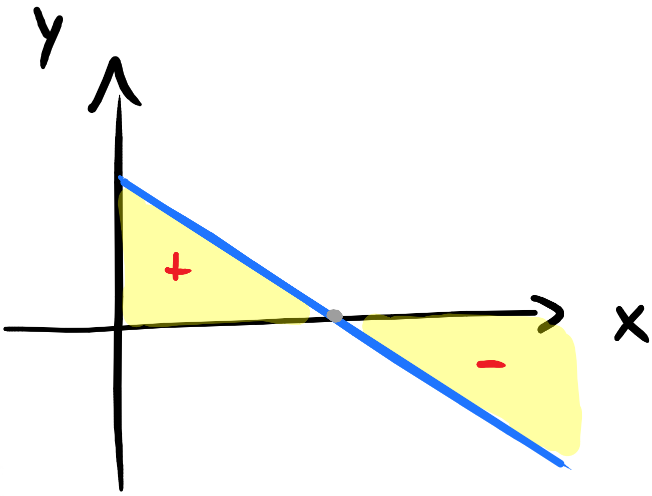
$$ax = -b \quad \Big) \div a$$

$$\boxed{x = -b/a}$$ → único raiz!





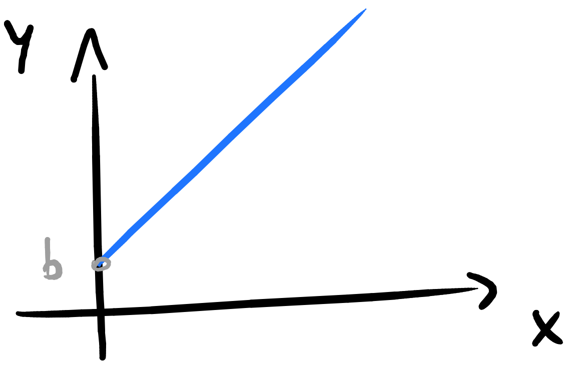


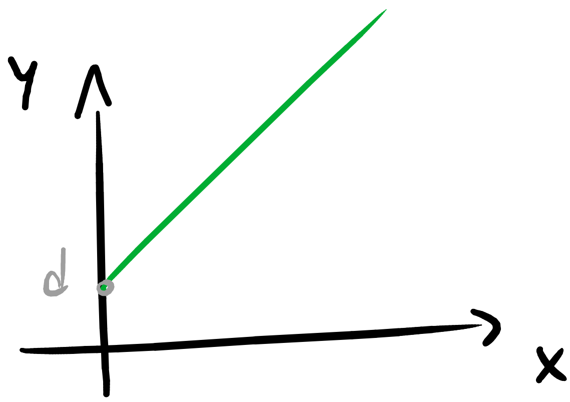


8.3.4 Igualdade de funções

$$\boxed{f(x) = ax + b}$$

$$\boxed{g(x) = c \cdot x + d}$$





$$f(x) = g(x)$$

$$\Downarrow$$

$$a = c \quad e \quad b = d$$

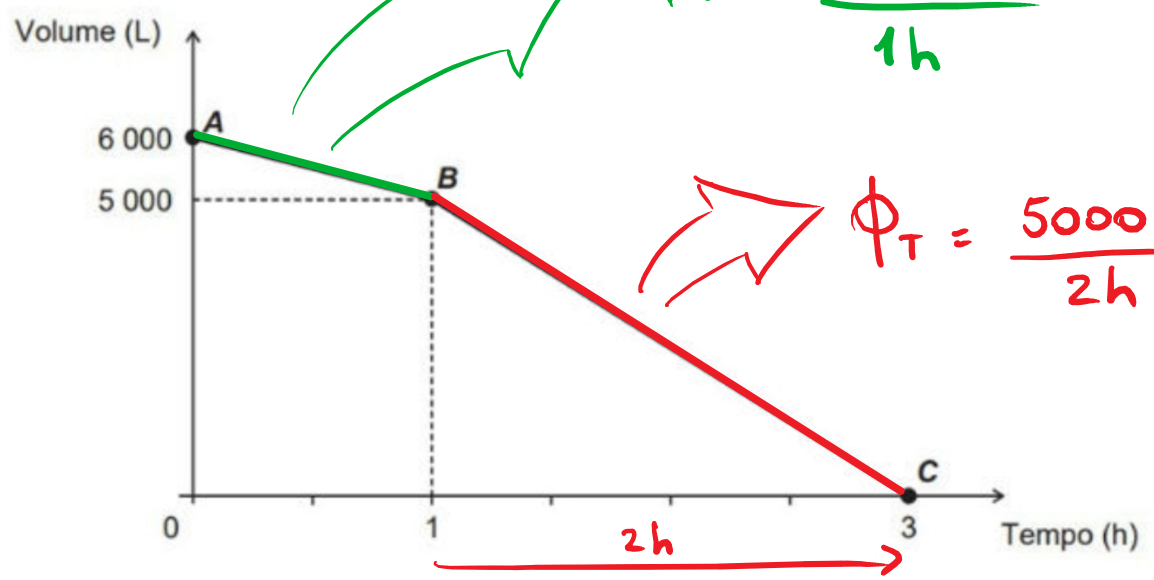
Exercício

$$\phi = \frac{Vol}{\Delta t}$$

(Enem - 2016) Uma cisterna de 6 000 L foi esvaziada em um período de 3h. Na primeira hora foi utilizada apenas uma bomba, mas nas duas horas seguintes, a fim de reduzir o tempo de esvaziamento, outra bomba foi ligada junto com a primeira. O gráfico, formado por dois segmentos de reta, mostra o volume de água presente na cisterna, em função do tempo.

