Geometria e Trigonometria

Parte 1: Geometria Plana

Quais são as regras desse jogo?

Os elementos

Noções comuns

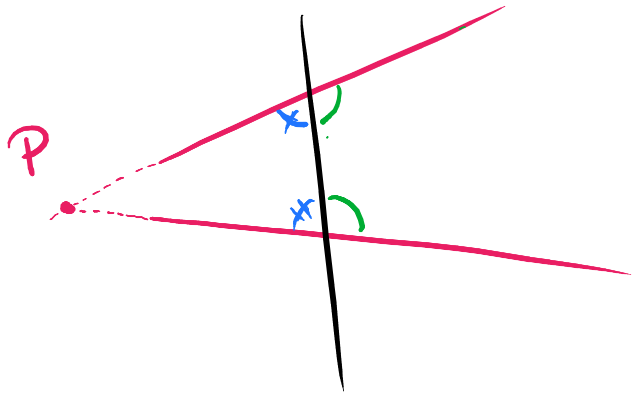
1. As coisas iguais à mesma coisa são também iguais entre si.
2. E, caso sejam adicionadas coisas iguais a coisas iguais, os todos são iguais.
3. E, caso de iguais sejam subtraídas iguais, as restantes são iguais.
[4. E, caso iguais sejam adicionadas a desiguais, os todos são desiguais.
5. E os dobros da mesma coisa são iguais entre si.
6. E as metades da mesma coisa são iguais entre si.]
7. E as coisas que se ajustam uma à outra são iguais entre si.
8. E o todo [é] maior do que a parte.
9. E duas retas não contêm uma área.

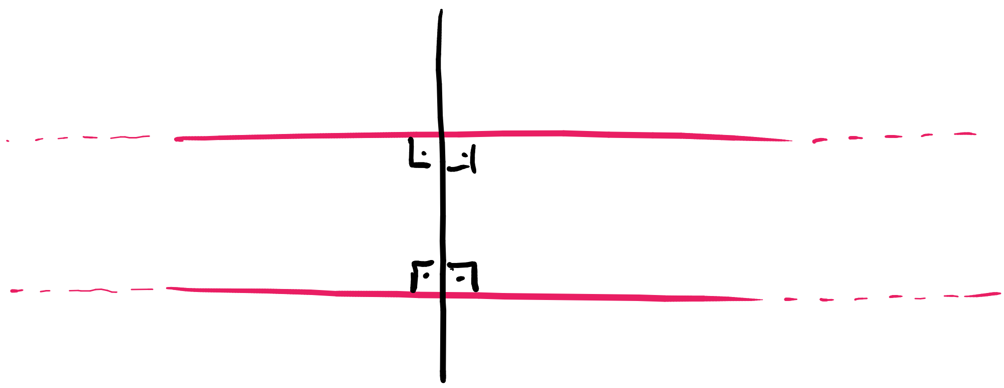
congruência e semelhança de triângulos.

Postulados

1. Fique postulado traçar uma reta a partir de todo ponto até todo ponto.
2. Também prolongar uma reta limitada, continuamente, sobre uma reta.
3. E, com todo centro e distância, descrever um círculo.
4. E serem iguais entre si todos os ângulos retos.
5. E, caso uma reta, caindo sobre duas retas, faça os ângulos interiores e do mesmo lado menores do que dois retos, sendo prolongadas as duas retas, ilimitadamente, encontrarem-se no lado no qual estão os menores do que dois retos.

↳ referentes à geometria apenas.

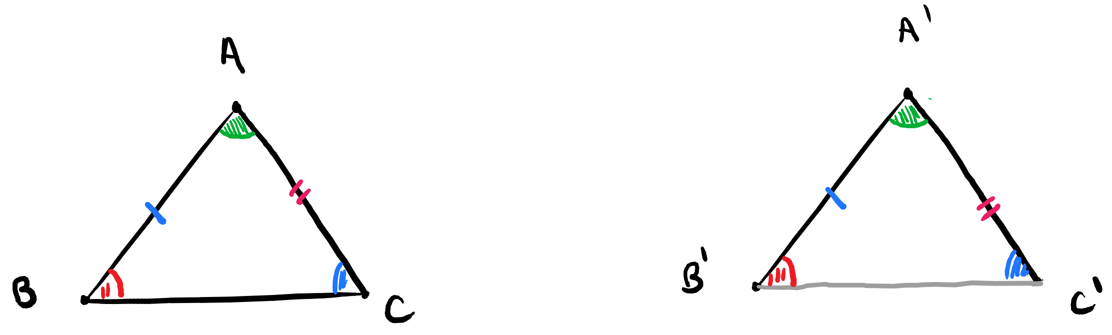




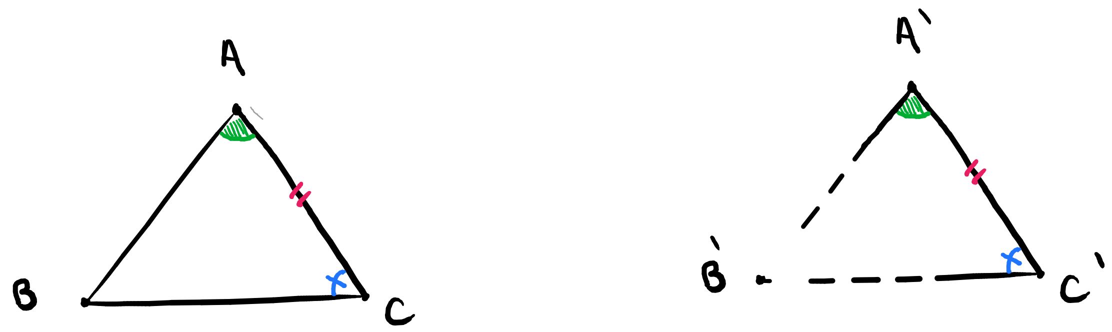
Congruência de triângulos

2 triângulos são ditos congruentes se eles são idênticos (ângulos todos iguais e lados todos iguais).

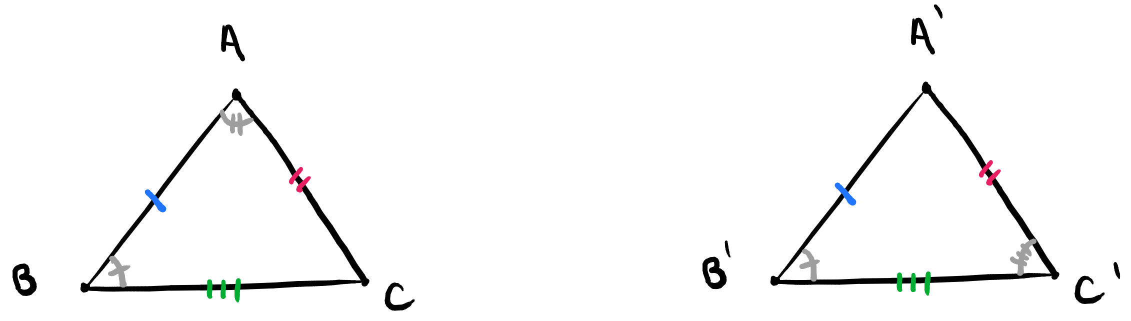
Caso 1: LAL



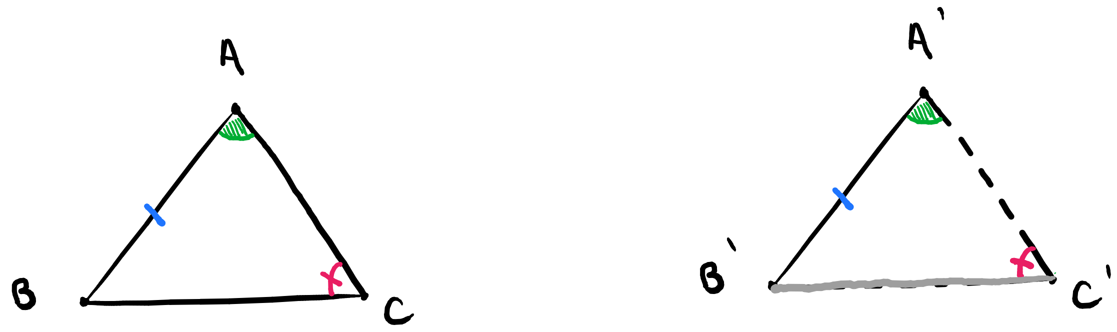
Caso 2: ALA



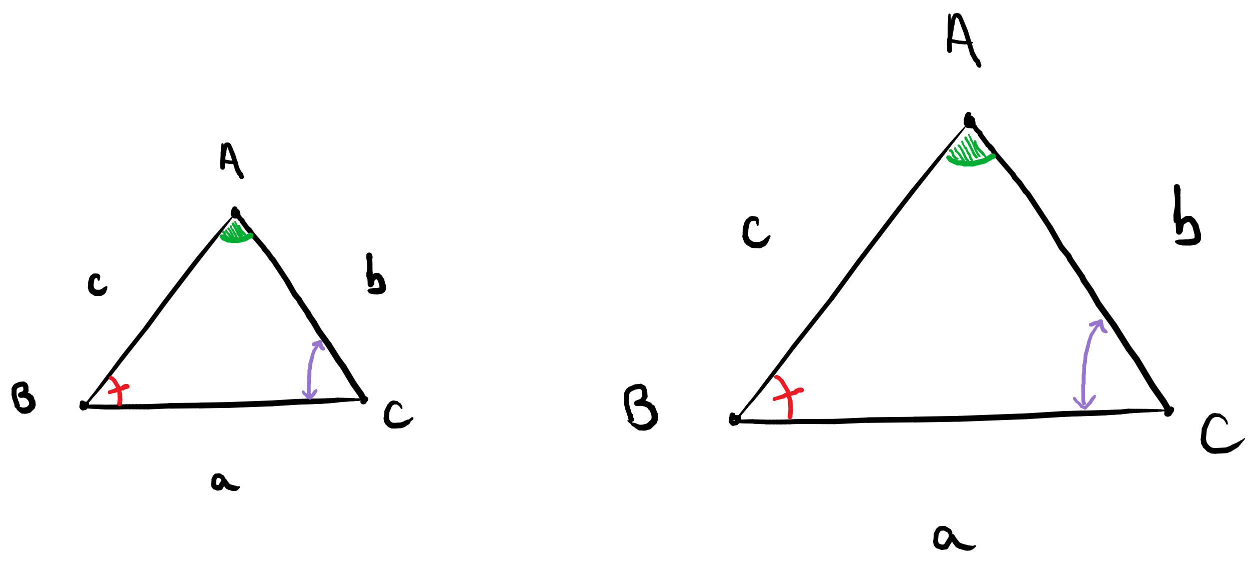
Caso 3 : L L L



Caso 4 : L A $A_{op.}$



Semelhança de triângulos



Os triângulos ΔABC e $\Delta A'B'C'$ são semelhantes

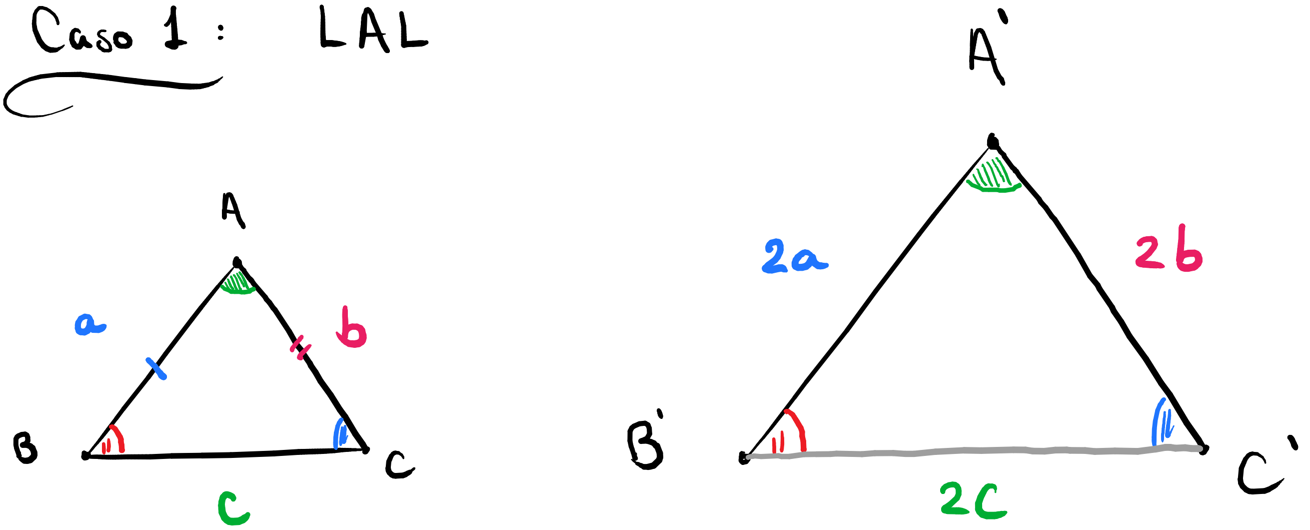
se $\dfrac{a}{a'} = \dfrac{b}{b'} = \dfrac{c}{c'} = K$

↳ razão de semelhança

- Congruência : $K = 1$

Os casos de semelhança

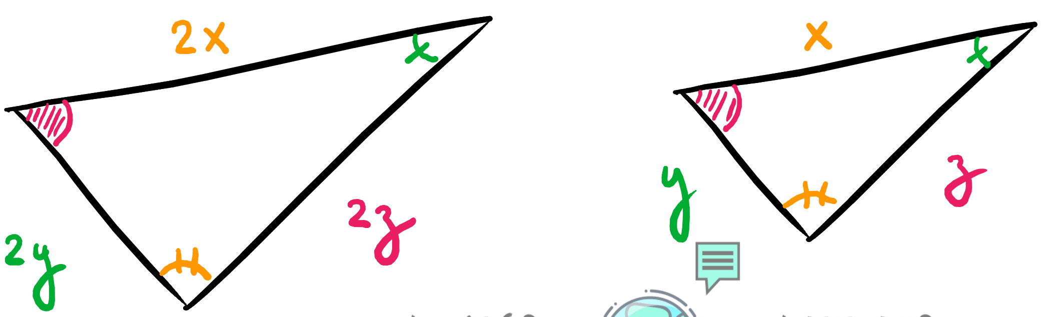
Caso 1 : LAL



... são os mesmos casos de congruência!

