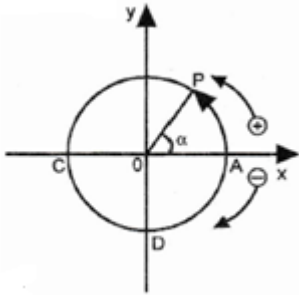
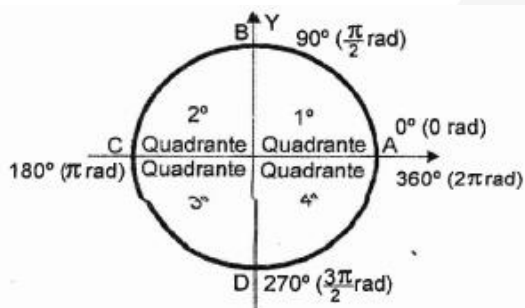
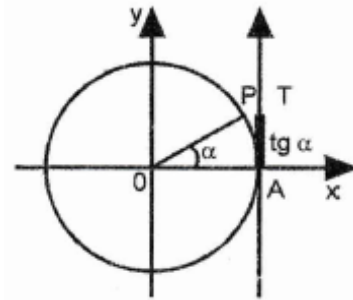
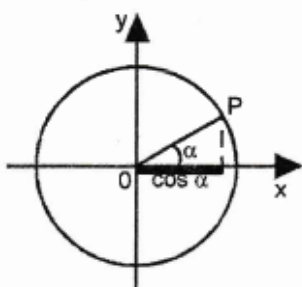
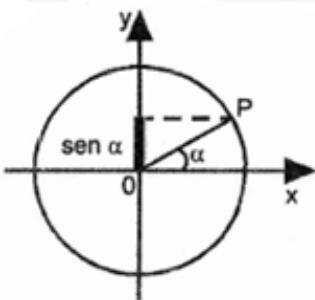


**MÓDULO 30**
**1. CÍRCULO TRIGONOMÉTRICO**

É a circunferência de raio unitário, com centro na origem de um sistema de coordenadas cartesianas XOY. A circunferência trigonométrica é orientada positivamente no sentido anti-horário a partir do ponto A(1,0).

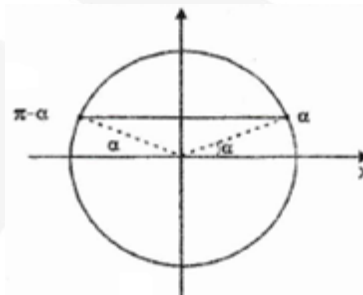
Cada ponto P dessa circunferência é extremidade de um arco AP, de medida  $\alpha$ .


**2. QUADRANTES**

**3. SENO E COSSENO**


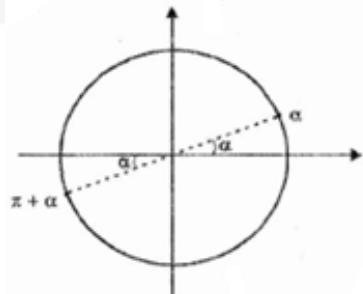
O seno de  $\alpha$  é a ordenada do ponto P.  
O cosseno de  $\alpha$  é a abscissa do ponto P.

**4. REDUÇÃO AO 1º QUADRANTE**

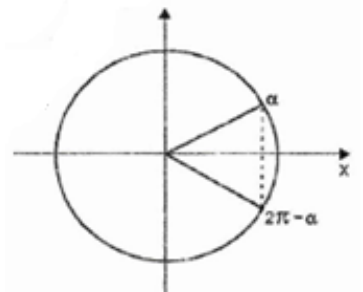
a) Redução ao 1º quadrante:



$$\begin{aligned} \text{sen}(\pi - \alpha) &= \text{sen } \alpha \\ \text{cos}(\pi - \alpha) &= -\text{cos } \alpha \\ \text{tg}(\pi - \alpha) &= -\text{tg } \alpha \end{aligned}$$



$$\begin{aligned} \text{sen}(\pi + \alpha) &= -\text{sen } \alpha \\ \text{cos}(\pi + \alpha) &= -\text{cos } \alpha \\ \text{tg}(\pi + \alpha) &= \text{tg } \alpha \end{aligned}$$