$$\phi_1 = \frac{1000\,L}{1\,h}$$

$$\phi_T = \frac{5000\,L}{2\,h} = \frac{2500\,L}{h}$$



Qual é a vazão, em litro por hora, da bomba que foi ligada no início da segunda hora?

a) 1 000
b) 1 250
c) 1 500
d) 2 000
e) 2 500

$$\phi_T = \phi_1 + \phi_2$$

$$\frac{2500\,L}{h} = \frac{1000\,L}{h} + \phi_2$$

$$\boxed{\phi_2 = \frac{1500\,L}{h}}$$

8.4 . função do 2º grau

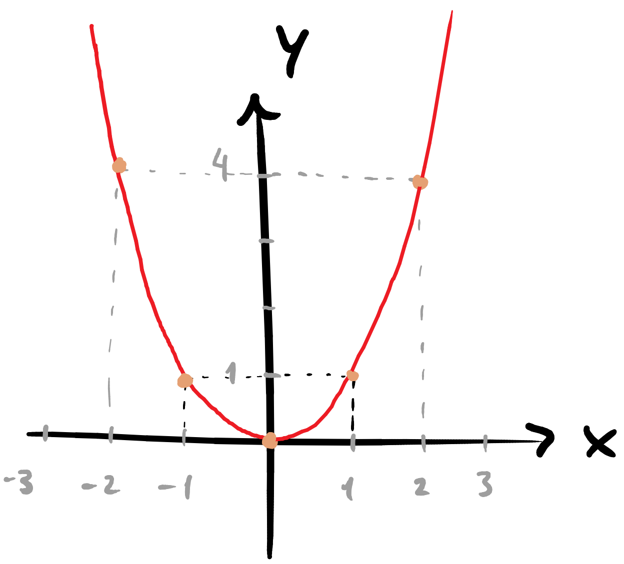
$$\boxed{f(x) = a.x^2 + b.x + c} \quad , \quad a \neq 0$$

Ex.:

$$f(x) = 2x^2 + 3x - 1 \quad \begin{cases} \cdot\ a = 2 \\ \cdot\ b = 3 \\ \cdot\ c = -1 \end{cases}$$

$$g(x) = -5x^2 - 2x + 4 \quad \begin{cases} \cdot\ a = -5 \\ \cdot\ b = -2 \\ \cdot\ c = 4 \end{cases}$$