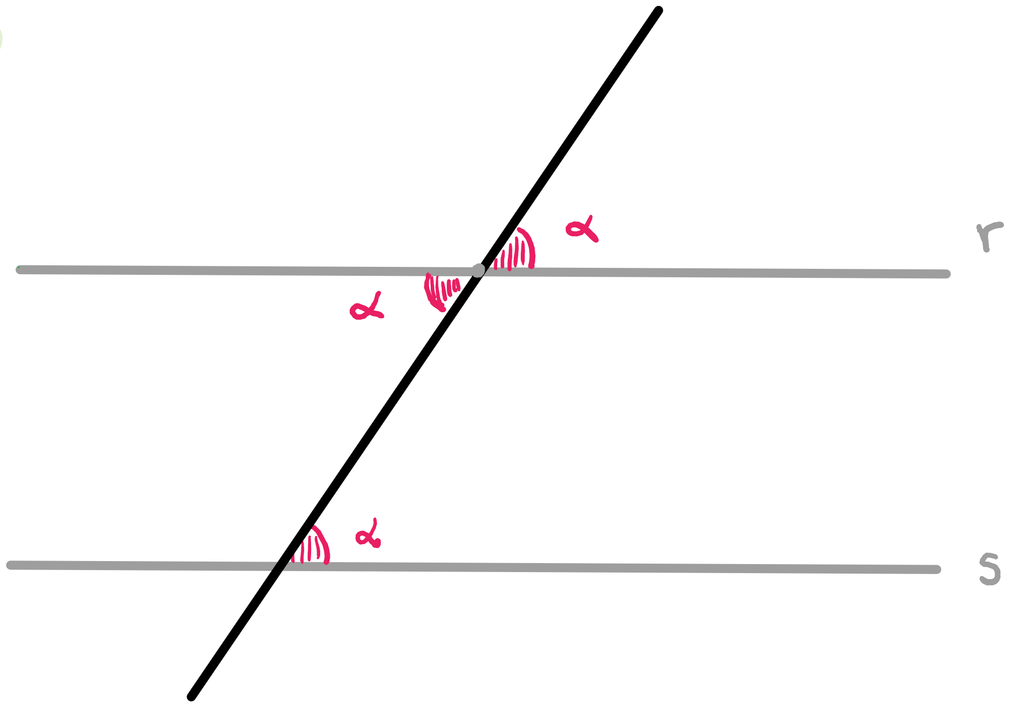
A congruência de triângulos é uma semelhança na qual a razão de semelhança vale 1.

Ex.: (ALA)