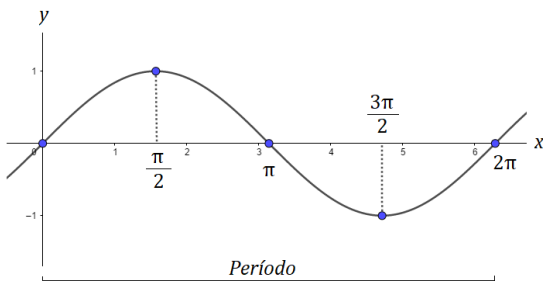
$$\begin{aligned} \text{sen}(2\pi - \alpha) &= -\text{sen } \alpha = \text{sen}(-\alpha) \\ \text{cos}(2\pi - \alpha) &= \text{cos } \alpha = \text{cos}(-\alpha) \\ \text{tg}(2\pi - \alpha) &= -\text{tg } \alpha = \text{tg}(-\alpha) \end{aligned}$$

b) Arcos complementares

$$\begin{aligned} \operatorname{sen}\left(\frac{\pi}{2} - \alpha\right) &= \cos \alpha \\ \cos\left(\frac{\pi}{2} - \alpha\right) &= \operatorname{sen} \alpha \\ \operatorname{tg}\left(\frac{\pi}{2} - \alpha\right) &= \operatorname{cot} g \alpha \\ \operatorname{sen}\left(\frac{\pi}{2} + \alpha\right) &= \cos \alpha \\ \cos\left(\frac{\pi}{2} + \alpha\right) &= \operatorname{sen} \alpha \\ \operatorname{tg}\left(\frac{\pi}{2} + \alpha\right) &= -\operatorname{cot} g \alpha \\ \operatorname{sen}\left(\frac{3\pi}{2} - \alpha\right) &= -\cos \alpha \\ \cos\left(\frac{3\pi}{2} - \alpha\right) &= -\operatorname{sen} \alpha \\ \operatorname{tg}\left(\frac{3\pi}{2} - \alpha\right) &= \operatorname{cot} g \alpha \\ \operatorname{sen}\left(\frac{3\pi}{2} + \alpha\right) &= -\cos \alpha \\ \cos\left(\frac{3\pi}{2} + \alpha\right) &= \operatorname{sen} \alpha \\ \operatorname{tg}\left(\frac{3\pi}{2} + \alpha\right) &= -\operatorname{cot} g \alpha \end{aligned}$$

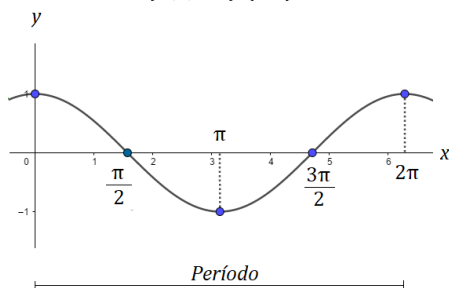
5. FUNÇÃO SENO

$$\begin{aligned} f: R &\rightarrow R \\ f(x) &= \operatorname{sen} x \\ D(f) &= R \\ CD(f) &= R \\ \operatorname{Im}(f) &= \{y \in R \mid -1 \leq y \leq 1\} \\ &\text{função ímpar} \\ f(-x) &= -f(x) \end{aligned}$$



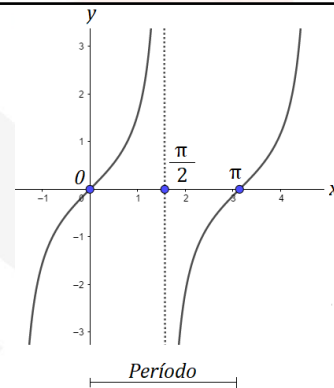
5. FUNÇÃO COSSENO

$$\begin{aligned} f: R &\rightarrow R \\ f(x) &= \cos x \\ D(f) &= R \\ CD(f) &= R \\ \operatorname{Im}(f) &= \{y \in R \mid -1 \leq y \leq 1\} \\ &\text{função par} \\ f(x) &= f(-x) \end{aligned}$$