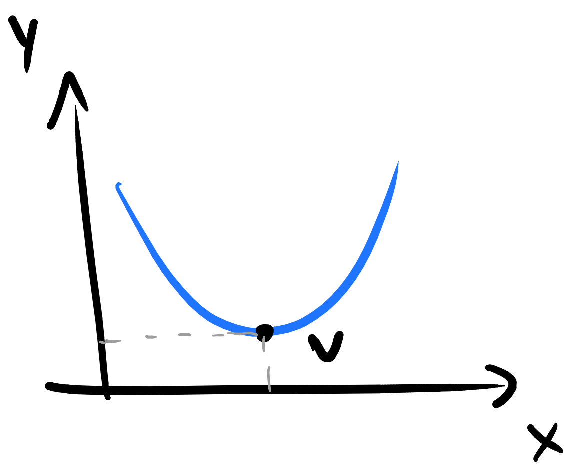
Gráfico: parábola

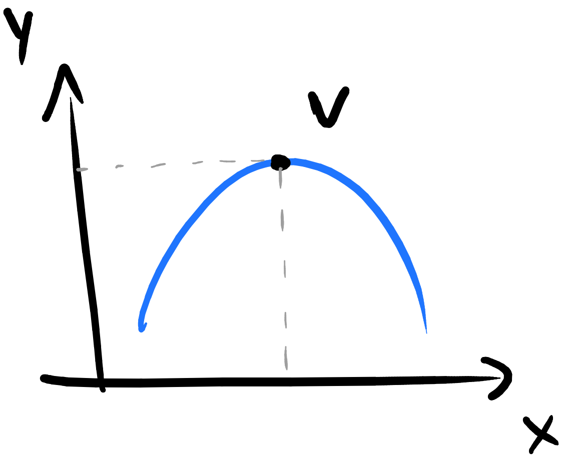


$f(x) = x^2$	x
9	−3
4	−2
1	−1
0	0
1	1
4	2
9	3

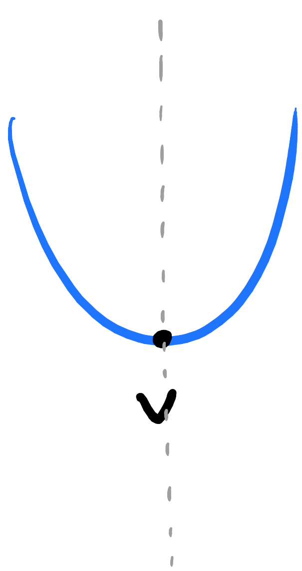
Propriedades:

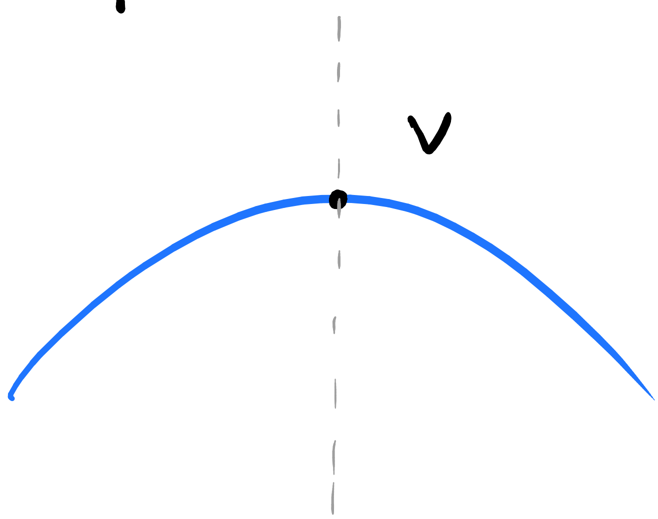
(i) Possui sempre **um** máximo ou mínimo (no seu vértice V)





(ii) É simétrica em relação ao seu eixo (reta vertical que passa pelo vértice)