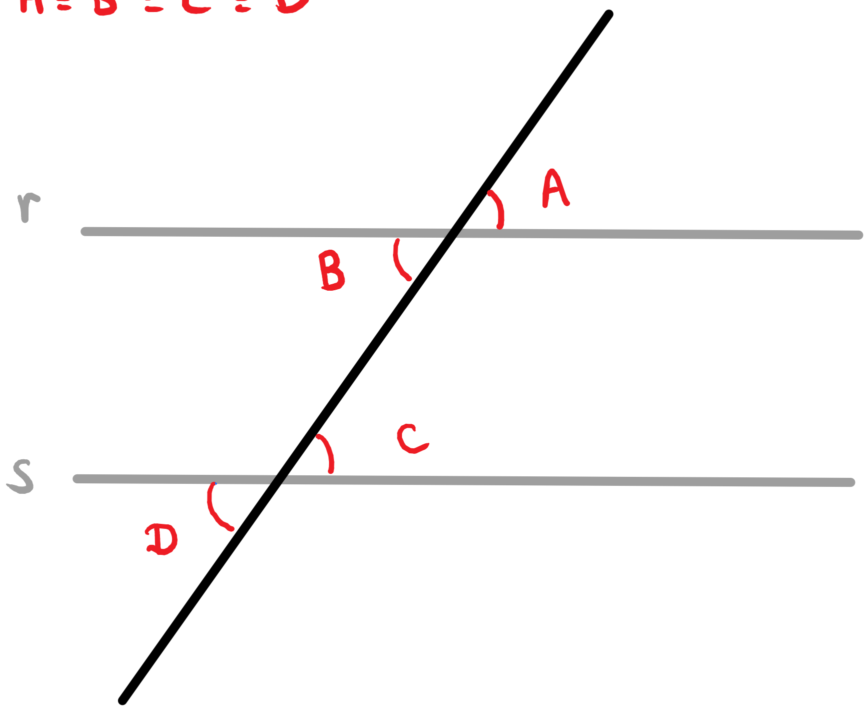


Paralelismo

$r // s$



$A = B = C = D$



- **Correspondentes**

A e C | B e D

- **Opostos pelo vértice**

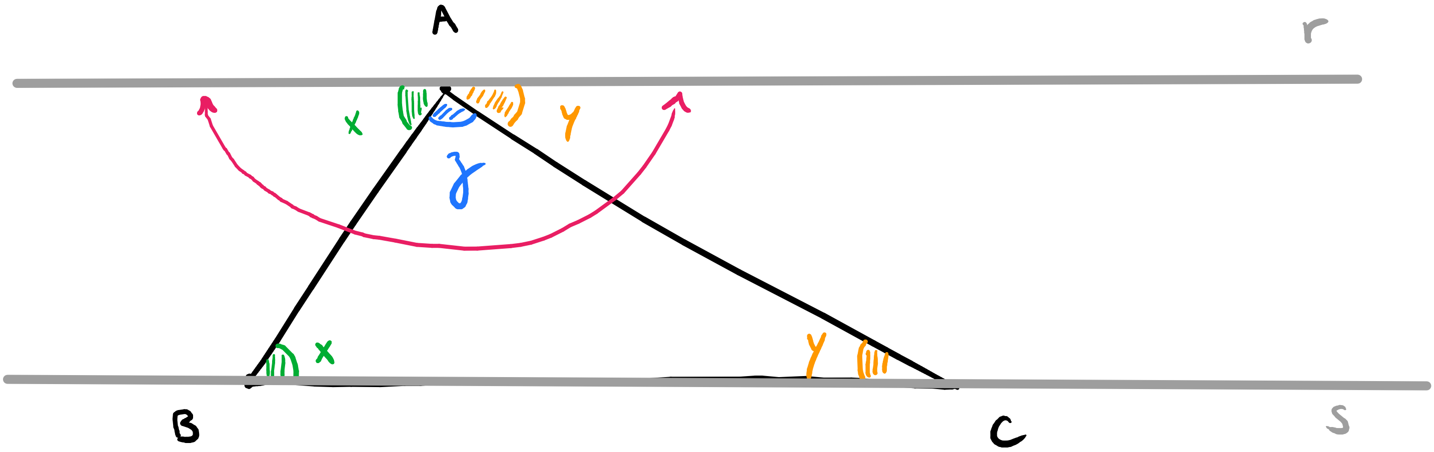
A e B | C e D

- **Alternos - internos**

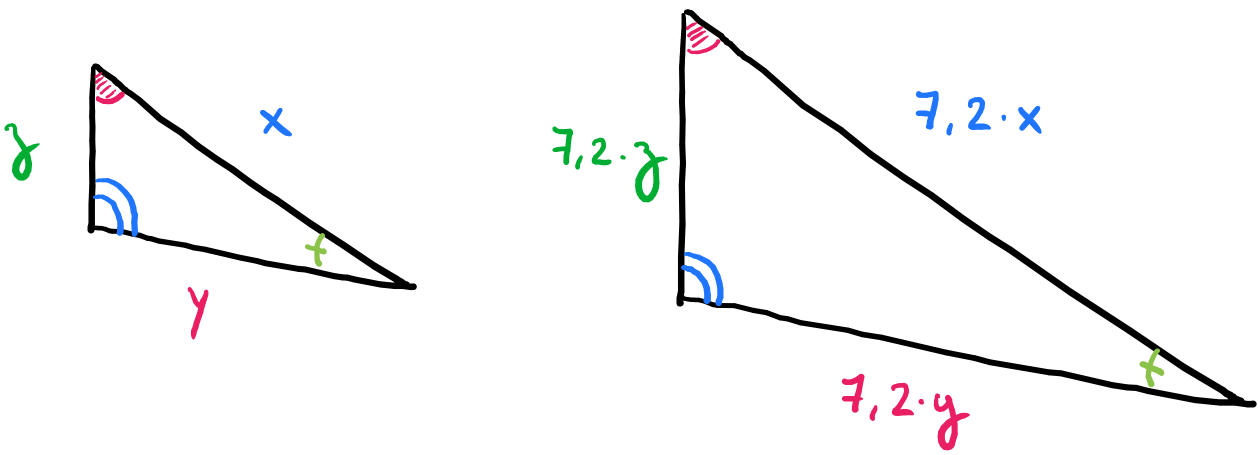
B e C

Obs.: Soma dos ângulos internos de um Δ

$$\boxed{x + y + z = 180°}$$



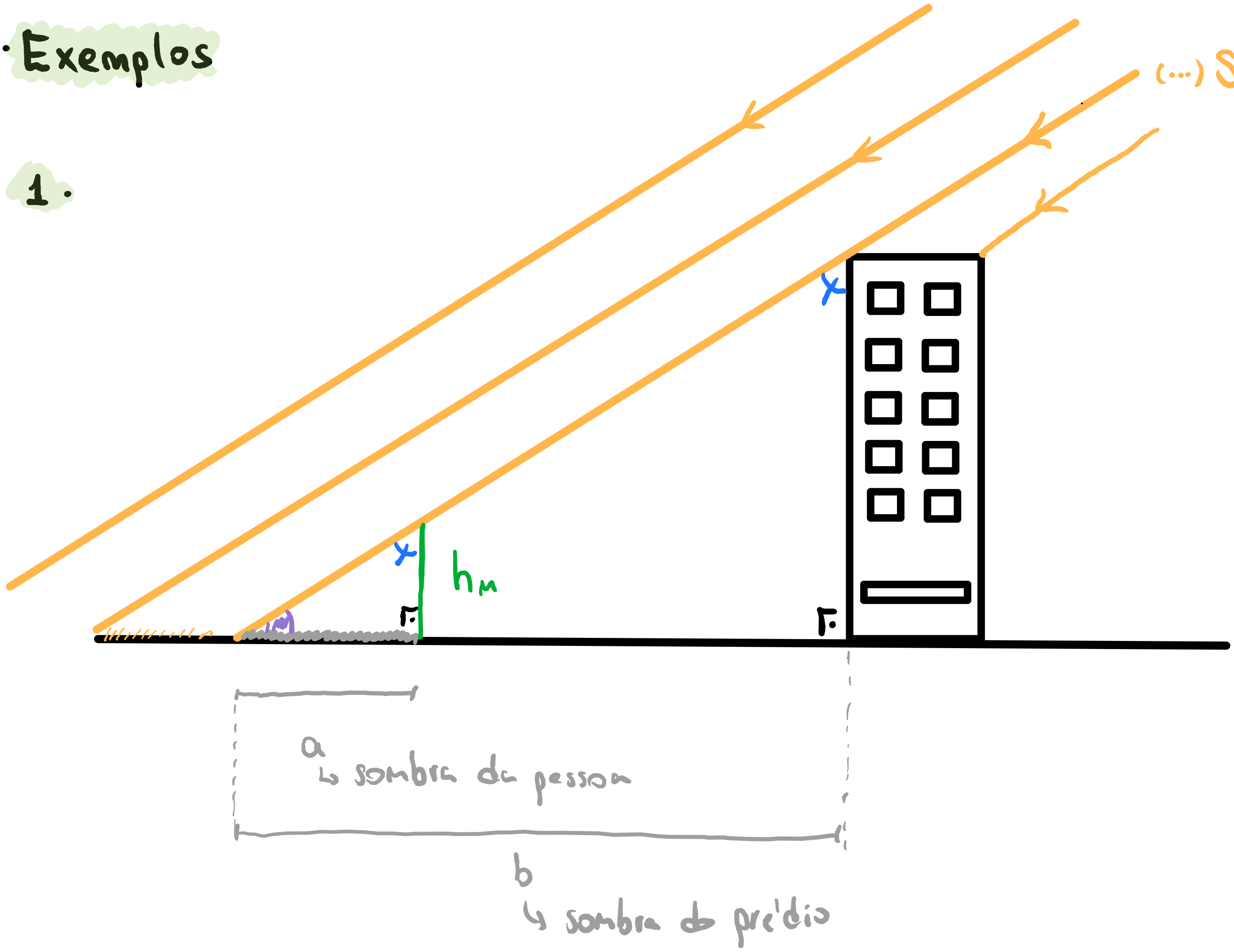
... um pouco mais sobre semelhança



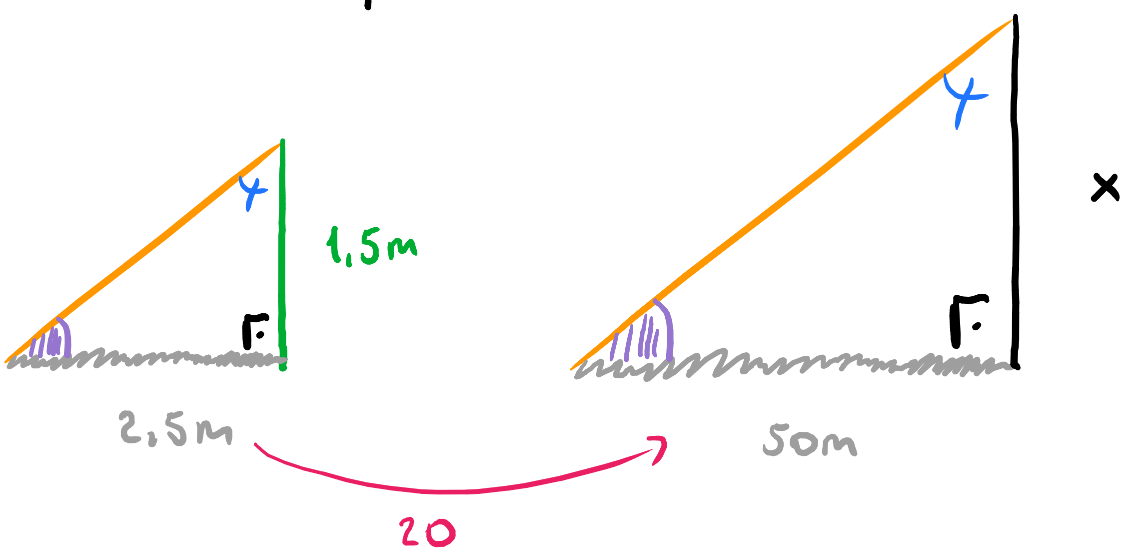
Se dois triângulos tem dois ângulos iguais então eles são semelhantes.

· Exemplos

1.

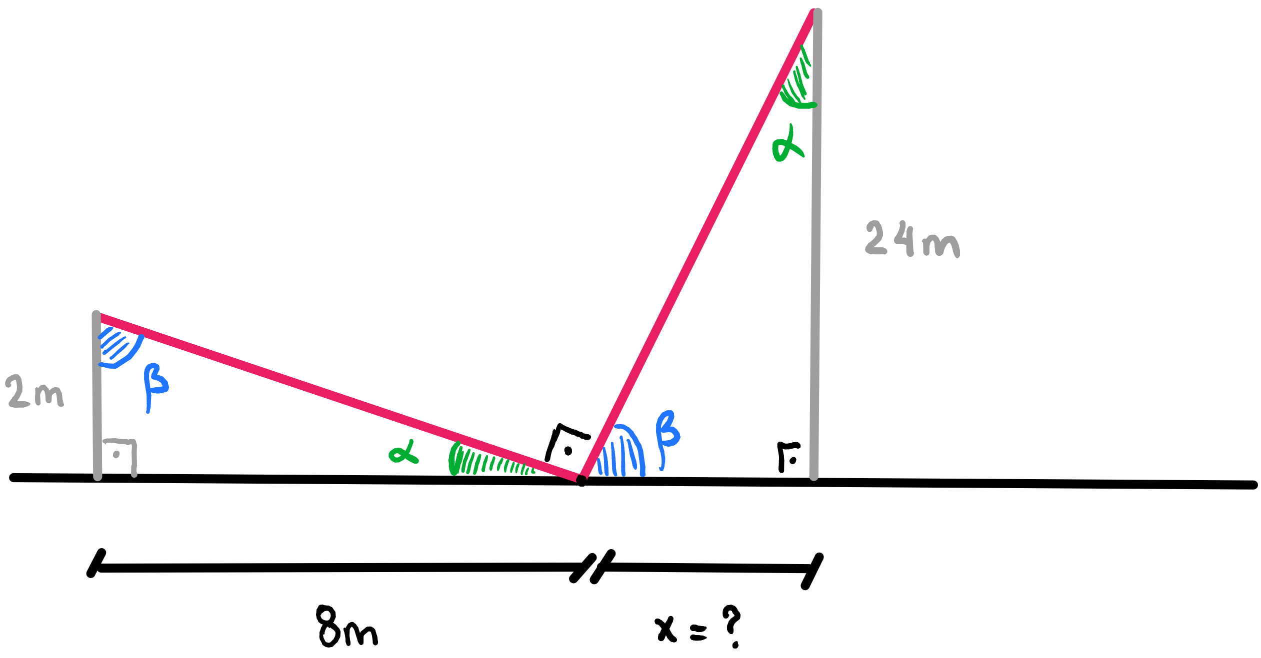


Se $a = 2,5m$, $b = 50m$ e $h_{menino} = 1,5m$, qual é o tamanho do prédio?



$$\frac{x}{1,5} = \frac{50}{2,5} \quad \therefore \quad x = 30m$$

2.



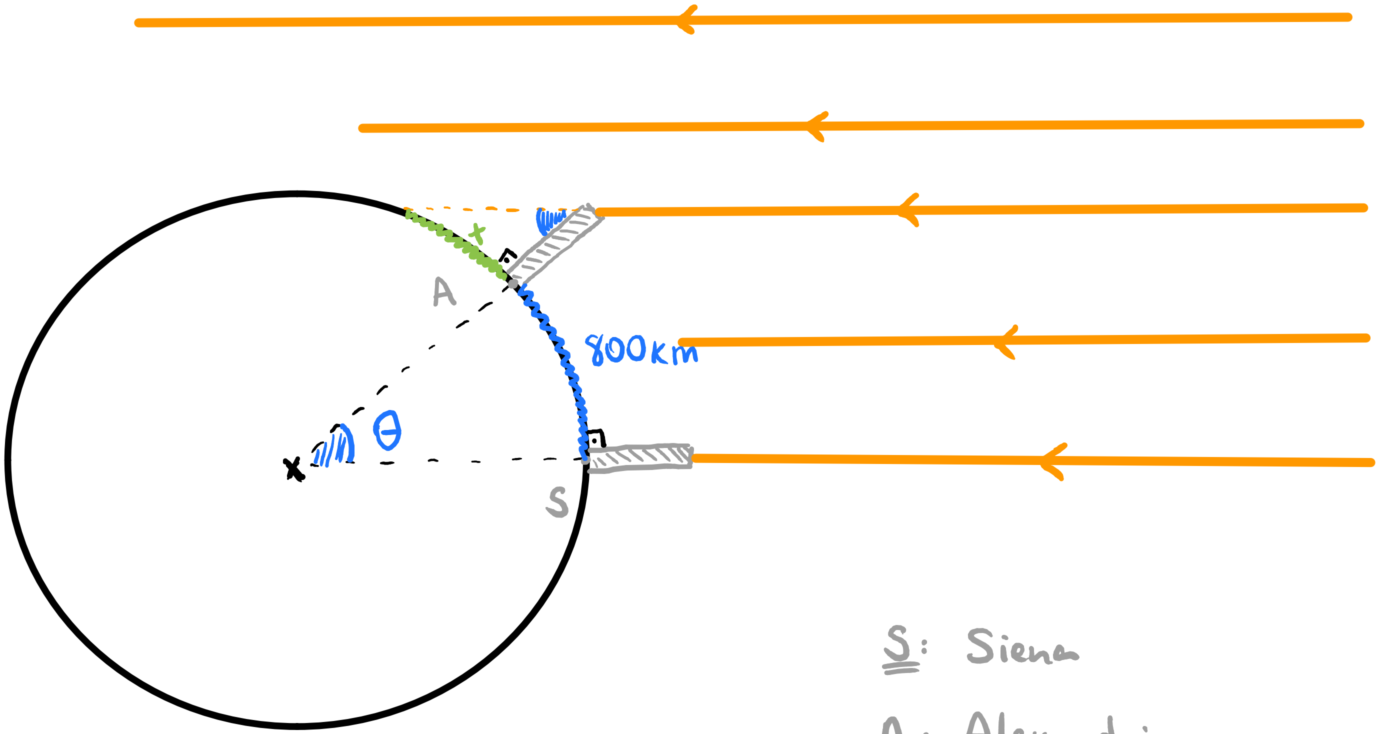
- $\alpha + 90 + \beta = 180 \quad \therefore \quad \boxed{\alpha + \beta = 90^\circ}$

- Semelhança: $\dfrac{24}{8} = \dfrac{x}{2} \quad \therefore \quad x = 2 \cdot 3$

$x = 6m$

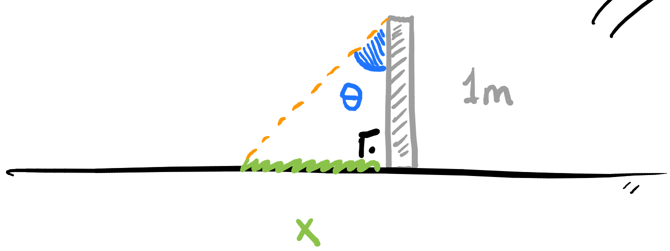
3. Eratóstenes mede o tamanho da terra

↳ séc. 2/3 A.C.



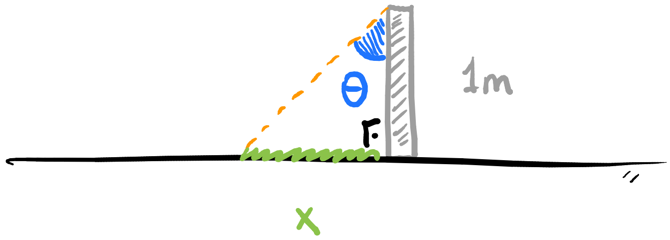
S: Siena

A: Alexandria

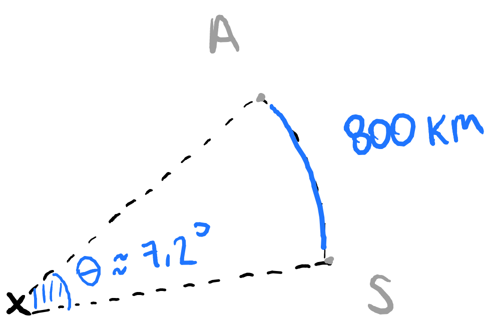


É possível obter o ângulo!

Complemento

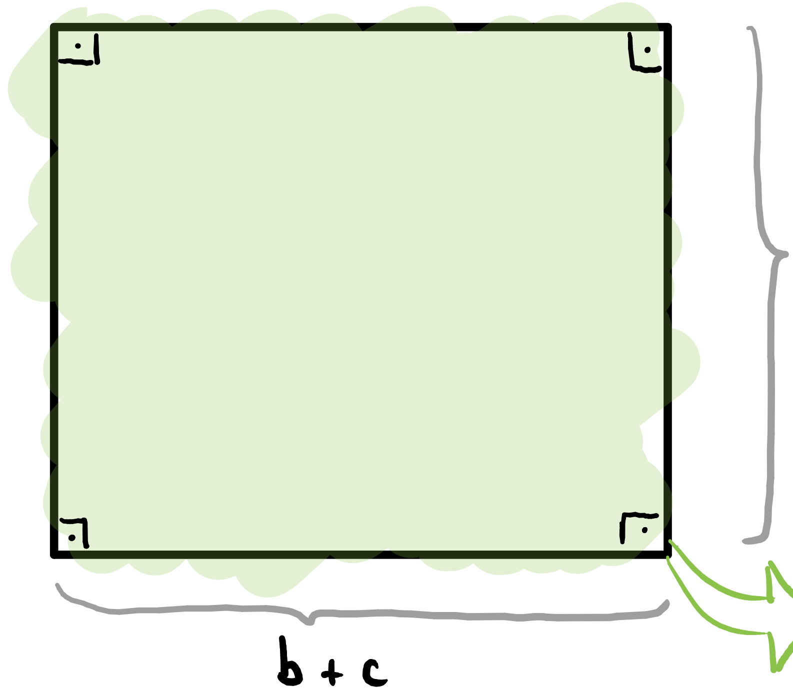


$$tg\,\theta = \frac{x}{1} \Rightarrow \text{é obtido o ângulo } \theta.$$

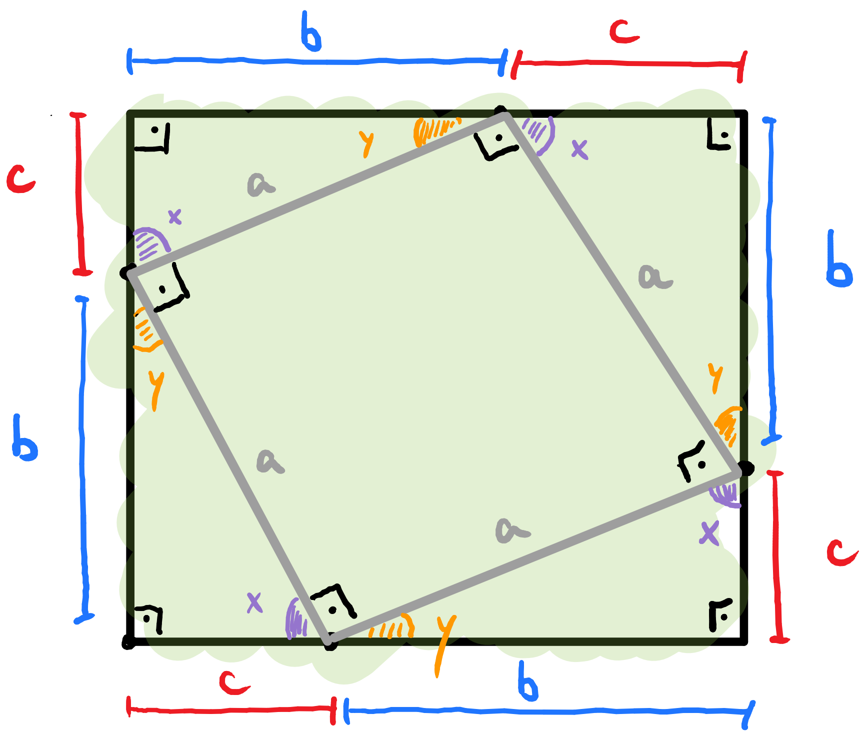


$$\frac{7{,}2^\circ}{360^\circ} = \frac{800\ km}{C} \Rightarrow C \approx 40.000\ km$$