7. FUNÇÃO TANGENTE

$$\begin{aligned} f: \{x \in R \mid x \neq \frac{\pi}{2} + k\pi, k \in Z\} &\rightarrow R \\ f(x) &= \operatorname{tg} x \\ D(f) &= \{x \in R \mid x \neq \frac{\pi}{2} + k\pi, k \in Z\} \\ CD(f) &= \operatorname{Im}(f) = R \\ &\text{função ímpar} \\ f(-x) &= -f(x) \end{aligned}$$

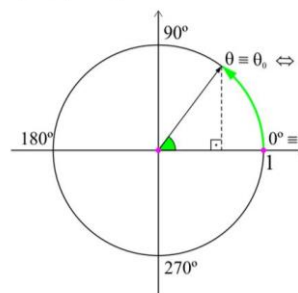


8. RELAÇÕES FUNDAMENTAIS

$$\begin{aligned} \operatorname{sen}^2 \alpha + \cos^2 \alpha &= 1 \\ \operatorname{tg}^2 \alpha + 1 &= \operatorname{sec}^2 \alpha \\ \operatorname{cot}^2 \alpha + 1 &= \operatorname{cosec}^2 \alpha \end{aligned}$$

9. ARCOS CÔNGRUOS

$$\theta = \theta_0 + k(360^\circ)$$



10. EXERCÍCIOS

1) (EEAR – 2017)

Ao somar as medidas angulares  $120^\circ$  e  $\frac{3\pi}{2} \text{ rad}$ , obtenha a medida de um arco pertencente ao \_\_\_ quadrante.

- a)  $1^\circ$
- b)  $2^\circ$
- c)  $3^\circ$
- d)  $4^\circ$

**2) (EEAR – 2006)**

Sejam as medidas de arcos trigonométricos:

- I.  $\frac{17\pi}{8} \text{ rad}$  e  $\frac{41\pi}{8} \text{ rad}$ ;  
 II.  $1490^\circ$  e  $-1030^\circ$ .

É correto afirmar que as medidas:

- a) em I são de arcos côngruos.  
 b) em I são de arcos suplementares.  
 c) em II são de arcos côngruos.  
 d) em II são de arcos complementares.

**3) (EEAR – 2005)**

 Existirá  $x \in \mathbb{R}$  que satisfaça a igualdade  $\text{sen } x = 2k - 5$  se, e somente se:

- a)  $1 < k \leq 3$   
 b)  $1 < k < 4$   
 c)  $2 \leq k < 4$   
 d)  $2 \leq k \leq 3$

**4) (EEAR – 2009)**

São negativas, no 4º quadrante, as funções:

- a) seno, cosseno e tangente  
 b) seno, cosseno e cotangente  
 c) cosseno, tangente e secante  
 d) seno, tangente e cossecante

**5) (EEAR 2017)**

 As funções  $f(x) = \text{sen } x$  e  $g(x) = \text{cos } x$ , no segundo quadrante, são, respectivamente:

- a) decrescente e decrescente  
 b) decrescente e crescente  
 c) crescente e decrescente  
 d) crescente e crescente

**6) (EEAR – 2006)**

O quadrante em que as funções seno, cosseno e tangente são, simultaneamente, crescente é o:

- a) 1º  
 b) 2º  
 c) 3º  
 d) 4º

**7) (EEAR – 2007)**

 Se  $0 < x < \frac{\pi}{2}$ , e  $y = \frac{\text{sen}(\frac{\pi}{2}-x) \cdot \text{cossec}(\frac{\pi}{2}-x)}{\text{cos}(\frac{\pi}{2}-x) \cdot \text{tg}(\frac{\pi}{2}-x)}$ , então  $y$  é igual

a:

- a)  $\text{tg } x$   
 b)  $\text{cos } x$   
 c)  $\text{sec } x$   
 d)  $\text{sen } x$

**8) (EEAR 2018)**

 O valor de  $\text{sen } 1270^\circ$  é igual a:

- a)  $-\text{cos } 10^\circ$   
 b)  $-\text{sen } 30^\circ$   
 c)  $-\text{sen } 10^\circ$   
 d)  $-\text{cos } 30^\circ$

**9) (EEAR – 2011)**

 Se  $A = \text{tg } 120^\circ$  e  $B = \text{tg } 240^\circ$ , então:

- a)  $B = A$   
 b)  $B = -A$   
 c)  $B = 2A$   
 d)  $B = -2A$

**10) (EEAR – 2010)**

 Para  $x \cdot y \neq 0$ , a

 expressão  $\frac{y^2 \cos 180^\circ - xy \text{sen } 270^\circ + y^2 \text{sen } 90^\circ}{x^2 \cos 0^\circ}$  equivale a:

- a)  $\frac{y}{x}$   
 b)  $\frac{2}{x}$   
 c)  $\frac{y}{x^2}$   
 d)  $\frac{y^2}{x^2}$

**11) (EEAR – 2014)**

 Se  $x$  é um arco do terceiro quadrante, tal que  $\text{tg } x = \frac{2}{3}$ ,

 o valor de  $\text{sen } x$  é:

- a)  $\frac{\sqrt{13}}{13}$   
 b)  $\frac{-\sqrt{13}}{13}$   
 c)  $\frac{-2\sqrt{13}}{13}$   
 d)  $\frac{-3\sqrt{13}}{13}$

**12) (EEAR – 2013)**

 Seja  $x$  um arco do 3º quadrante tal que  $\text{sen } x = -\frac{1}{3}$ .

 Então o valor de  $\text{cos } x$  é:

- a)  $-\frac{2\sqrt{2}}{3}$   
 b)  $-\frac{\sqrt{2}}{3}$   
 c)  $\frac{2\sqrt{2}}{3}$   
 d)  $\frac{\sqrt{2}}{3}$

**13) (EEAR – 2019)**

 Simplificando a expressão  $\text{sen}(2\pi - x) + \text{sen}(3\pi + x)$ , obtém-se:

- a)  $\text{sen } x$   
 b)  $-\text{sen } x$   
 c)  $2 \text{sen } x$   
 d)  $-2 \text{sen } x$

**14) (EEAR – 2006)**

 O domínio da função  $f(x) = 3 \cdot \text{tg}\left(x + \frac{\pi}{4}\right)$  é:

- a)  $\left\{x \in \mathbb{R} \mid x \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\}$   
 b)  $\left\{x \in \mathbb{R} \mid x \neq \frac{\pi}{4} + k\pi, k \in \mathbb{Z}\right\}$   
 c)  $\left\{x \in \mathbb{R} \mid x \neq \frac{\pi}{2} + 2k\pi, k \in \mathbb{Z}\right\}$   
 d)  $\left\{x \in \mathbb{R} \mid x \neq \frac{\pi}{4} + 2k\pi, k \in \mathbb{Z}\right\}$

**15) (EEAR – 2012)** Sejam as sentenças:

- I. período  $p = \pi$ ;
- II. domínio  $D = \mathbb{R}$ ;
- III. conjunto imagem  $Im = [-1,1]$ .

Em relação à função tangente, é (são) verdadeira(s) a(s) sentença(s).

- a) I
- b) III
- c) I e II
- d) II e III

**11. GABARITO**

- 1) A
- 2) C
- 3) D
- 4) D
- 5) A
- 6) D
- 7) C
- 8) C
- 9) B
- 10) A
- 11) C
- 12) A
- 13) D
- 14) B
- 15) A

**12. ANOTAÇÕES**

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