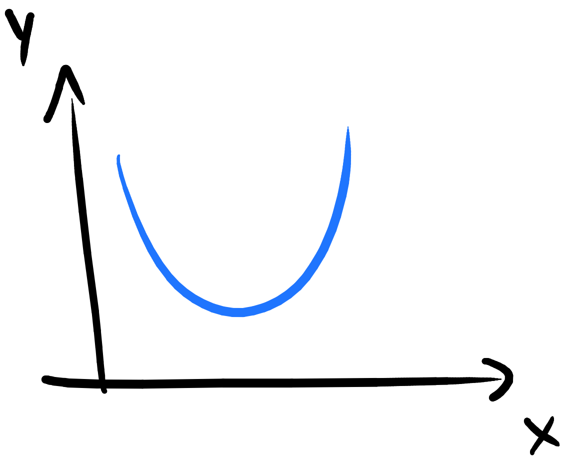


8.4.1 Concavidade

$$\boxed{f(x) = a.x^2 + b.x + c}$$

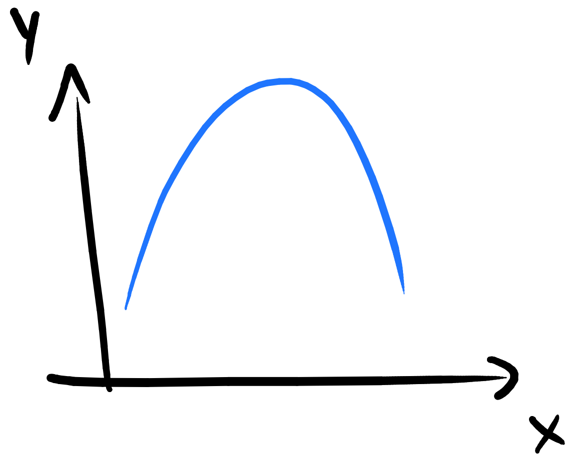
$\boxed{a > 0}$

CONCAVIDADE P/ CIMA

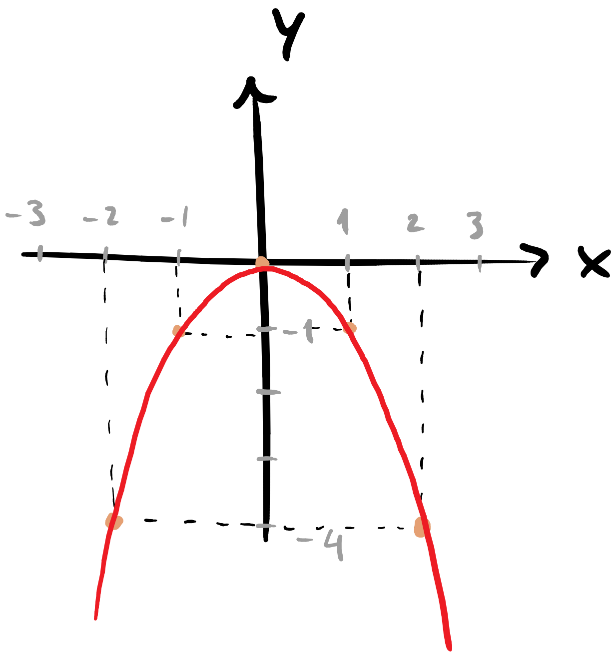


$\boxed{a < 0}$

CONCAVIDADE P/ BAIXO



Ex :



$f(x) = -x^2$	x
-4	-2
-1	-1
0	0
-1	1
-4	2

8.4.2 Raízes

↳ Valores de x para os quais $f(x) = 0$:

$$f(x) = \underbrace{ax^2 + bx + c = 0}_{\text{equação do segundo grau}}$$

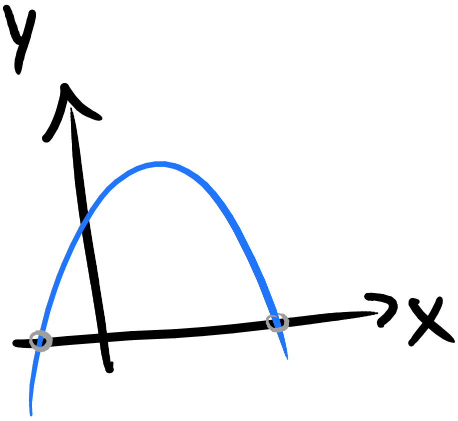
Solução:

$$\boxed{x = \frac{-b \pm \sqrt{\Delta}}{2a}}$$

onde $\Delta = b^2 - 4ac$

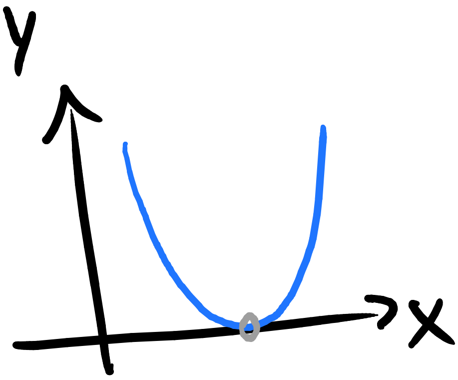
Interpretação geométrica

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$



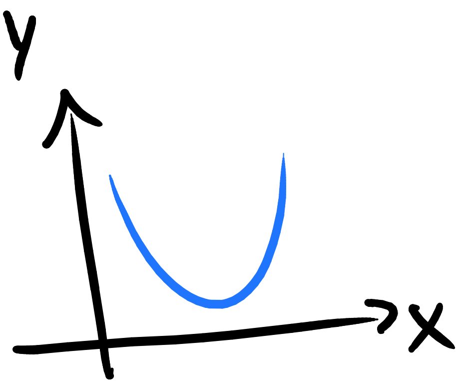
$\boxed{\Delta > 0}$

↳ 2 raízes distintas



$\boxed{\Delta = 0}$

↳ 2 raízes iguais
(única raiz)