Teorema de Pitágoras



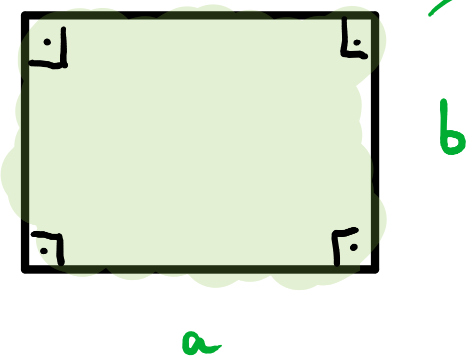
$A = (b+c)^2$



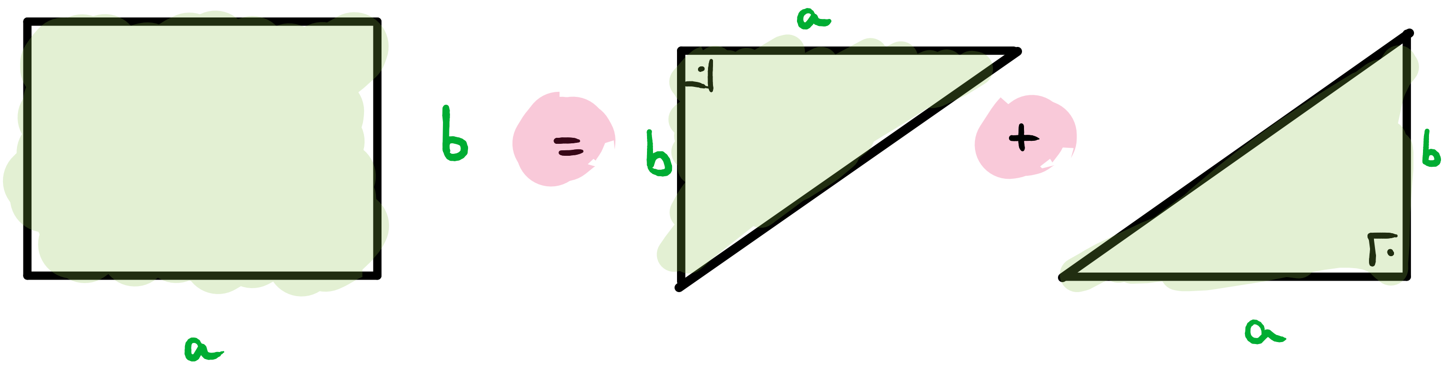
$x + y = 90°$

$180°$

Obs.:



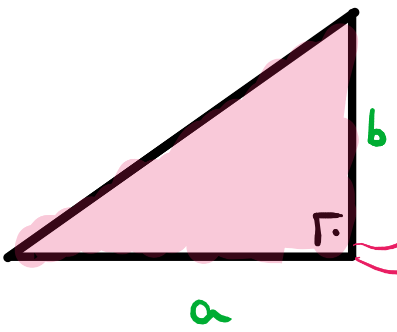
Área = a.b

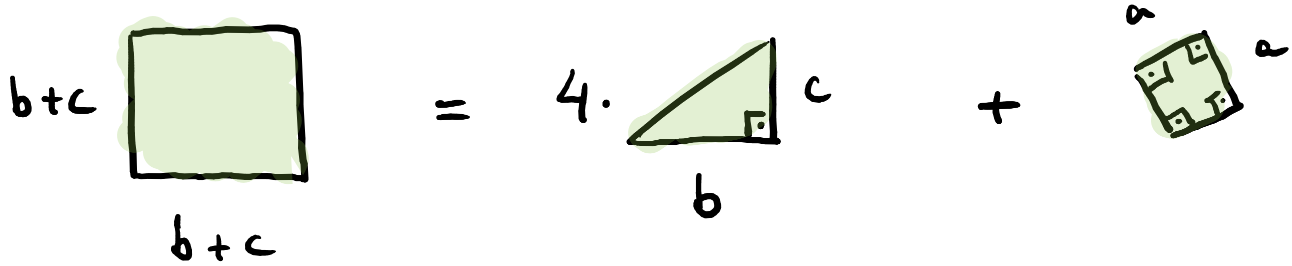


$$A_{\square} = A_{\triangle} + A_{\triangle}$$

$$a \cdot b = 2 \cdot A_{\triangle}$$

$$\boxed{A_{\triangle} = a \cdot b / 2}$$





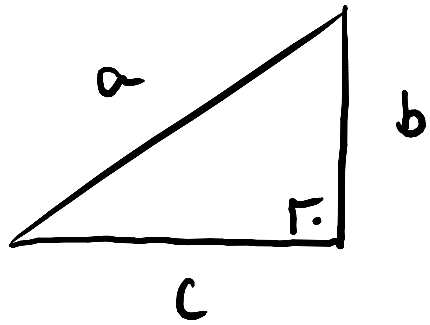
$$(b+c)\cdot(b+c) = 4\cdot\frac{b\cdot c}{2} + a^2$$

$$(b+c)^2 = 2bc + a^2$$

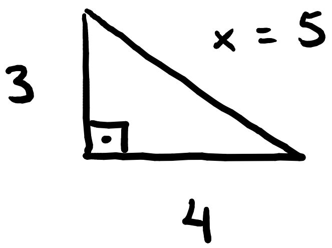
$$b^2 + \cancel{2\cdot b\cdot c} + c^2 = \cancel{2bc} + a^2$$

$$\boxed{a^2 = b^2 + c^2}$$

↳ teorema de Pitágoras



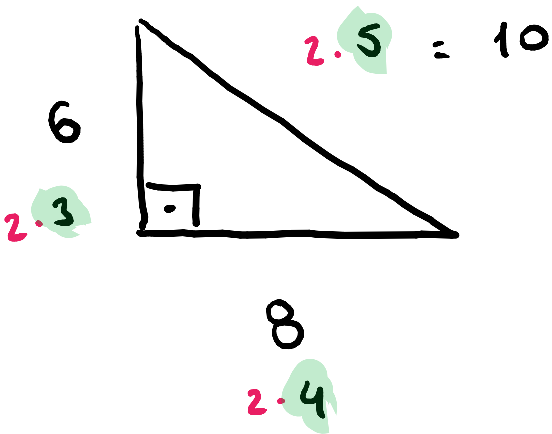
Obs.: triângulos pitagóricos

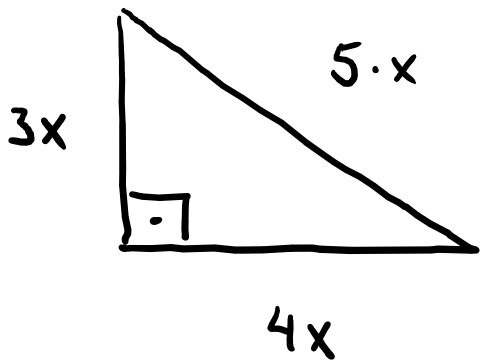


$$x^2 = 3^2 + 4^2 = 9 + 16 = 25$$

$$x^2 = 25$$

$$x = 5$$



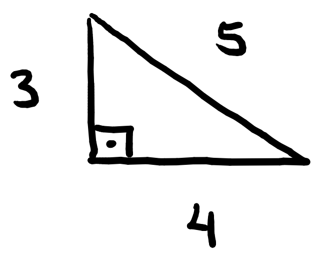


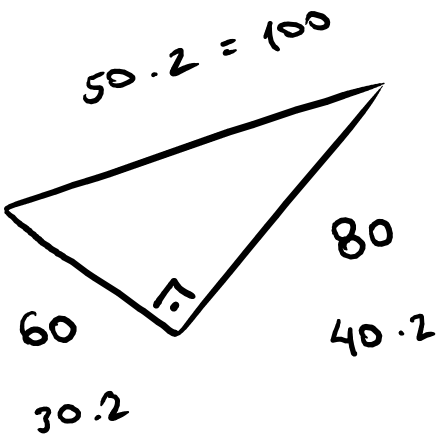
$$(5x)^2 = (3x)^2 + (4x)^2$$

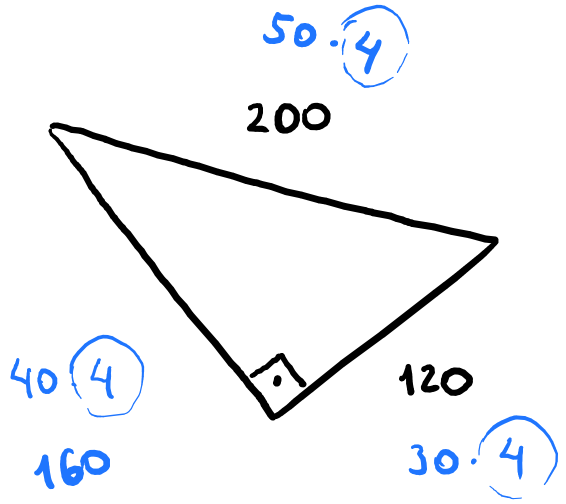
$$25x^2 = 9x^2 + 16x^2$$

$$25x^2 = 25x^2 \checkmark$$

Exemplos







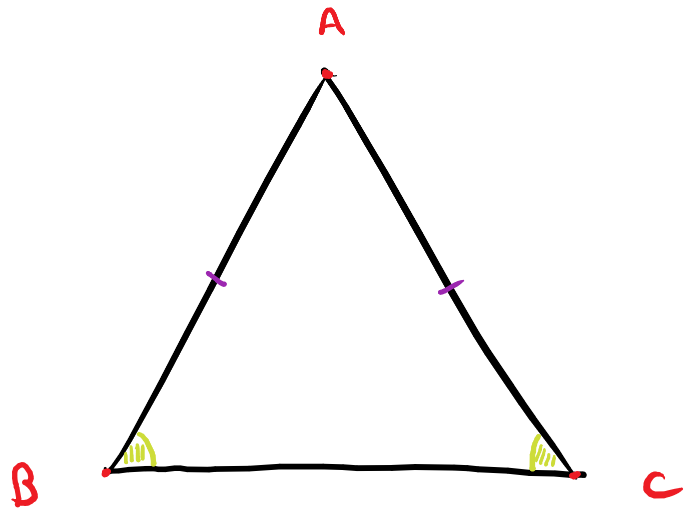
3, 4, 5

×40 ×40 ×40

120, 160, 200

Aplicação: O triângulo isósceles

⇝ Num triângulo isósceles os ângulos da base são iguais



AB = BC

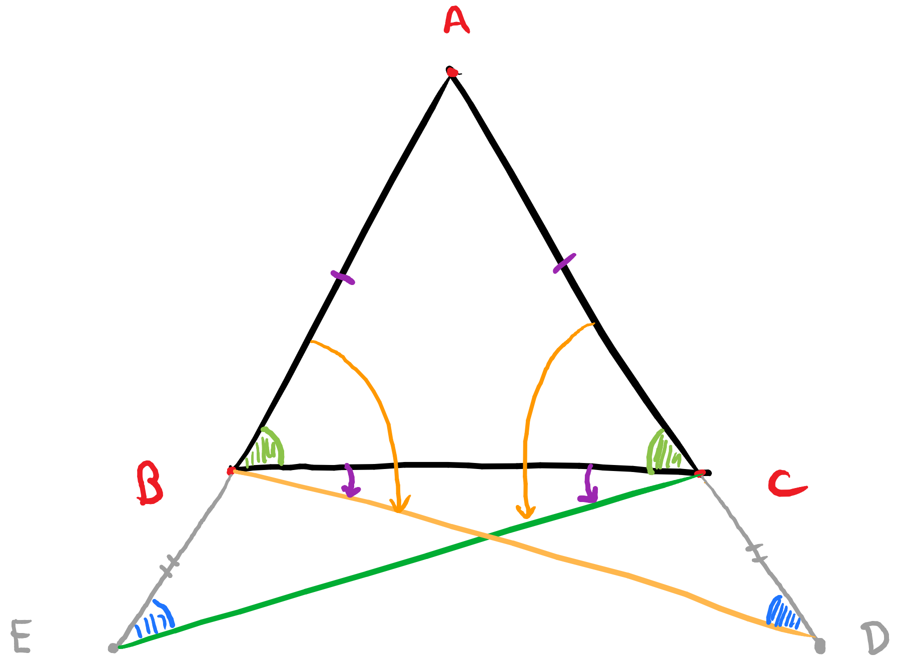
· **Passo 1**: prolongar B até E e C até D de modo que BE = CD
(postulado 1)

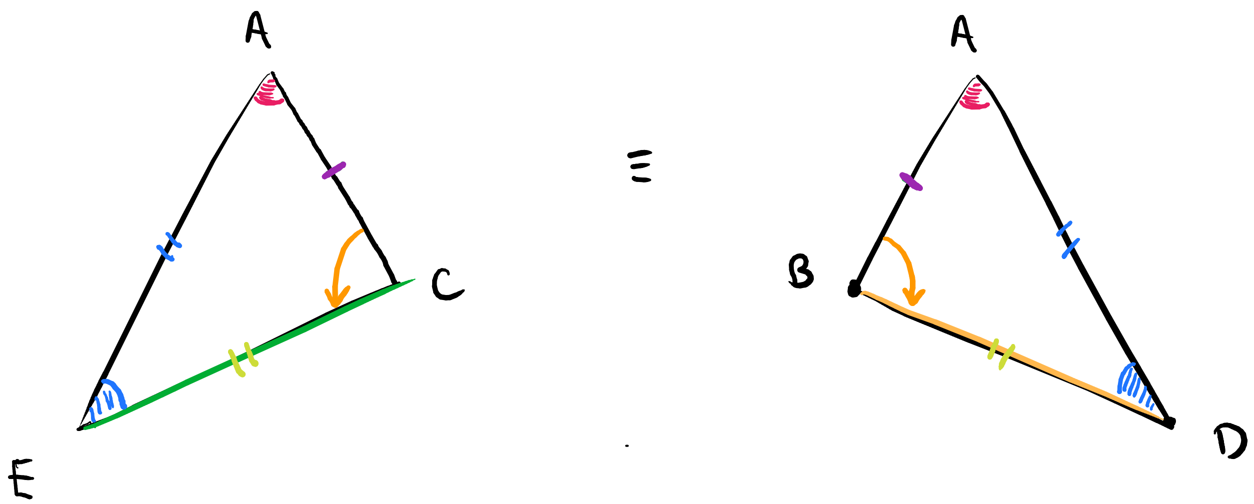
· **Passo 2**: ΔACE e ΔADB são iguais $\Rightarrow$ CE = BD
$\Downarrow$
$A\hat{D}B = A\hat{E}C$
(congruência → I.4)

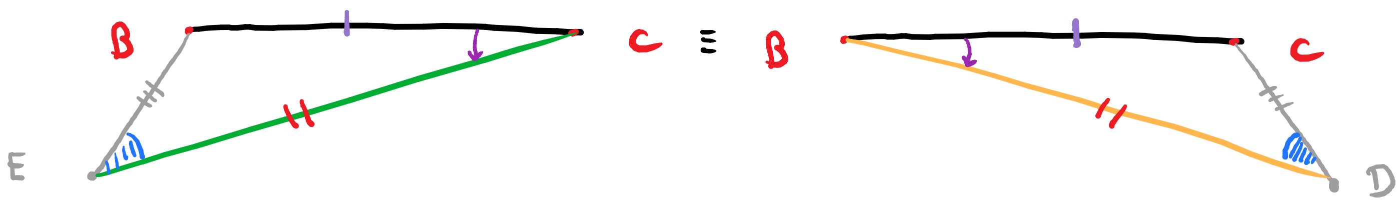
· **Passo 3**: $\Delta BEC = \Delta BCD \Rightarrow B\hat{C}E = C\hat{B}D$
(congruência → I.4)

· **Passo 4**: Subtrair tais ângulos:

(noção comum 3)

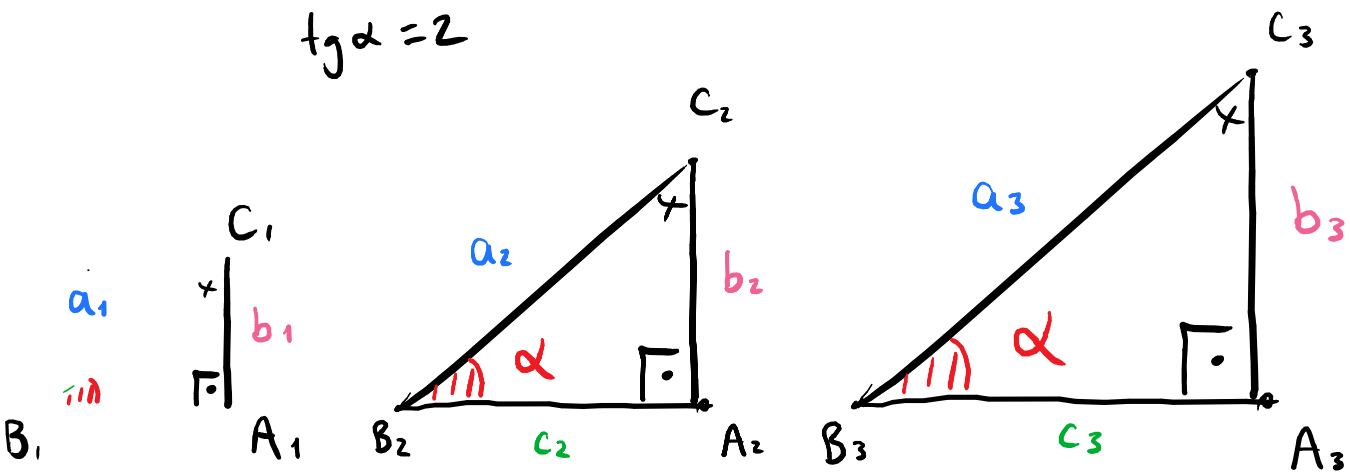

A
B
C
E
D


A
C
E
≡
A
B
D


B
C
E
≡
B
C
D

Parte 2 : Trigonometria

Razões trigonométricas no triângulo retângulo



Semelhança de Δ's :

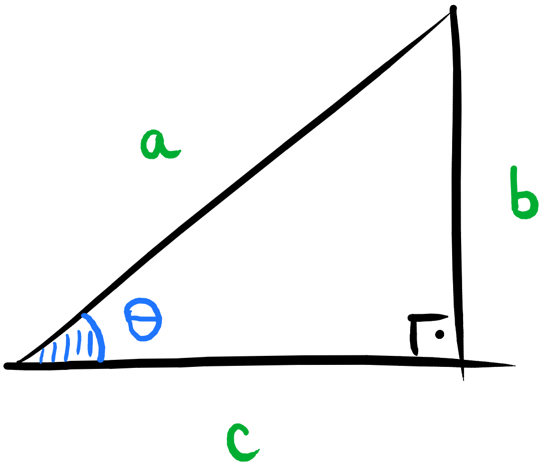
$$\frac{b_1}{b_2} = \frac{a_1}{a_2} \therefore \boxed{\frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \frac{b_4}{a_4}} = \text{sen}\,\alpha$$

$$\frac{c_1}{c_2} = \frac{a_1}{a_2} \therefore \boxed{\frac{c_1}{a_1} = \frac{c_2}{a_2} = \frac{c_3}{a_3} = \frac{c_4}{a_4}} = \cos\alpha$$

$$\frac{b_1}{b_2} = \frac{c_1}{c_2} \therefore \boxed{\frac{b_1}{c_1} = \frac{b_2}{c_2} = \frac{b_3}{c_3} = \frac{b_4}{c_4}} = \text{tg}\,\alpha$$

$(\tan\alpha)$

Definições :



b : cateto oposto a θ

c : cateto adjacente a θ

a : hipotenusa

Razões trigonométricas no triângulo retângulo

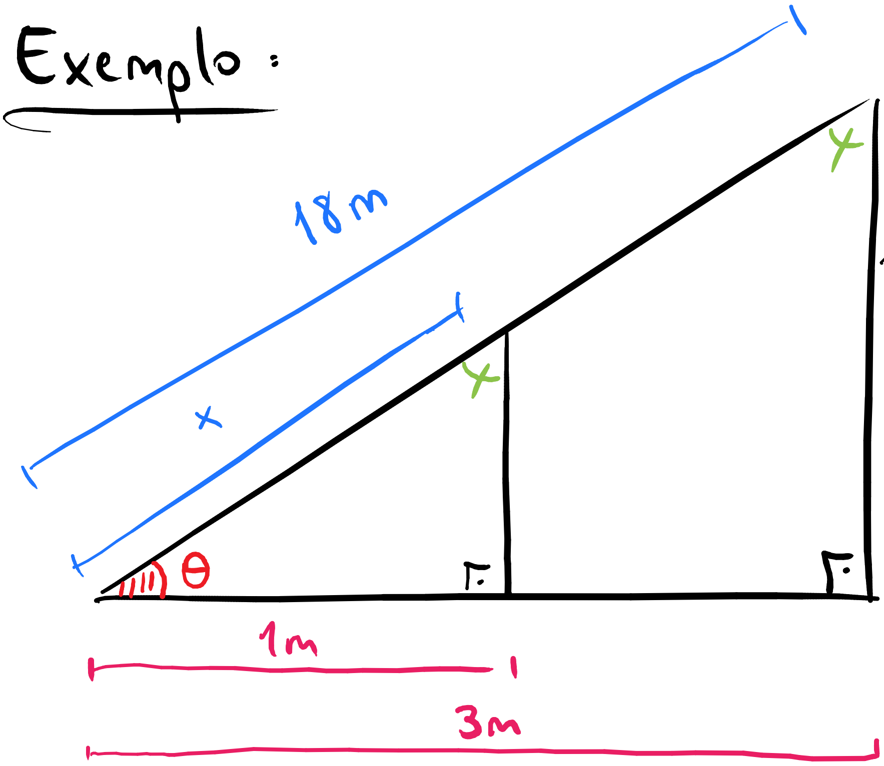
$$\boxed{\operatorname{sen}\theta = \frac{\text{cat. oposto}}{\text{hipotenusa}} = \frac{b}{a}}$$

$$\boxed{\cos\theta = \frac{\text{cat. adj.}}{\text{hipotenusa}} = \frac{c}{a}}$$

$$\operatorname{tg}\theta = \frac{\operatorname{sen}\theta}{\cos\theta}$$

$$\boxed{\tan\theta = \frac{\text{cat. oposto}}{\text{cat. adj.}} = \frac{b}{c}}$$

Exemplo:

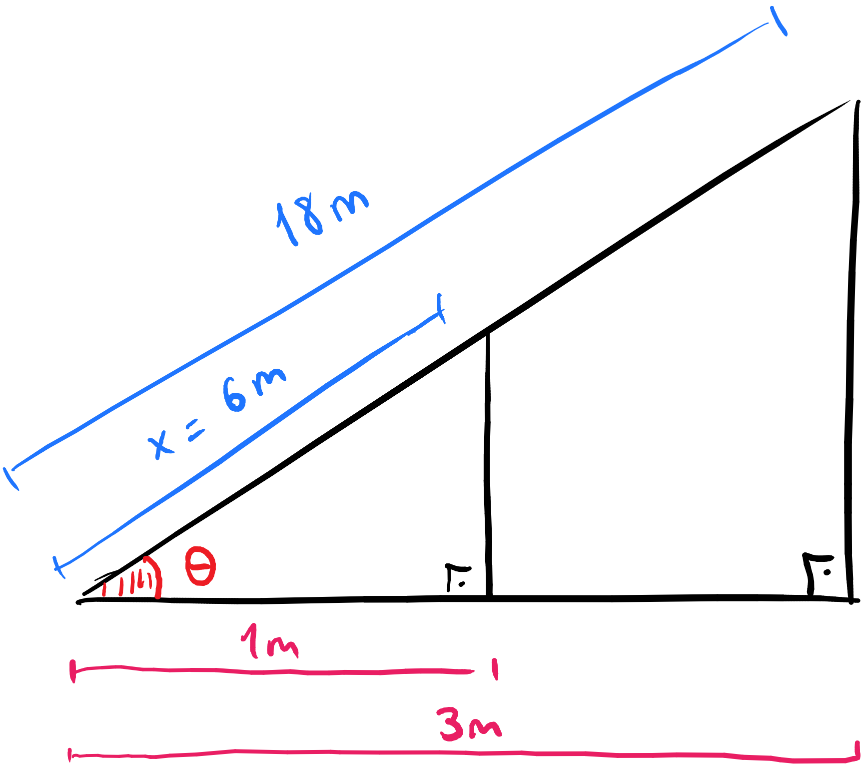


Semelhança:

$$\cancel{18} \cdot \frac{x}{\cancel{18}} = \frac{1}{3} \cdot 18$$

$$\boxed{x = 6\,m}$$

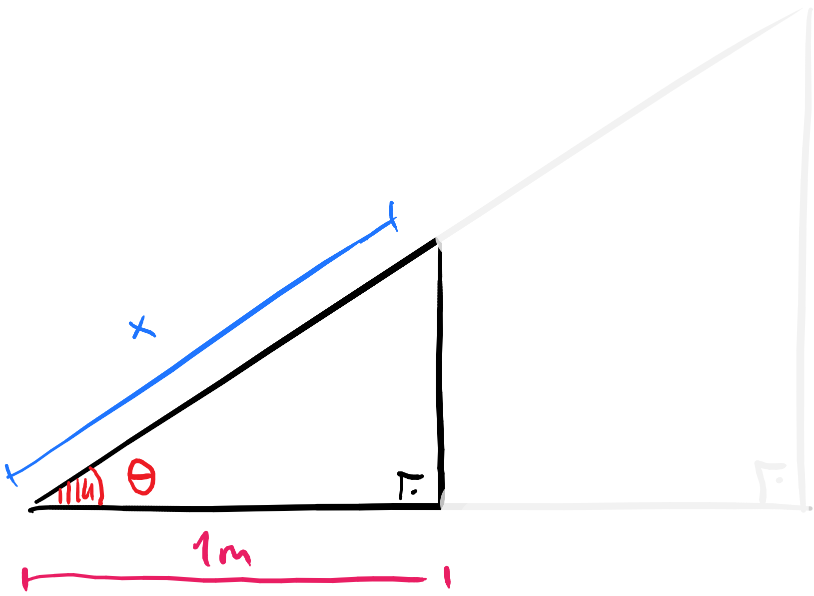
Outra solução:



$$\cos\theta = \frac{3}{18} = \frac{1}{6}$$

$$\boxed{\cos\theta = \frac{1}{6}}$$

$$\cos\theta = \frac{1}{6}$$



$$\cos\theta = \frac{1}{x} = \frac{1}{6}$$

$$x = 6\,m$$

sen, cos, tg

↳ é uma propriedade do ângulo!