$\boxed{\Delta < 0}$

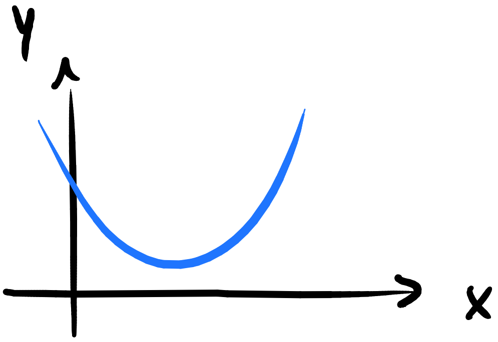
↳ não há raízes reais

Exemplo

$$f(x) = x^2 + 1$$

$$f(x) = 0 \quad \therefore \quad x^2 + 1 = 0$$

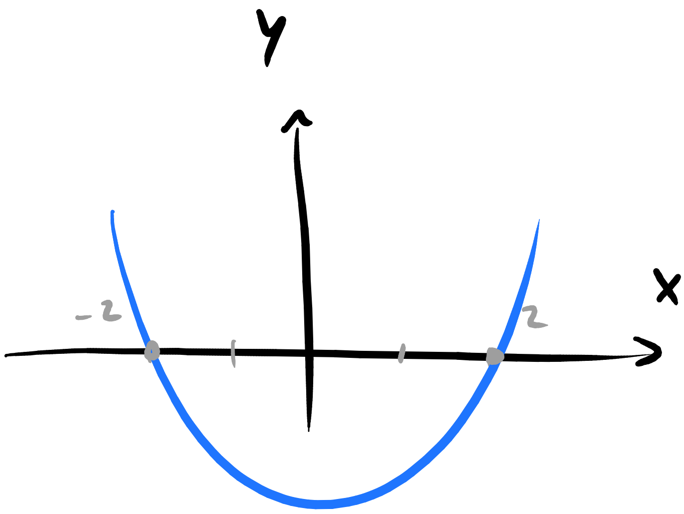
$$x^2 = -1$$



$$g(x) = x^2 - 4$$

$$g(x) = 0 \quad \therefore \quad x^2 - 4 = 0$$

$$x^2 = 4 \quad \begin{cases} \boxed{x_1 = +2} \\ \boxed{x_2 = -2} \end{cases}$$



8.4.3 Coeficiente "c"

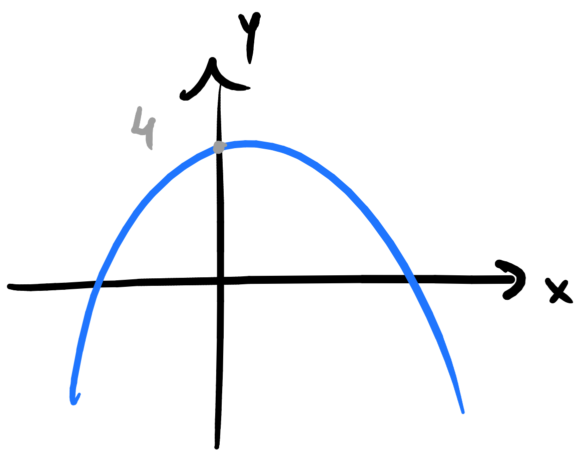
$f(x) = ax^2 + bx + c$

↳ **"c"**: representa onde a parábola corta o eixo vertical (valor de y quando x é zero)