Ângulos notáveis

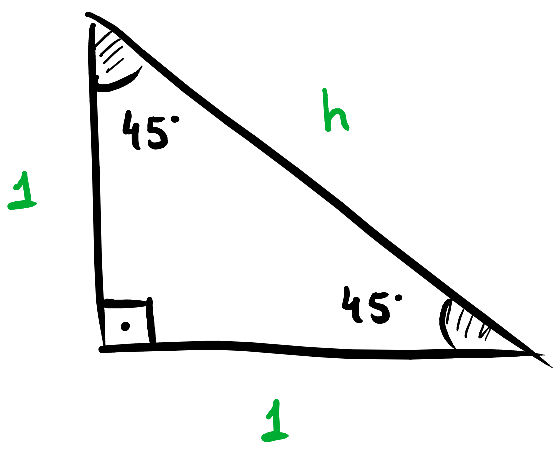
$$tg\,\theta = \frac{sen\,\theta}{cos\,\theta}$$

	30°	45°	60°
sen	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
cos	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$
tg	$\sqrt{3}/3$	1	$\sqrt{3}$

$$sen(90 - x) = cos\,x$$

Mas da onde vem isso?

Caso 1

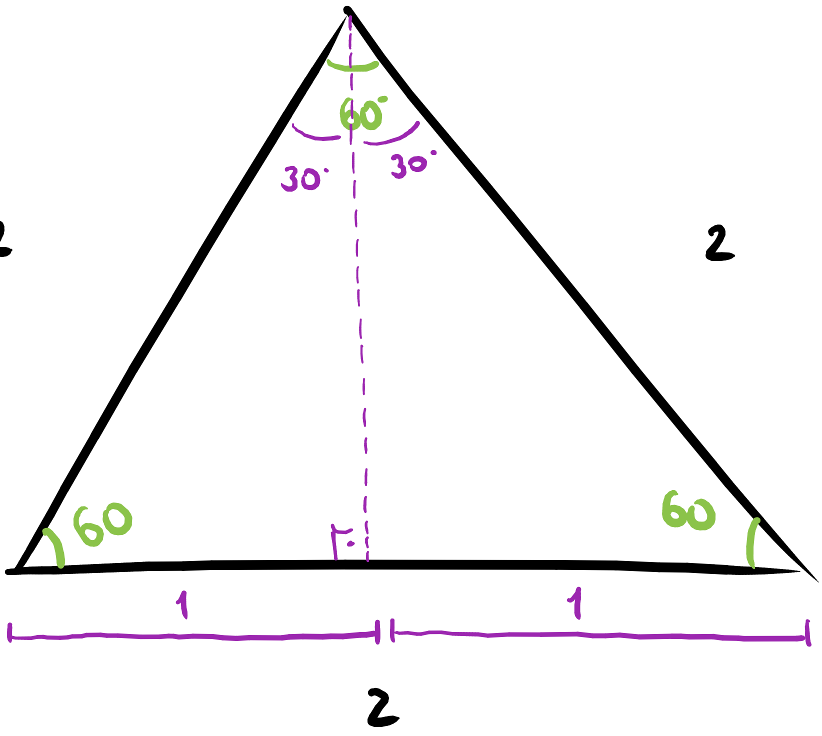


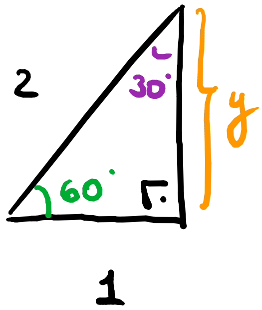
Pitágoras: $h^2 = 1^2 + 1^2 = 2$

$h = \sqrt{2}$

- $sen\,45° = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ ✓
- $cos\,45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ ✓
- $tg\,45 = \frac{1}{1} = 1$ ✓

Caso 2

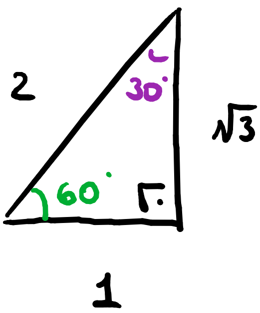




Pitágoras:

$2^2 = 1^2 + y^2$

$4 = 1 + y^2 \quad \therefore \quad y = \sqrt{3}$

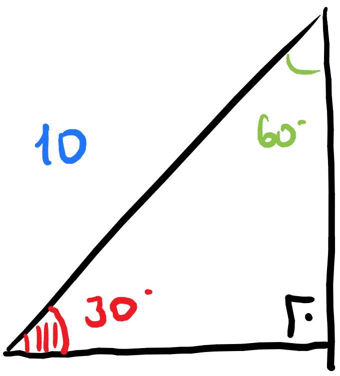


$\cdot\ \text{sen } 60° = \dfrac{\sqrt{3}}{2}$, $\cos 60° = 1/2$, $\text{tg } 60° = \sqrt{3}/1$

$\cdot\ \text{sen } 30° = 1/2$, $\cos 30 = \dfrac{\sqrt{3}}{2}$, $\underbrace{\text{tg } 30° = \dfrac{1}{\sqrt{3}}}$

$\dfrac{1}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

Ex ∴ Calcule x



$x = ?$

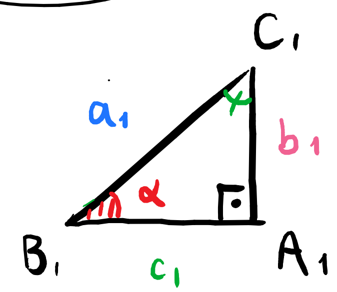
* $\text{Sen } 30 = \dfrac{1}{2} = \dfrac{x}{10} \quad \therefore \quad x = \dfrac{10}{2} = 5m$

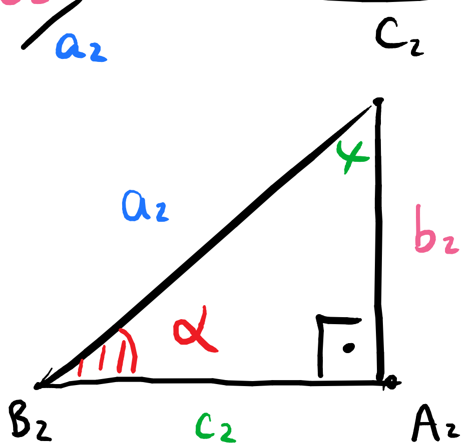
* $\cos 60° = \dfrac{1}{2} = \dfrac{x}{10} \quad \therefore \quad x = 5m$

Obs.: Pq o Δ precisa ser retângulo?

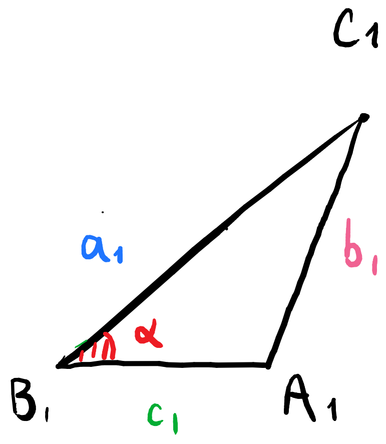
↳ Para dois triângulos retângulos serem semelhantes basta que eles tenham um (outro) ângulo em comum.

$$\operatorname{sen}\alpha = \frac{b_1}{a_1} \rightarrow \frac{b_1}{a_1} = b_2/a_2 \leftarrow \operatorname{sen}\alpha = b_2/a_2$$

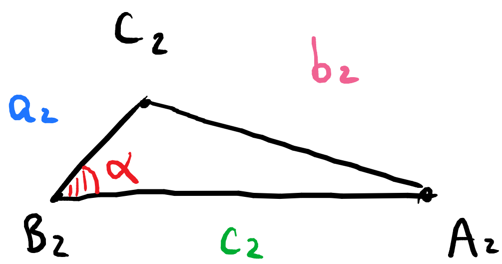




$$\frac{b_1}{b_2} = \frac{a_1}{a_2}$$

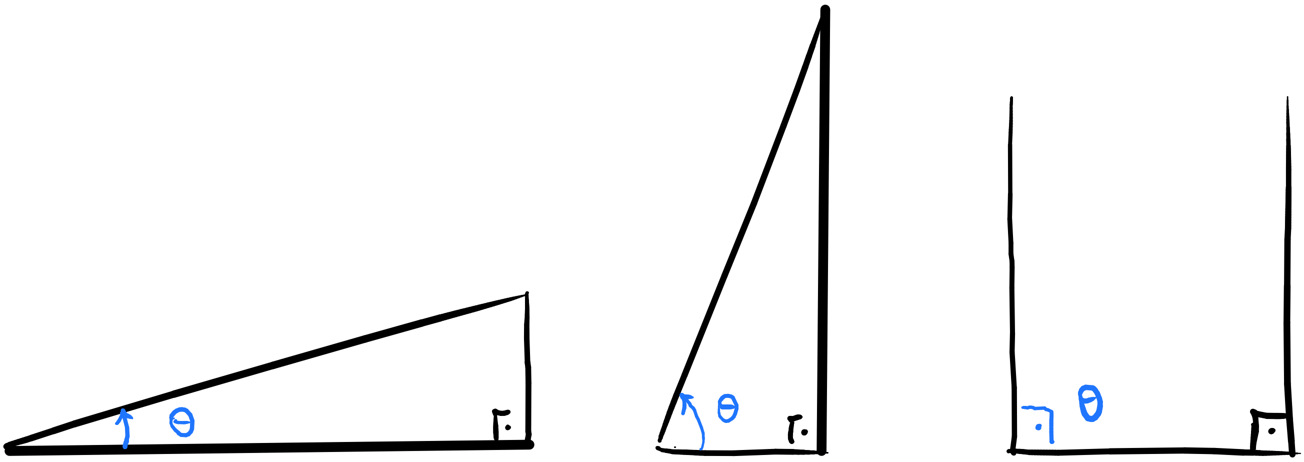


$$\frac{b_1}{b_2} \neq \frac{a_1}{a_2}$$

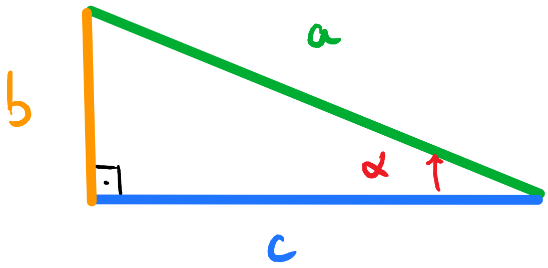


faz sentido falar em sen, cos e tg

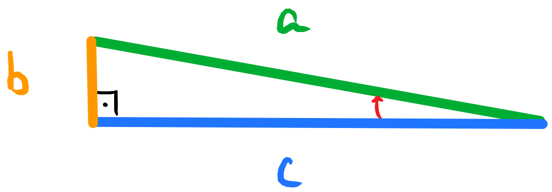
p/ ângulos maiores que 90°?

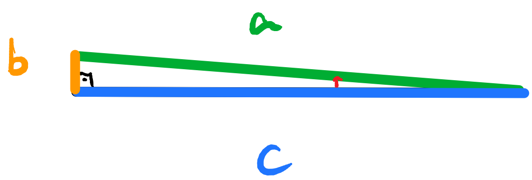


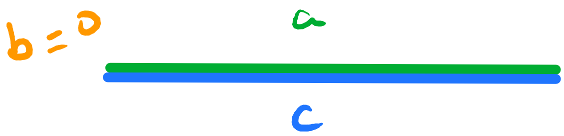
Se $\alpha \approx 0$:



- $\text{sen}\,\alpha = b/a$
- $\cos\alpha = c/a$
- $\text{tg}\,\alpha = b/c$

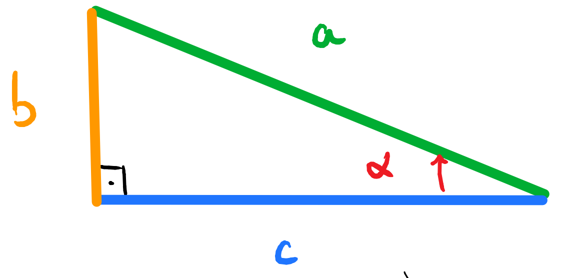




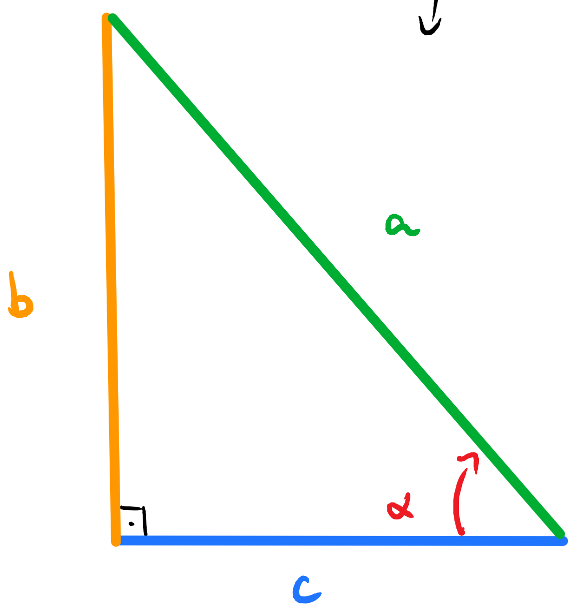


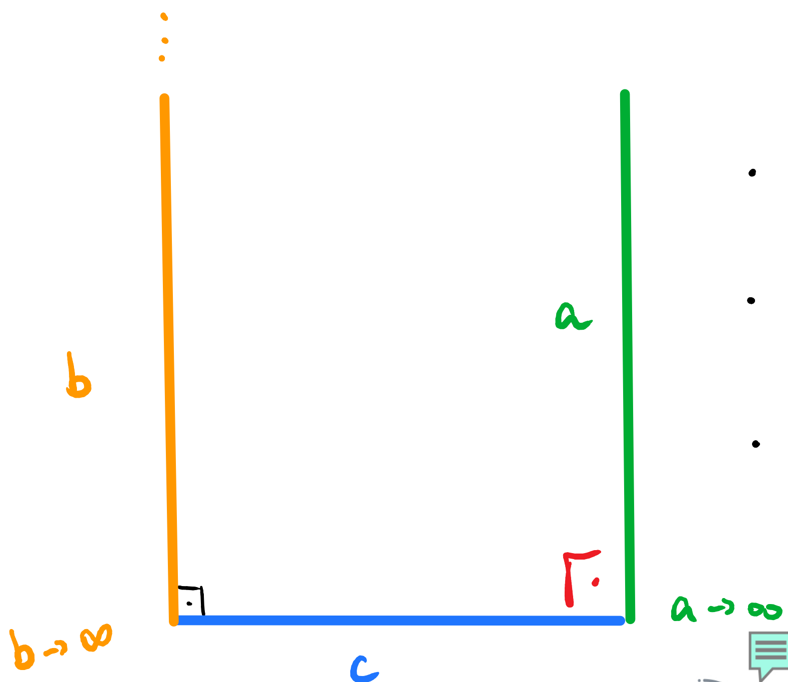
- $\text{sen}\,0 = b/a = 0$
- $\cos 0 = c/a = 1$
- $\text{tg}\,0 = b/c = 0$

Se $\alpha \approx 90°$



- $\operatorname{sen}\alpha = b/a$
- $\cos\alpha = c/a$
- $\operatorname{tg}\alpha = b/c$



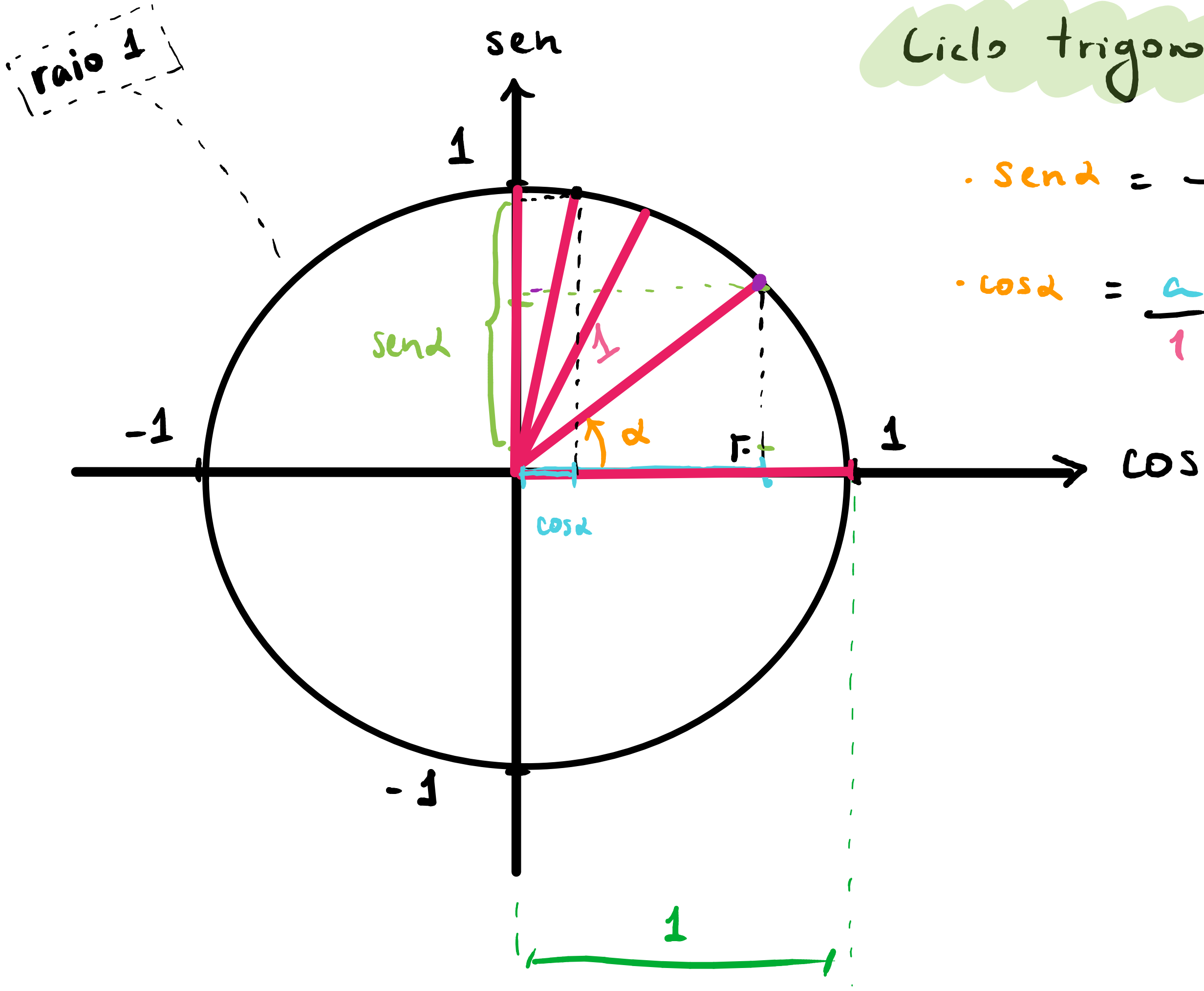


- $\operatorname{sen} 90° = b/a = 1$
- $\cos 90° = c/a = 0$
- $\operatorname{tg} 90° = b/c = \infty$

Ciclo trigonométrico

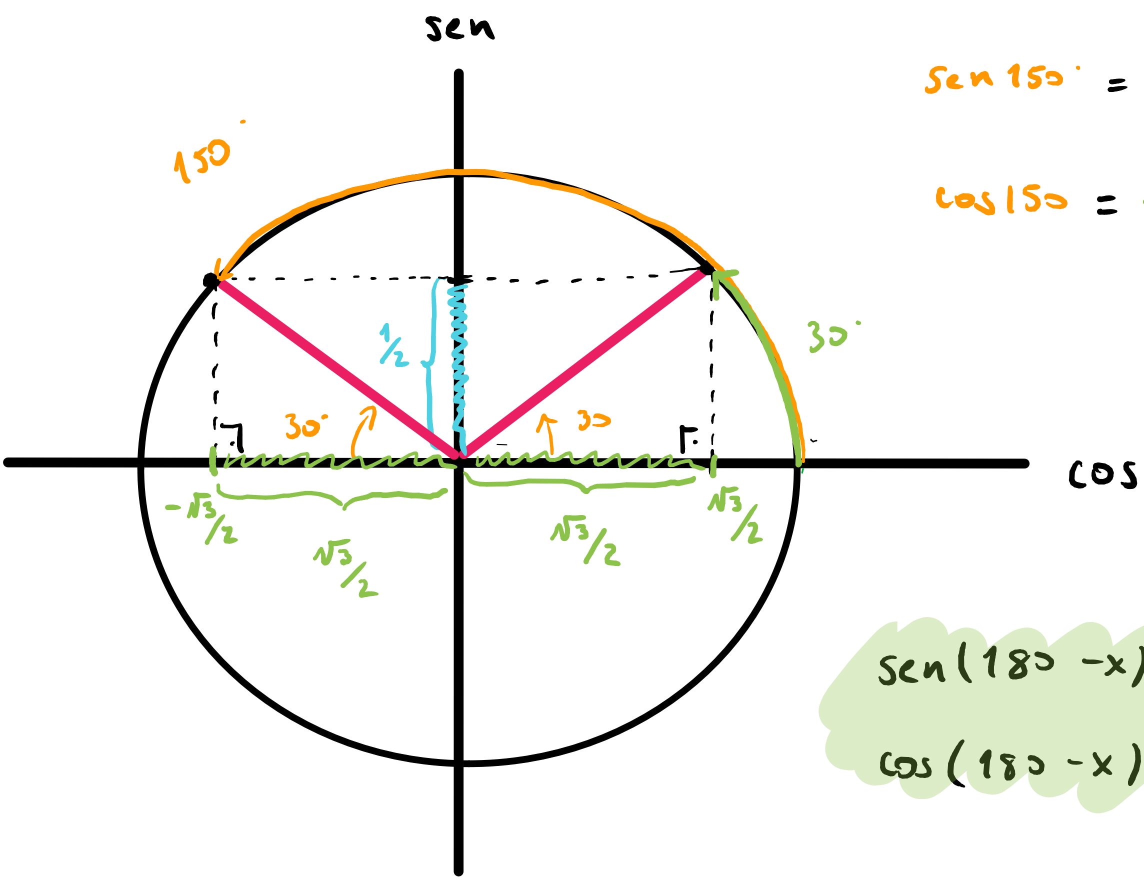
$\cdot \ \text{sen}\,\alpha = \dfrac{b}{1} = b$

$\cdot \ \cos\alpha = \dfrac{a}{1} = a$



$\text{sen}\,150^\circ = \text{sen}\,30 = 1/2$

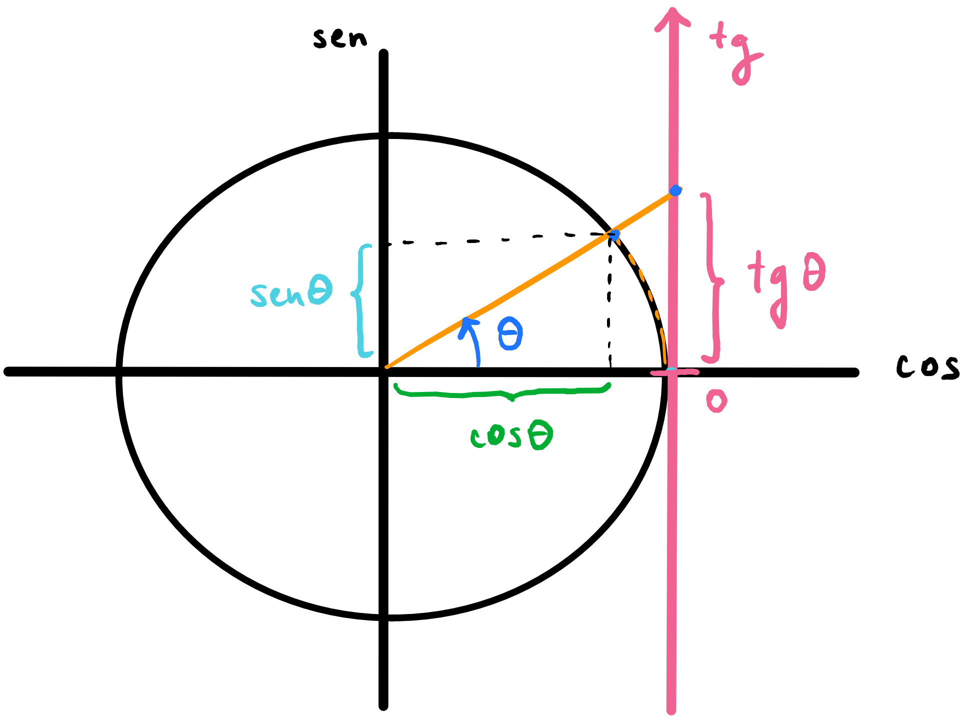
$\cos 150 = -\cos 30 = -\dfrac{\sqrt{3}}{2}$