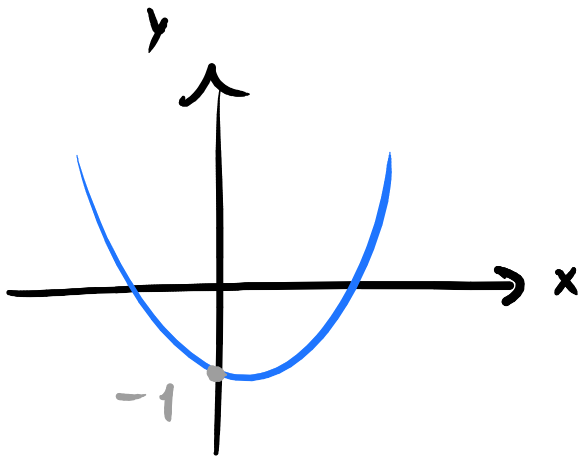
$$f(x) = ax^2 + bx + c$$

$$f(0) = \cancel{a}\cdot 0^2 + \cancel{b}\cdot 0 + c$$

$$f(0) = c$$



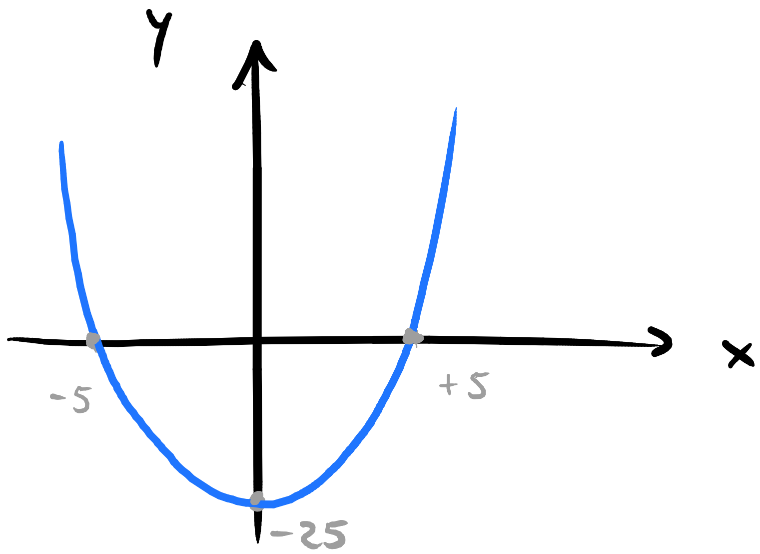
$f(x) = ax^2 + bx + 4$



$f(x) = ax^2 + bx - 1$

Exemplos

(i) Esboce o gráfico de $f(x) = x^2 - 25$



↳ RAÍZES: $f(x) = 0$

$x^2 - 25 = 0$

$x^2 = 25$

$\boxed{x = \pm 5}$

1 CONCAVIDADE
- ↗ a(+) ∪
- ↘ a(−) ∩

2 RAÍZES → intercepto com o eixo $\underline{x}$.

3 TERMO INDEPENDENTE → intercepto com o eixo $\underline{y}$.

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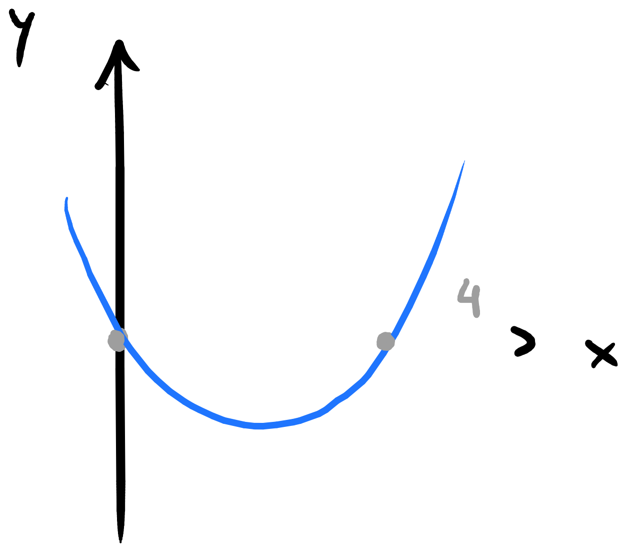
(ii) Esboce o gráfico de $f(x) = x^2 - 4x$