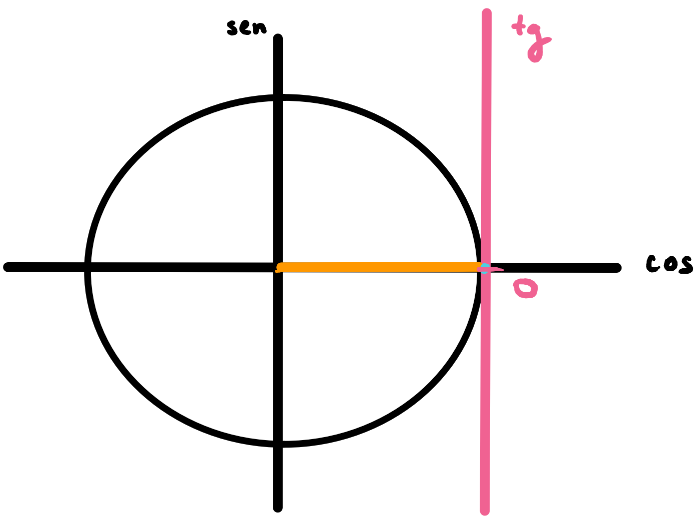


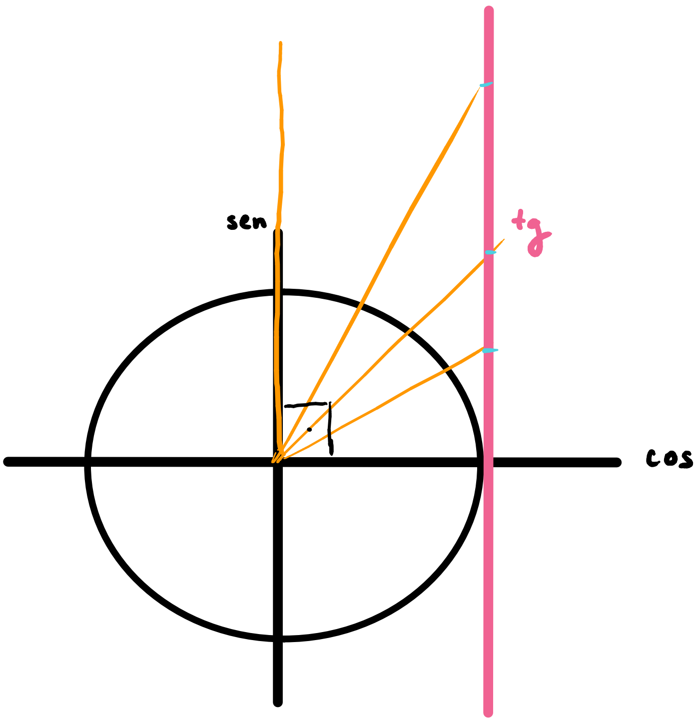
$\text{sen}(180 - x) = \text{sen}\,x$

$\cos(180 - x) = -\cos x$

tangente no ciclo unitário





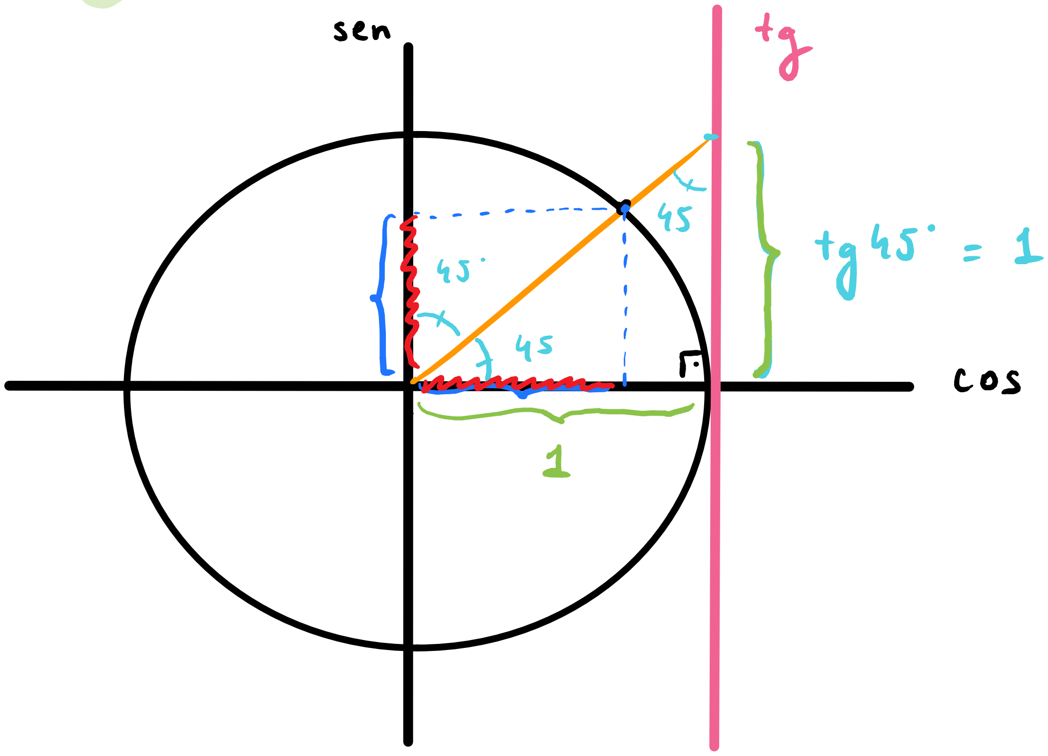


$tg\,0 = 0$

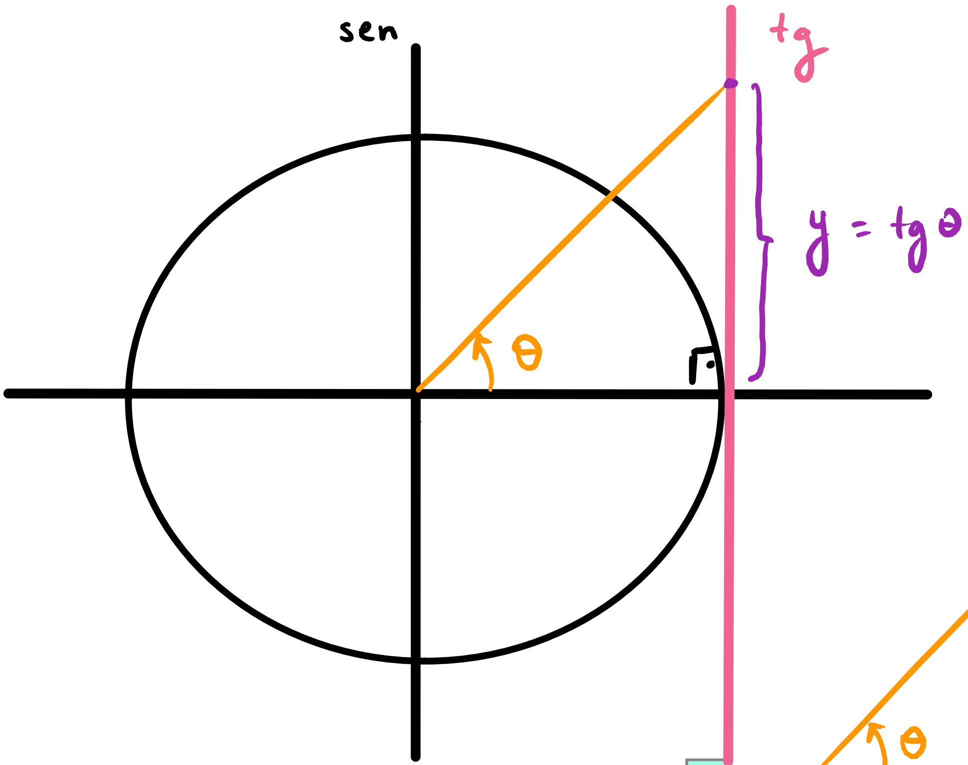
$tg\,90° = \infty$

Exemplos

1. 45°

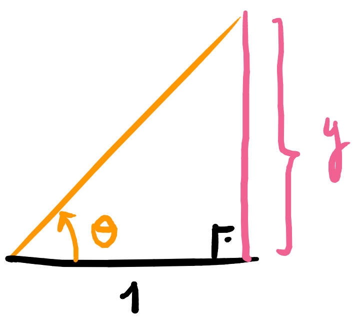


2. ângulo qualquer

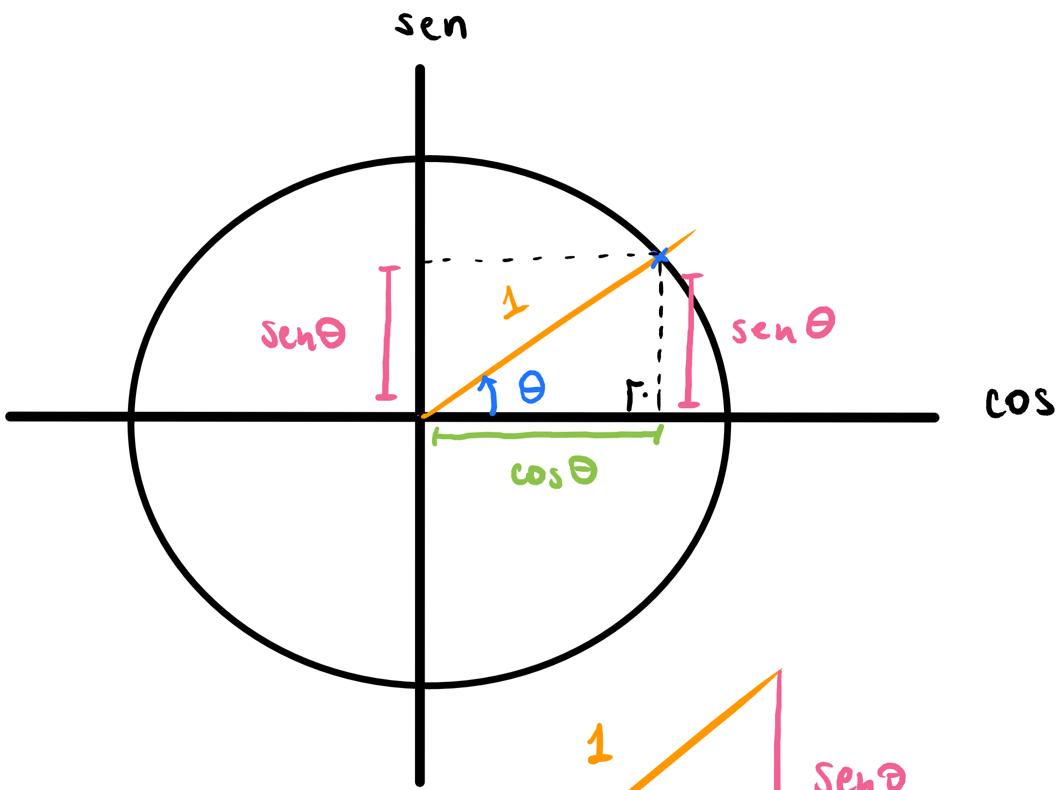


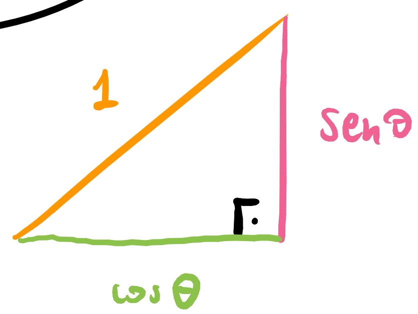
$$tg\,\theta = \frac{y}{1}$$

$$y = tg\,\theta$$



Relação fundamental da trigonometria





Teo. de Pitágoras:

$$1^2 = \text{sen}^2\theta + \cos^2\theta \qquad \cancel{\text{sen}\,\theta^2}$$

$$\boxed{\text{sen}^2\theta + \cos^2\theta = 1}$$

↳ Relação fundamental da Trigonometria

$$(\text{sen}\,30)^2 = \text{sen}^2 30 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Exercícios

1. $\cos 30° = \sqrt{3}/2$, $\text{sen}\, 30 = ?$

$\hookrightarrow \quad \text{sen}^2 30 + \cos^2 30 = 1$

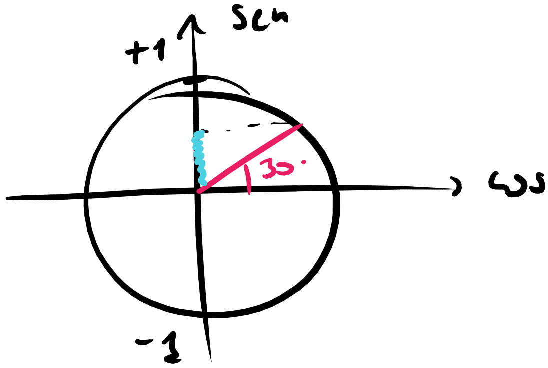
$$\text{sen}^2 30 + \left(\sqrt{3}/2\right)^2 = 1$$

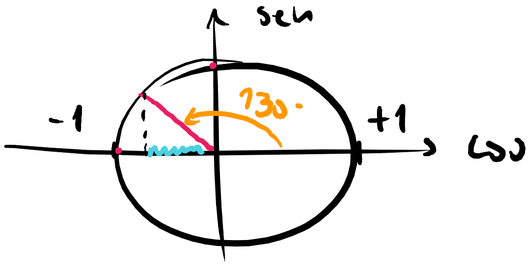
$$\text{sen}^2 30 + \frac{3}{4} = \frac{4}{4}$$

$$\text{sen}^2 30 = \frac{4-3}{4} = \frac{1}{4}$$

$$\sqrt{\text{sen}^2 30} = \frac{\sqrt{1}}{\sqrt{4}}$$

$$\text{sen}\, 30° = +\, 1/2$$





2. $\text{sen}\, 130° = 1/3$, $\cos 130° = ?$

$$\text{sen}^2 x + \cos^2 x = 1$$

$$\left(1/3\right)^2 + \cos^2 x = 1$$

$$1/9 + \cos^2 x = 9/9$$

$$\cos^2 x = 9/9 - 1/9 = \frac{9-1}{9}$$

$$\cos^2 x = \frac{8}{9} = \frac{4 \cdot 2}{9}$$

$$\sqrt{\cos^2 x} = \frac{\sqrt{4} \cdot \sqrt{2}}{\sqrt{9}}$$

$$\cos x = -\frac{2}{3}\sqrt{2}$$