RAÍZES: $f(x) = 0$

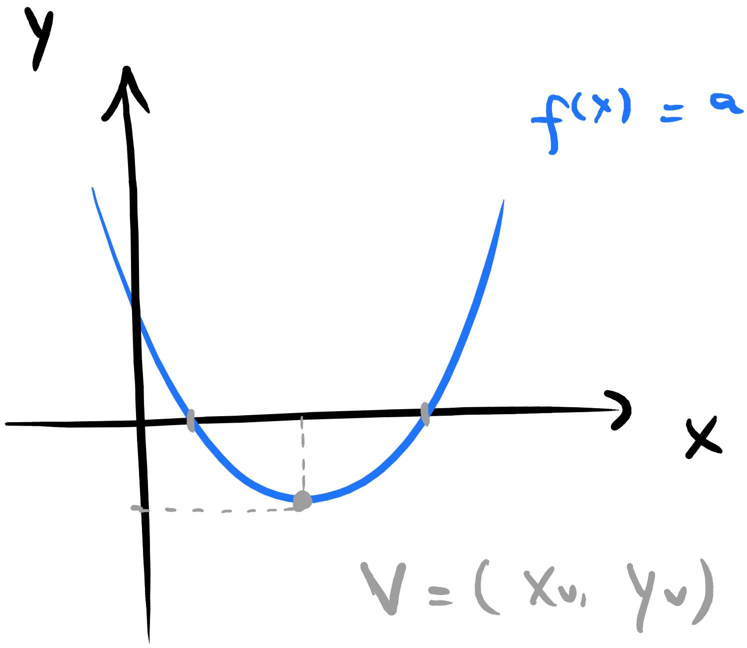
$$x^2 - 4x = 0$$

$$x \cdot (x - 4) = 0$$

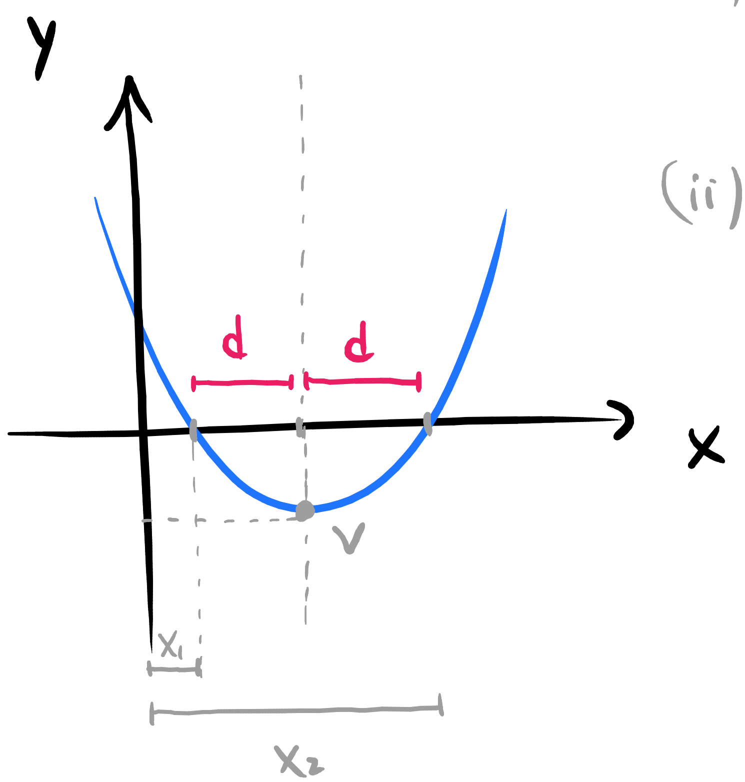
$\boxed{x = 0}$ ou $\boxed{x = 4}$

8.4.4 Coordenadas do Vértice

$$f(x) = ax^2 + bx + c$$



$$V = (x_v, y_v)$$



(i) $\boxed{x_v = x_1 + d}$

(ii) $2d = x_2 - x_1$

$$\boxed{d = \frac{x_2 - x_1}{2}}$$

$$X_v = x_1 + \frac{2 - x_1}{2} \quad \therefore \quad X_v = \frac{x_1 + x_2}{2}$$

$$\left.\begin{aligned} x_1 &= \frac{-b - \sqrt{\Delta}}{2a} \\[2ex] x_2 &= \frac{-b + \sqrt{\Delta}}{2a} \end{aligned}\right\} \quad \frac{x_1 + x_2}{2} = \frac{\dfrac{-b - \cancel{\sqrt{\Delta}}}{2a} + \dfrac{-b + \cancel{\sqrt{\Delta}}}{2a}}{2}$$

$$= \frac{-\cancel{2}b/\cancel{2}a}{2} = \frac{-b/a}{2}$$

$$\boxed{X_v = -b/2a} \quad \therefore \quad y_v = f(x_v) = a x_v^2 + b x_v + c$$

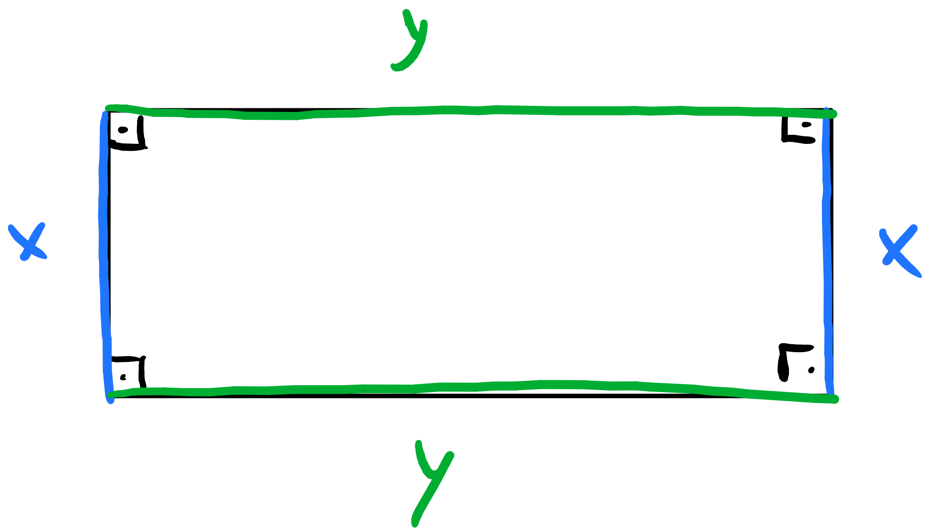
O vértice da parábola está no ponto

$V = (x_v, y_v)$ tal que:

$$\boxed{X_v = -b/2a} \qquad \boxed{y_v = -\frac{\Delta}{4a}}$$

Exemplo

Perímetro $= 100$. Máxima área $= ?$



(i) $P = 2x + 2y = 100 \therefore \boxed{x + y = 50}$

(ii) $A = x \cdot y \therefore A = x \cdot (50 - x)$

MÁX.

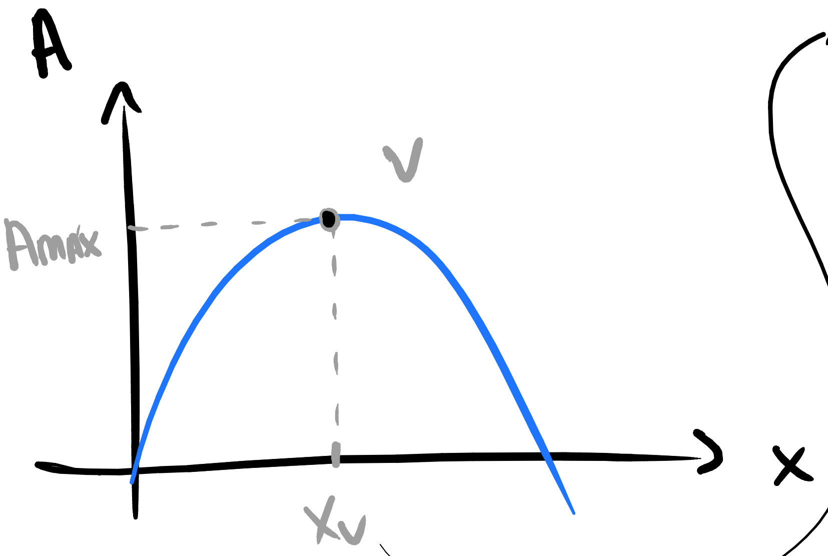
$$A = x \cdot (50 - x)$$

$$A = 50x - x^2$$

$$y = bx - ax^2$$

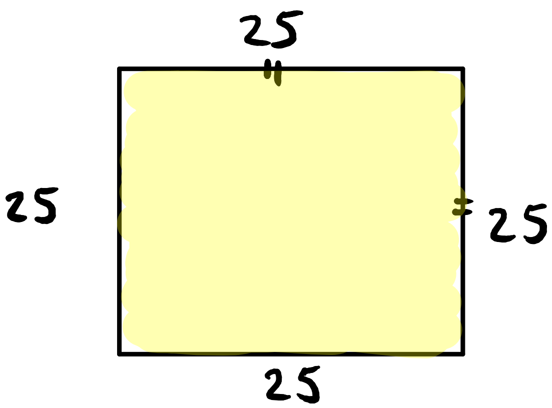
- $b = 50$
- $a = -1$

$$\boxed{A(x) = 50x - x^2}$$



$$X_v = \frac{-b}{2a} = \frac{-50}{2(-1)}$$

$$\boxed{X_v = 25}$$



$$\boxed{x + y = 50}$$

$$\text{Amáx:} \begin{cases} x = 25 \\ y = 25 \end{cases}$$